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SYNCHROTRON IMPROVEMENTS WITH SHED WAVEFORM*

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Summary

In synchrotrons for ion beam fusion one needs a small bucket area to limit the longitudinal emittance, a small synchrotron frequency to avoid synchrobetatron resonance, a large bunching factor to reduce space charge tune depression, and a high synchronous voltage to complete acceleration without ion-ion collisions. It is proposed to use a trigonometric series approximation to a new "shed-like" waveform to replace the usual pure sine wave (fundamental). The shed waveform divides the interval $(0,2\pi)$ into two parts with a crossing in between. The right portion contains the particles and is linear, while the left portion is merely an "area dump." A fit to a shed with three properly phased sine waves is demonstrated to give almost as good results as the original. In the present application a fundamental frequency range 12.5 to 50.0 MHz is required. Four cavities and four rf systems are utilized to produce the fundamental and two harmonics up to 150 MHz. The frequency range limit of a cavity is imposed by properties of the ferrite, voltage requirement, and the operating frequency. Each cavity covers a factor of 3 in frequency. A "dropping down" scheme is described so that a single cavity may be used in more than one range.

Waveform and Bucket Calculations

Acceleration in a synchrotron is usually carried out by an electric field waveform which is a pure sine wave at a radian frequency $\omega_{rf} = h\omega_{rev}$. In choosing the synchronous phase one simultaneously determines the synchronous accelerating voltage V_s, the bucket area A_{bu}, the bunching factor B_F, and the synchrotron period τ_y. Assuming that the bucket is eventually filled by nonlinearities, the final longitudinal emittance will be determined by the bucket area. The set of parameters is optimal when the effective portion of the wave is linear. Remembering that the net area under an ac wave must be zero, one sees that a shed-like waveform (Fig. 1) is indicated, with the slope of the shed made as small as practicality allows.

The differential equation for synchrotron oscillation is

$$\ddot{\phi} + \frac{hq\ \eta c}{2\pi\ R^2\ A\chi\gamma}\left[V(\phi) - V(\phi_s)\right] = 0 \quad .$$

See Table I for notation.

The variable canonically conjugate to ϕ is taken to be

$$W = \Delta T/\omega_{RF} = -\ A\chi\gamma\ R^2\ e\ \dot{\phi}/h^2\ \eta c \quad .$$

It is convenient to introduce the "potential"

$$G(\phi) = \int_{\phi_s}^{\phi}\left[V(\phi) - V(\phi_s)\right] d\phi \quad ,$$

which has a trough with minimum at ϕ_s flanked by two peaks at ϕ_1 and ϕ_2 where

$$V(\phi_1) = V(\phi_2) = V(\phi_s) \quad .$$

The bucket extends from ϕ_1 to ϕ_2. Phase plane trajectories may be labeled by G_0 and are given by

$$W = \frac{qeR}{h}\left(\frac{A}{q}\frac{\chi\gamma}{\pi h\ \eta c}\right)^{1/2}\left(G_o - G(\phi)\right)^{1/2} \quad .$$

Note that ϕ_1 and ϕ_2 depend on G_0. The maximum G_0 for stable motion is denoted by G_m and the conventional notation is

$$\alpha = \frac{1}{4\sqrt{2}}\int_{\phi_{1m}}^{\phi_{2m}}\left(G_m - G(\phi)\right) d\phi \quad ;$$

the bucket area is given by

$$\frac{1}{e}\ h\ A_{bu} = 8\sqrt{2}\ \alpha\ R\ q\left(\frac{A\chi\gamma}{\pi qh\ \eta c}\right)^{1/2} \quad ,$$

The bunching factor is the ratio of average to peak particle density. For uniformly filled buckets the average particle density equals some constant times the area divided by the length $A_{bu}/2\pi$, while the peak density equals the same constant times the peak height of the bucket

$$2W_m = \frac{2qeR}{h}\left(\frac{A\chi\gamma}{\pi qh\ \eta c}\right)^{1/2}\left(G_m - G_{min}\right)^{1/2} \quad .$$

Then $B_F = A_{bu}/4\pi W_m$.

The period of oscillation of the trajectory G_0 is

$$\tau_y = 2R\left(\frac{\pi A\chi\gamma}{hq\ \eta c}\right)^{1/2}\int_{\phi_1}^{\phi_2}\left[G_o - G(\phi)^{-1/2}\right] d\phi \quad ,$$

with tune $\nu_1 = 2\pi/\omega_{rev}\,\tau_y$.

A shed waveform is defined on $(\phi_2 - 2\pi,\ \phi_2)$ as

$$V(\phi) = \begin{cases} -\ 2V_1 & \phi_2 - 2\pi < \phi < \phi_1 \\ V_1 + V'\ (\phi - \phi_2) & \phi_1 < \phi < \phi_1 \end{cases} \quad ,$$

with $\phi_s = 1/2\ (\phi_1 + \phi_2)$. If $\Delta V = V(\phi_1) - V(\phi_2)$, we find from

$$\int_{\phi_2 - 2\pi}^{\phi_2} V(\phi)\ d\phi = 0 \quad ,$$

that

$$\phi_2 - \phi_1 = \frac{2\pi}{3}\cdot\frac{1 + \Delta V/2V_s}{1 + \Delta V/3V_s} \quad .$$

The shed waveform is approximated by

$$V(\phi) = \sum_{n=1}^{N}\left(A_n \cos n\phi + B_n \sin n\phi\right) \quad .$$

Application

This problem arose in connection with the proposed Argonne National Laboratory Beam Development Facility[2] where the synchrotron has parameters:

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$q = 8$ $A = 131.3$ $R = 25$ $h = 109$

$\beta_{in} = .0602$ $\beta_{out} = 0.383$ $\eta_{in} = 1.0$ $\eta_{out} = 0.857$

$V_s = 33$ keV

The set (A_n, B_n) was chosen by cut-and-try beginning with the trigonometric series with N = 2, 3, 4 for a shed. Case N=3 was much better than N=2, while N=4 showed no significant improvement over N=3, so we list only the latter case in Table II.

Figure 1 shows the maximum stable buckets for the waveforms described by a fundamental and a shed. The functions $V(\phi)$ for the fundamental-plus-two harmonics and for the shed are shown in Fig. 2a, with the corresponding buckets in Fig. 2b. In Table II the synchrotron tune ν_1 is given for the case $G_0 = 0.5\ G_m$.

RF System

We have designed a system using three ferrite loaded cavities to produce a three harmonic accelerating waveform for about 50% of the acceleration cycle (Fig.3). It is also capable of producing two harmonics at reduced levels for 71% of accelerating cycle and one harmonic for the remainder of cycle. Studies have shown that it is not necessary to carry these harmonics the full acceleration cycle. Improved bucket area and bunching factors are maintained if three-harmonics are applied for about 50% of the cycle. Other cavities could be added as required, however, at increased cost and complexity.

The cavity design itself is similar to the FNAL booster cavity design.[3] It uses 6 side attached ferrite tuners per cavity, with two symmetric about center accelerating gaps. The cavity is driven at the center by a directly attached rf amplifier. There will be one amplifier for each cavity. Table III shows the parameters of the cavities using representative ferrites. Reverse ferrite biasing at the beginning of cycle is required in cavity #3 to achieve the large frequency swing. Reverse bias could be used in the other cavities to achieve improved performance.

Table I. Notation

- h = harmonic number
- q = charge state
- η = momentum compaction factor
- R = ring radius
- m_p = proton mass
- χ = $m_p\ c/e$
- γ = relativistic factor
- A = atomic weight
- B_F = bunching factor

Table II. Buckets and Waveforms

	Fundamental	Shed	Fund. + 2(A)	Fund. + 2(B)
$\frac{1}{e}$ hA	4.341 ev-sec	1.815	1.838	1.287
BF	0.3231	0.4810	0.4156	0.4018
ν_1	0.120	0.0217	0.0326	0.2068
A_1	0	X	-8.863 kV	-8.024
B_1	66 kV	X	56.09	55.18
A_2	0	X	30.29	29.80
B_2	0	X	6.731	6.622
A_3	0	X	2.104	2.207
B_3	0	X	-8.274	-8.278
$\Delta V/V$	X	0.2	X	X
ϕ_s	30°	30°	27.5°	30°

Note: $V_s = 33$ kV $V(\phi) = (A_n \cos n\phi + B_n \sin n\phi)$

TABLE III. Cavity Parameters

Cavity #	Frequency Range (MHz)	Peak Voltage (kV)	Cavity rf Power (kW)	Cavity Length (m)	Ferrite Type
1	12.5 - 40	56.1	53	0.77	$M4C_{21A}$
2	25.0 - 80	56.8	110	0.40	$M4C_{21A}$
3	37.5 - 112.5	8.6	5	0.76	$M4D_{21A}$

References

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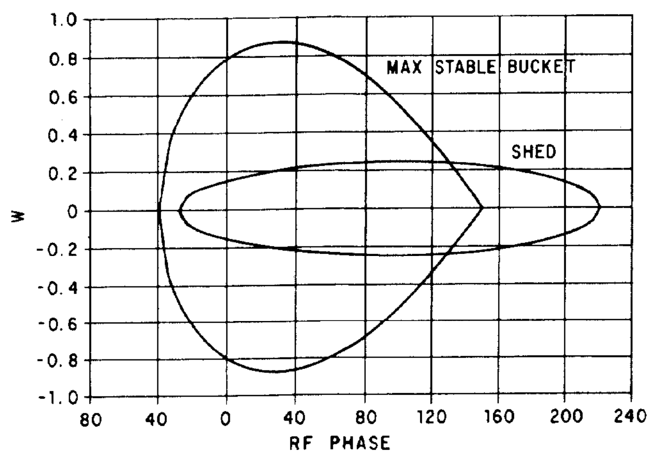

Fig. 1 Buckets for Fundamental-Only vs Shed.

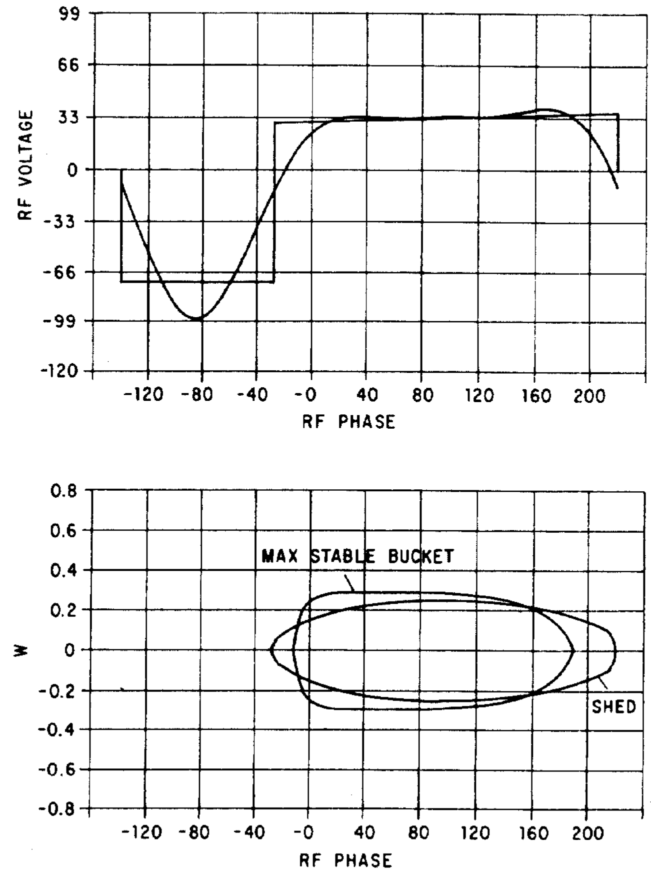

Fig. 2 Waveforms and Buckets for Shed vs Fundamental-Plus-2 Harmonics.

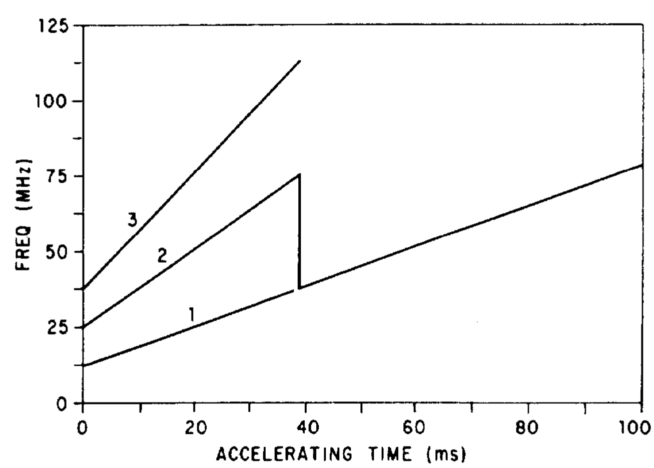

Fig. 3 Frequency Profile
(Note: numbers refer to cavities)