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BEAM OPTICS FOR A 12-GeV ISOCHRONOUS RING CYCLOTRON

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Summary

This paper discusses the tolerances required for isochronous cyclotrons accelerating protons to kaon and $\overline{p}$ producing energies. The beam traverses many more imperfection resonances and low order intrinsic resonances than in existing cyclotrons since the isochronous radial tune $\nu_r \approx \gamma$. We anticipate an incoherent radial amplitude of 1 mm at extraction. The turns will overlap and the net accelerating voltage must be flat-topped to ensure that all particles make the same number of turns. With these conditions a 1 G imperfection at the final integer resonance develops 3 mm coherent amplitude, and although some non-linear stretching takes place, a clear space of 0.8 mm permits insertion of a septum. To preserve 1 mm amplitude the operational imperfection fields and gradients at other resonances must be less than 0.10 G and 0.10 G/cm. A third order intrinsic resonance will cause some filamentation and impair extraction efficiency, although this can be partially controlled.

Introduction

The TRIUMF cyclotron accelerates more than 130 μA of negative hydrogen ions to 520 MeV. Recently we have studied several schemes to take the beam from TRIUMF and accelerate it to energies above 8 GeV to produce copious beams of secondary particles K, $\overline{p}$ and ν. We have considered post accelerators based on synchrotrons and also on a cyclotron or series of cyclotrons. Cyclotron post accelerators have the advantage that their time structure is completely compatible with the TRIUMF beam. The reference design consists of two isochronous superconducting ring cyclotrons. The first would accelerate the protons to 3.5 GeV,[1] the second to 9, 12 or 15 GeV. In this report we consider the optics of two proposed versions of this second stage (see Table I): a 30-sector 9 GeV and a 42-sector 12 GeV cyclotron. A plan view of the 12 GeV machine is shown in Fig. 1.

Table I
Characteristics of the two proposed versions of the final stage.

Injection energy	3.5 GeV	3.5 GeV
Extraction energy	9 GeV	12 GeV
# sectors	30	42
Radius (max)	20.6 m	41.3 m
Radius (min)	20.2 m	40.5 m
Energy gain per turn	25 MeV	50 MeV
RF frequency	69.5 MHz	115 MHz
Beam time width	0.5 ns	0.3 ns
Peak sector field	5 T	5 T
Radial beam size at extraction ($\beta\gamma\varepsilon = 2\pi$ mm mrad)	1.2 mm	1.2 mm

In an isochronous cyclotron, the radial focusing frequency ν_r, is approximately equal to the relativistic parameter γ (Fig. 2). Hence, many integer and half-integer imperfection resonances and some non-linear intrinsic resonances are traversed. The axial focusing frequency, ν_z, depends upon sector shape and work is in progress to minimize the number of axial and coupled resonances. It appears, however, that there will be at least one axial integer resonance and that the $\nu_r = 2\nu_z$ coupling resonance is unavoidable. Some thought must also be given to the placement of RF cavities so that resonances are not driven by the accelerating field.

We have been able to derive order of magnitude estimates for the construction tolerances of such cyclotrons. These tolerances depend also upon the rf system. In particular, a large energy gain per turn is desirable so that resonances are quickly traversed. Also, harmonic cavities will be necessary to flat-top the wave form, provide single turn operation and avoid precessional mixing.

Integral Resonances

The chief effect of an integral resonance is to introduce a similar coherent amplitude for all particles in the phase space region. This may be represented as a vector in r-p_r space. Studies with the accelerated orbit code GOBLIN have shown that for the 30-sector machine, a coherent amplitude of 5 mm would be introduced by a 1 G 5th harmonic at 4.2 GeV or by a 2 G 12th harmonic at 8.7 GeV. This is in excellent agreement with the values expected from simple theory, namely that the presence at $\nu_r = n$ of an nth harmonic field error B_n (in c.u.) results in a coherent oscillation of amplitude (also in c.u.)

$$A = \frac{\pi}{\sqrt{\nu_t}} \frac{\beta}{n\gamma} B_n$$

where ν_t is the change in ν_r per turn.

The beam traverses nine such integral resonances in the 12 GeV machine. It is expected that second harmonic cavities would be used to flat-top the wave form over the 12° of phase expected to contain 100 μA and thus yield single turn conditions for the 240 turns expected.

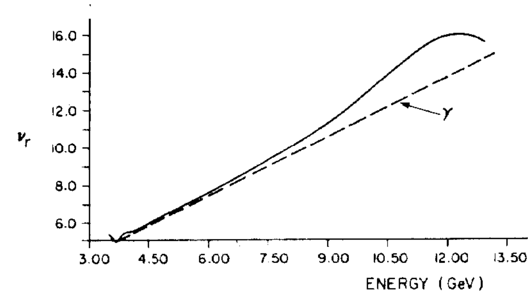

Fig. 2. The radial focusing frequency as a function of energy for the 42-sector cyclotron. The relativistic parameter γ is shown for comparison.

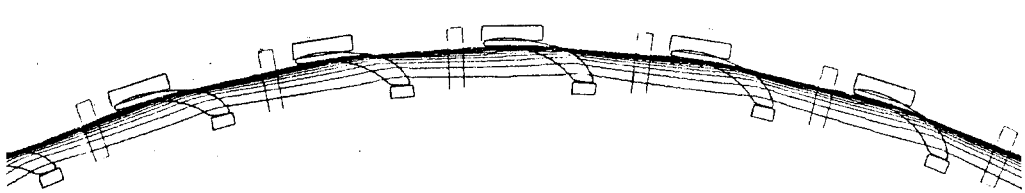

Fig. 1. Five sectors from the 40 m radius 42 sector cyclotron showing magnets, RF cavities and proton orbits.

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In principle, the total coherent amplitude may be compensated at any point, e.g. at a resonance before extraction. Should the flat-topping not be complete, say because of deviations from isochronism, then precessional mixing may transform a coherent amplitude into an effective incoherent amplitude.

Another consideration is the size of the linear region in r-p_r space; for, if the coherent oscillations are at any time larger than this size, the phase space ellipse will be distorted and it will no longer be possible to completely compensate for the amplitude gain. We find that whereas the linear region is fairly large at low energies (> 10 times the beam size), it is rather small at the highest energy. In particular, near extraction where the beam size in either of the cyclotrons is ~1 mm, the linear region is only ~2 mm in extent. This places stringent tolerances on the cyclotron design because the magnetic field error incurred by misplacing one sector by Δr is proportional to $\beta\gamma^3\Delta r$. In particular, if ν_r=n is the largest integer resonance before extraction and the sector positions are given by $r=r_n\cos n\theta$ then the largest tolerable value of r_n is given by

$$r_n = \frac{1}{\pi n}\sqrt{\frac{E_1}{E_0}}\,A$$

where E_1 is the energy gain per turn, E_0 is the particle rest energy and A is the largest tolerable coherent amplitude. To keep A<1 mm in either cyclotron, the maximum allowable r_n is ~0.005 mm. A positioning harmonic of this amplitude will be expected to arise for example from randomly positioning the sectors within a tolerance of ±0.04 mm. It is probably not possible to achieve such a tolerance. Nonetheless, it may be practical to meet a positional tolerance of ±0.4 mm and to achieve the final tolerance by adjusting harmonic coils to a precision of ~0.1 G.

Half-integral Resonances

Here the increase in radial amplitude is linearly dependent on the amplitude entering the resonance. The effect is to stretch the radial phase space and produce a subsequent mismatch between the phase space and cyclotron acceptance. Again we assume single turn conditions and no precessional mixing. GOBLIN runs show that a gradient 0.1 G/cm 15th harmonic introduced where ν_r=7.5 stretches the ellipse by 10% and a gradient 0.4 G/cm 31st harmonic introduced where ν_r= 15.5 stretches the ellipse by 5%. Calculations based on equilibrium orbit transfer matrices yield amplitude gains of only 0.1% in either case. Furthermore, we find from GOBLIN runs that the amplitude gain is proportional to the size of the imperfection gradient of the magnetic field while the standard theory (eg. Gordon[2]) predicts that the amplitude gain is proportional to the square of the size of the imperfection gradient. The discrepancy occurs because the amplitude growth is caused by a non-adiabatic mismatch between the beam phase space ellipse and the stable 'static' ellipse which stretches considerably before and after the n/2 stop band. GOBLIN runs were also made for an isochronous but axially non-focusing cyclotron with no flutter ($B=(1-r^2)^{-1/2}$ in c.u.). These gave amplitude gains through n/2 imperfection resonances which agreed with the above results for the 42-sector machine. This indicates that the phenomenon of non-adiabatic amplitude gain is not peculiar just to the types of cyclotrons which we are considering.

To get a feel for the construction tolerances imposed by passage through half-integer resonances, we again imagine a cyclotron constructed of imperfectly placed perfect sectors. For the 42-sector cyclotron, there are ~20 half-integer resonances. We expect no correlation between the incoming stretched ellipse and the direction of stretching at each passage. We desire to limit the accumulated growth to 10%. One can show then that the amplitude of a given positional harmonic must be limited to ~0.04 mm. This situation can arise from, for example, randomly positioning the sectors within a tolerance of ±0.3 mm. We see that half integer resonances are more forgiving than the integer resonances. Nevertheless, one would still expect to provide gradient trim coils to empirically compensate errors.

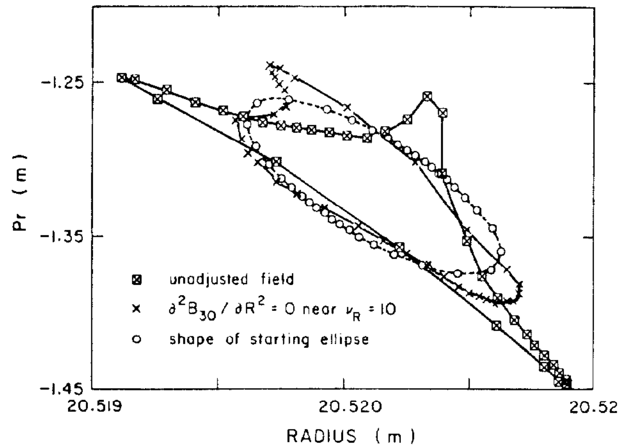

Fig. 3. 7 turns before to 7 turns after ν_r=30/3 (o) starting ellipse, (⊠) uncorrected field, (x) field with $\partial^2B_{30}/\partial r^2$=0.

Intrinsic Non-linear N/3 Resonance

The lowest order intrinsic resonance occurring in the 30-sector machine is ν_r=30/3 at 7.5 GeV. Figure 3 shows the distortion in phase space for a beam of 1π mm-mrad accelerated through this resonance. The calculations started 7 turns before the resonance where ν_r=9.45 at 7.30 GeV and ended 7 turns after where ν_r=10.7 at 7.66 GeV. The forces depend on a second derivative of the intrinsic 30-fold symmetric field; hence it may not be easily shimmed out. They also depend on the square of the radial oscillation amplitudes and the situation is worse if previous resonances have increased this.

To reduce the driving term the 30th harmonic was smoothed numerically to give a zero second derivative over 20 mm around ν_r=10. CYCLOP showed that the change in tune was less than 0.7 in ν_z and 0.5 in ν_r. The results of tracing particles through this modified field are shown in Fig. 3. While the emittance is still distorted with respect to the ideal ellipse, the degree of distortion has been reduced. Widening the region of smoothed field to 90 mm to include all orbit scalloping rendered ν_z imaginary. Smoothing in two regions, one above the inner excursion of the scalloped beam, the other at the outer excursion, gave results similar to the 20 mm wide smoothing around the mean radius.

Extraction

For efficient extraction it is proposed to exploit an integer resonance and develop a coherent betatron amplitude, resulting in complete turn separation at the chosen radius. Several runs were made with a 1 G 12th harmonic imperfection imposed on the 30-sector cyclotron and an energy gain per turn of 25 MeV. Figure 4 shows the detailed phase space history for one of these cases. It appears that for a flat-topped acceleration mode enough turn separation can be developed for the scaled TRIUMF emittance to enter a septum cleanly.

An electric deflector with a field of 8 kV/mm and 1 m long will deflect 12 GeV protons through 0.6 mrad. Two

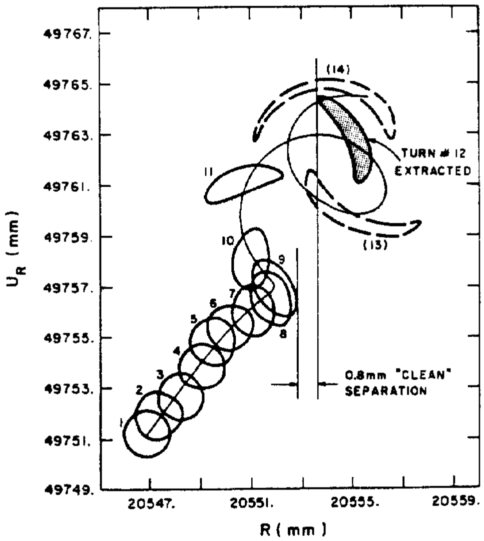

Fig. 4. Radial phase space history of successive turns near the ν_r=12 resonance showing how one turn (at 8.775 GeV) may be clearly separated for extraction by means of a 1 G 12th harmonic field. U is a function of r and P_r chosen to give a circular starting ellipse. The normalized emittance is 2π mm-mrad.

or three such devices would enable the beam to enter an active magnetic channel. The conductors of these channels are arranged in such a way that the stray field in the inner, beam side of the device, is very small, <0.1% of the field in the channel. Room temperature devices[3] have provided field changes of 0.35 T, proposed superconducting devices[4] hope to achieve 2 T. Channels 1 m long would provide kicks of 0.008 rad and 0.046 rad respectively. To pass between the outer yokes the beam must emerge at an angle of 0.17 rad with respect to the equilibrium orbit. This may be difficult to achieve. It may be easier to direct the beam between two of the smaller inner yokes into the free space inside the ring.

It should be noted that the beam is unlikely to traverse the intrinsic and imperfection resonances and retain the pure elliptical shape of Fig. 4, and thus some loss must be anticipated in the extraction system. An extensive study will be necessary to minimize this loss and devise appropriate remote handling techniques. Also evident from Fig. 4 is the non-linear stretching that takes place because the coherent oscillation is larger than the size of the linear regime. This in itself is expected to cause the effective emittance of the extracted beam to be larger than the original emittance of 2π mm-mrad.

Conclusions

It would appear that the tolerances set by field imperfections affecting the radial motion can be met, although empirical adjustments to cancel field errors would be necessary. This could most easily be done if one could observe the individual turns. It is also likely that we could extract the beam if the initial radial quality could be maintained. However, we are unlikely to cross the N/3 or N/4 intrinsic resonances without some filamentation. A 1% beam loss at extraction not only causes component activation but is also a 10 kW energy source in the neighbourhood of the superconducting magnet-coils. In Fig. 3 about 12% of phase space area of the partly compensated ellipse lies outside the area the matched ellipse would occupy in the absence of the resonances. The amount of beam lost would depend on the details of the magnet field and extraction geometry. The 42-sector machine has N/4 and N/3 resonances at 8.4 and 10.6 GeV. The turns remain separated to 9.4 GeV.

References

1. M.K. Craddock, C.J. Kost and J.R. Richardson, IEEE NS-26, 2065 (1979); M.K. Craddock, C.J. Kost, G.H. Mackenzie and J.R. Richardson, Proc. 9th Cyclotron Conf. (Les ed. de Physique, Les Ulis, 1982), p. 209; J.I.M. Botman et al., this conference.
2. M.M. Gordon, Ann. Phys. 50, 571 (1968).
3. R.E. Berg and H.G. Blosser, IEEE 12, 392 (1965).
4. W. Wilhelm et al., SuSe Project, private communication 1982.