

## EMITTANCE CALCULATIONS FOR THE STANFORD LINEAR COLLIDER INJECTOR\*

J. C. SHEPPARD, J. E. CLENDENIN, R. H. HELM, M. J. LEE AND R. H. MILLER

*Stanford Linear Accelerator Center  
Stanford University, Stanford, California 94305*

C. A. BLOCKER  
*High Energy Physics Laboratory  
Harvard University Cambridge, MA*

A series of measurements have been performed to determine the emittance of the high intensity, single bunch beam that is to be injected into the Stanford Linear Collider. On-line computer programs were used to control the Linac for the purpose of data acquisition and to fit the data to a model in order to deduce the beam emittance. This paper will describe the method of emittance calculation and present some of the measurement results.

### Introduction

One of the goals of the Stanford Linear Collider (SLC) injector tests has been to measure the transverse components of the beam sigma matrix<sup>1</sup> in the Collider Injector Development (CID) region. The measured beam sigma are used in TRANSPORT calculations to determine the strengths of injector quadrupoles required for efficient matching of the CID beam into the SLC FODO array. In addition to lattice calculations, a determination of the emittance of the CID beam is of interest as a figure of merit for CID injector operation. A brief description of the philosophy behind the measurements, the experimental apparatus used to gather the data, and the user interface to the measurement controlling software is given, followed by detailed renditions of the mathematics used in the emittance calculations. TRANSPORT notation is used unless otherwise stated;  $X$  is used to denote the horizontal direction and  $Y$  indicates the vertical.

The basic idea of the emittance measurement is to determine the horizontal and vertical beam size  $\sigma_{11}^{1/2}$  and  $\sigma_{33}^{1/2}$  as a function of the strength(s) of an upstream quadrupole(s). Assuming no  $X$ - $Y$  coupling, knowledge of the beam size in  $X$  or  $Y$  at a particular location in the beamline for three different quadrupole strengths is sufficient to calculate the beam matrix anywhere along the beamline. In practice, more than three independent measurements are acquired and a linear least squares fit to the data is done to deduce the sigma matrix elements. The assumption of negligible  $X$ - $Y$  coupling is valid if the beam was round when transversing rotating elements such as solenoids; this is thought to be true for the CID region.

Beam width data is collected using a  $32 \times 32$  array of photodiodes to measure the spatial intensity profile of visible radiation generated when the beam intercepts a fluorescent screen placed in the beamline. The  $32 \times 32$  array of digitized diode signals is sent off to the SLC VAX control computer wherein the data is processed. Basic data manipulation takes the form of background subtraction, axis scaling, and gaussian fitting to the subtracted and scaled data.

Subtraction of background noise is accomplished by turning off the gun trigger (but not the klystron trigger) and recording a diode scan. This scan is saved in computer memory and is subtracted element by element from scans collected with the gun trigger enabled. For convenience, a numerical fit of a gaussian to the scaled and subtracted data as a function of spatial

dimension is performed to determine the mean and standard deviations of the beam profiles. To do the fitting in a given dimension, the data are first integrated over the other direction. Beam widths are taken to be equal to the standard deviations of the integrated profiles; the mean positions of the profiles are retained in the computer memory for diagnostic purposes.

A series of programs have been written for on-line control of the emittance measurements. A user communicates to these programs via a touch panel. After selecting the particular quadrupole to be varied and then entering a list of magnet strengths, pushing the appropriate buttons allows the user to select a given quadrupole setting, examine the beam profile, observe the gaussian fit to the data, and enter the fitted beam widths into a FORTRAN common block. After collecting the beam data, one may try to fit a beam sigma matrix to the information saved in the common block. The computer returns the conserved emittance and two  $2 \times 2$  beam sigma matrices ( $X$  and  $Y$  planes) at one of several, user selected points along the beamline. The fitted  $\sigma_{11}$  and  $\sigma_{33}$ , transported to the profile monitor as a function of the strength of the varied quadrupole, may be displayed with the acquired data as a means of checking the analysis. Pertinent fit information is retained in the computer for use by other modeling programs.<sup>2</sup>

Figure 1 illustrates the region of the CID beamline in which the emittance measurements are made. Single bunches of electrons leave the CID disk-loaded waveguide with an energy of about 40 MeV. A triplet of quadrupoles are used to vary the beam size at a beam profile monitor. Processing electronics transfer the beam intensity information from the diode array to the SLC control computer. The user communicates with the computer via the Console On Wheels (COW); control of the quadrupole strengths is facilitated with the necessary accelerator control modules. The analysis of beam profile data to determine beam sizes and the fitting of an initial beam sigma matrix to those beam sizes are discussed below.

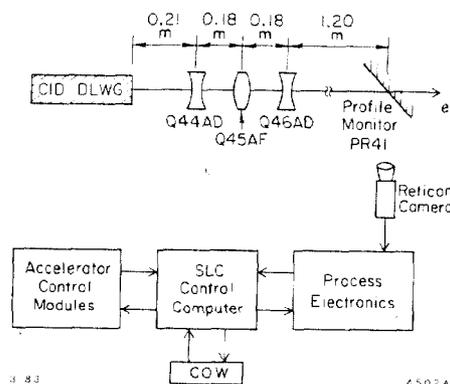


Fig. 1. CID beamline and apparatus used for measurement of the beam emittance.

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### Beam Width Fitting

The width of the beam is determined by fitting a gaussian to the projection of the profile monitor data onto the horizontal or vertical axis. As is well known, the form of a gaussian,  $G(x)$ , is

$$G(x) = \frac{A}{(2\pi\sigma^2)^{1/2}} \exp\left(-\frac{(X - X_0)^2}{2\sigma^2}\right) \quad (1)$$

in which  $X$  is the independent variable,  $X_0$  is the mean,  $\sigma$  is the width, and  $A$  is the area under the function. In this form, the gaussian function is nonlinear and very difficult to fit analytically. However, taking the natural logarithm of both sides of Eq. (1) gives

$$\ln(G) = \ln\left(\frac{A}{(2\pi\sigma^2)^{1/2}}\right) - \frac{(X - X_0)^2}{2\sigma^2} \quad (2)$$

By defining a new variable  $Z = \ln(G)$ , Eq. (2) becomes

$$Z = a_0 + a_1 X + a_2 X^2 \quad (3)$$

where  $a_0$ ,  $a_1$ , and  $a_2$  are the following functions of  $X_0$ ,  $\sigma$ , and  $A$ :

$$\begin{aligned} a_0 &= \ln\left(\frac{A}{(2\pi\sigma^2)^{1/2}}\right) - \frac{X_0^2}{2\sigma^2} \\ a_1 &= \frac{X_0}{\sigma^2} \quad a_2 = \frac{-1}{2\sigma^2} \end{aligned} \quad (4)$$

Equation (3) is a linear equation in the parameters  $a_0$ ,  $a_1$ , and  $a_2$ , and a standard linear least squares fit<sup>3</sup> can be done to obtain the parameters  $a_0$ ,  $a_1$ , and  $a_2$ . The parameters  $X_0$ ,  $\sigma$ , and  $A$  are then calculated from the following formulas:

$$\begin{aligned} \sigma^2 &= \frac{-1}{2a_2} \quad X_0 = \frac{-a_1}{2a_2} \\ A &= (2\pi\sigma^2)^{1/2} \exp\left(a_0 + \frac{X_0^2}{2\sigma^2}\right) \end{aligned} \quad (5)$$

In order to accommodate fluctuations in the beam width due to noise in the Linac and the measuring apparatus, it has been decided to implement an averaging procedure. This averaging takes two forms: (1) Diode array signals of a single monitor scan are integrated over one plane to obtain the beam profile in the other direction. Since the diode signals are conveniently binned, the integration reduces to a summation of diode array values over the appropriate index. (2) The measured beam width, for constant quadrupole settings, is taken to be the arithmetic mean of the  $\sigma$ 's fitted to a series  $N$  of repeated profile scans:

$$\langle \sigma \rangle = \sum_{k=1}^N \frac{\sigma_k}{N} \quad (6)$$

in which the  $\sigma_k$  are the results of a gaussian fit to the  $k$ th measurement of the beam profile. The error in  $\langle \sigma \rangle$  is taken to be the standard deviation of the individual measurements,  $\Delta$ :

$$\Delta = \left( \sum_{k=1}^N \frac{(\sigma_k - \langle \sigma \rangle)^2}{N} \right)^{1/2} \quad (7)$$

When only one profile scan is taken, the beam width is set to the fitted  $\sigma$  and the uncertainty in the measurement is arbitrarily taken as  $\sigma/2$ .

### Emittance Fitting

In the absence of  $X - Y$  coupling, the square of the beam size at the profile monitor ( $\sigma_{11}$  and  $\sigma_{33}$  for the  $X$  and  $Y$  planes, respectively) is related to the beam sigma matrix,  $\sigma_{ij}^0$ , at an upstream location via

$$\sigma_{11} = R_{11}^2 \sigma_{11}^0 + 2R_{11}R_{12} \sigma_{12}^0 + R_{12}^2 \sigma_{22}^0 \quad (8a)$$

and

$$\sigma_{33} = R_{33}^2 \sigma_{33}^0 + 2R_{33}R_{34} \sigma_{34}^0 + R_{34}^2 \sigma_{44}^0 \quad (8b)$$

with the  $R_{ij}$  being the  $ij$  elements of the TRANSPORT matrix describing the Linac lattice between the initial upstream location and the profile monitor. In the remainder of this discussion, only those equations pertaining to the  $X$ -plane are shown explicitly. Analogous expressions for the  $Y$ -plane have been included in the on-line system. The equations for the vertical plane may be written by inspection of those given for  $X$  by: (1) changing "X" to "Y"; (2) changing subscripts "1" to "3"; and (3) changing subscripts "2" to "4".

Equations (8) are generalized to account for multiple measurements of  $\sigma_{11}$  for different values of the  $R_{ij}$ :

$$\sigma_i = A_{ij} \sigma_j^0 + \xi_i \quad (9)$$

where summation over similar indices is understood;  $\sigma_1^0 = \sigma_{11}^0$ ,  $\sigma_2 = \sigma_{12}^0$ ,  $\sigma_3^0 = \sigma_{22}^0$ ;  $A_{i1} = (R_{11}^2)_i$ ,  $A_{i2} = (2R_{11}R_{12})_i$ , and  $A_{i3} = (R_{12}^2)_i$ ; and the  $\xi_i$  are the unknown errors in the measurements of the  $\sigma_i$ . The  $\sigma_i$  are the measurements of  $\sigma_{11}$  for the  $i$ 'th setting of the lattice quadrupoles.

Given the  $A_{ij}$ 's, the  $\sigma_j^0$ 's are determined using a weighted least squares fit:

$$[\sigma'] = [A'] [\sigma^0] \quad (10)$$

yielding

$$[\sigma^0] = ([A']^t [A'])^{-1} [A']^t [\sigma'] \quad (11)$$

wherein  $A'_{ij} = w_i A_{ij}$ ,  $\sigma'_i = w_i \sigma_i$ , with  $w_i$  = the weight factor of the  $i$ 'th measurement of the square of the beam width, and  $[A']^t$  is the transpose of  $[A]$ .

From the calculated  $\sigma_i^0$ , the  $X$ -plane emittance is given as  $\epsilon_x$ :

$$\epsilon_x = (\sigma_1^0 \sigma_3^0 - (\sigma_2^0)^2)^{1/2} \quad (12)$$

The error in the determination of  $\epsilon_x$  is taken to be the standard deviation of  $\epsilon_x$  and is given as

$$\begin{aligned} ERROR(\epsilon_x) &= ((\sigma_1^0)^2 B_{33} + (\sigma_3^0)^2 B_{11} - 4(\sigma_2^0)^2 B_{22} \\ &\quad + 2\sigma_1^0 \sigma_3^0 B_{13} - 2\sigma_1^0 \sigma_2^0 B_{23} - 2\sigma_2^0 \sigma_3^0 B_{12})^{1/2} / 2\epsilon_x \end{aligned} \quad (13)$$

in which the  $B_{ij}$  are the  $ij$  elements of the covariance matrix,

$$[B] = \left[ \frac{([\sigma'] - [A'] [\sigma^0])^t ([\sigma'] - [A'] [\sigma^0])}{M - 3} ([A'] [A']^t)^{-1} \right] \quad (14)$$

in which  $M$  is the number of measurements of  $\sigma$ . The weight factors,  $w_i$ , are given as

$$w_i = (2\sigma_i^{1/2} \Delta_i)^{-1} \tag{15}$$

in which  $(\sigma_i)^{1/2}$  are the measured beam sizes and the  $\Delta_i$  are the errors in the measured beam sizes.

**Results**

Figure 2 illustrates a "typical" gaussian fit to the horizontal beam size data. The "dots" represent the subtracted and scaled data; each point represents the summation of diode signals over the vertical direction. In Fig. 2, the calculated sample standard deviation of the integrated data is 0.81 mm; the fitted gaussian has a standard deviation of 0.71 mm.

Figure 3 illustrates the resultant fit of the beam sigma matrix to the measured beam data. The "dots" represent the square of the averaged beam size observations at the profile monitor; the solid lines are the values of the fitted matrix elements. The upper trace represents the horizontal plane and the lower shows the vertical plane results. For Fig. 3, the outer pair of triplet quadrupoles were varied together.

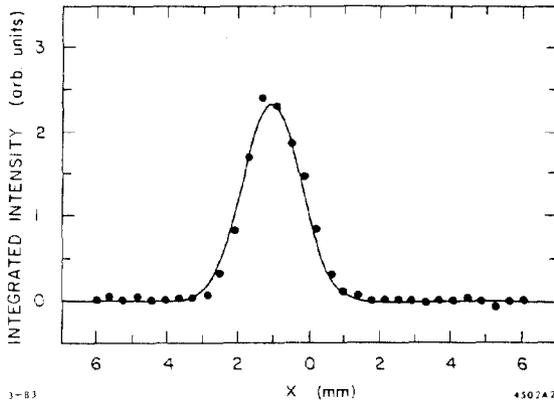


Fig. 2. Horizontal plane beam profile: the points represent the beam intensity, integrated over the Y-plane, and the solid line is the gaussian fit to the data.

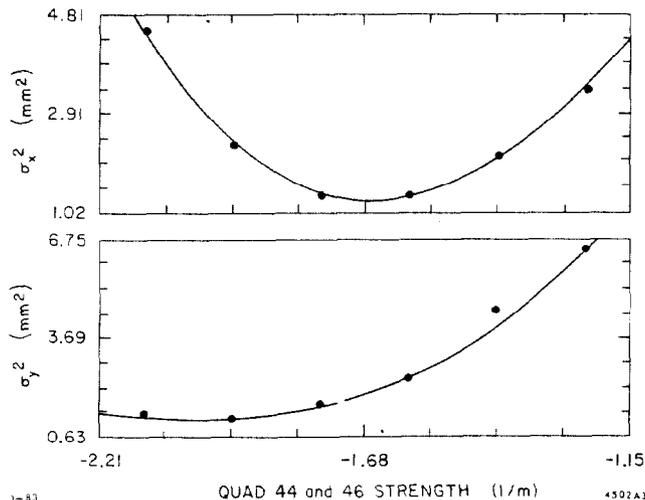


Fig. 3. Calculated  $\sigma_x^2 = \sigma_{11}$  and  $\sigma_y^2 = \sigma_{33}$  as a function of the strength of the varied quadrupoles along with the measured data. Errors on the measured data are smaller than the drawn points.

Figure 4 illustrates the compendium of emittance measurements, made during the Spring 1982 accelerator cycle. Error bars have been included for those cases in which there is sufficient information to calculate a standard deviation. While the emittance area is seen to vary systematically as a function of current, it has been observed that the orientation of the 2-dimensional phase space ellipse representations of the beam sigma did not vary significantly with beam current. The observed beam emittance is within SLC design specifications for injection into the emittance Damping Rings.<sup>4</sup>

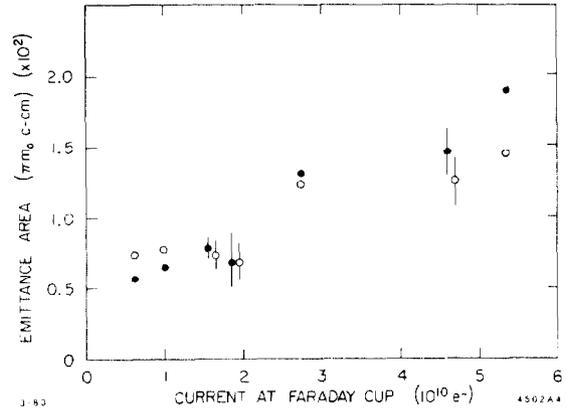


Fig. 4. Compendium of emittance measurement results: the solid points represent the horizontal plane while the hollow ones indicate vertical plane measurements.

**Summary**

A computer code has been implemented for on-line acquisition and analysis of data for emittance measurements of the SLC injector beam. The beam emittances have been determined experimentally using this code; measured beam emittance values have been found to be within expectations. When the system operates in the automatic mode, an emittance measurement takes less than two minutes. This emittance measurement method has been thoroughly tested and has been extended to other regions of the SLC system. Resultant beam sigma matrices are used in the lattice design models to calculate the strengths of quadrupoles required for the optical matching of the CID beam into the SLC Linac.

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