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CALCULATIONS OF CAPTURE EFFICIENCY OF THE DEBUNCHED STACK IN ISABELLE*

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Summary

A computer program which simulates particle behavior in the longitudinal phase space has been developed to investigate the rebunching process and capture efficiency of the debunched stack in ISABELLE. A simple expression for a capture efficiency for an arbitrary initial distribution has been derived and used to estimate capture efficiencies for several assumed distributions.

Introduction

Bunches from AGS will be stacked in momentum space in ISABELLE by a stacking rf system. The stack will be rebunched by a separate rf system operating at the third harmonic frequency 234.5 kHz, and then accelerated by the system up to the final energy. The injected beam will be stacked in the mode of "stacking at the top", which gives a low momentum tail to the stack. This will provide for stability against any self-bunching in the stack. It, however, obviously reduces the capture efficiency and consequently increases uncaptured particles which will have to be scraped off in the early part of the acceleration cycle.

Synchrotron Equation with Beam-Induced Cavity Voltage

The beam current is expanded in the Fourier Series of the phase φ of the external voltage

$$i = i_o + \sum_{n=1}^{\infty} |i_n| \sin(n\varphi + \theta_n). \tag{1}$$

The expressions of $|i_n|$ and θ_n are given by[1]

$$|i_n| = \frac{I_o}{\pi}\frac{\beta}{\beta_o}\left[\left(\int_o^{2\pi}\cos(n\varphi)\,d\varphi_o\right)^2 + \left(\int_o^{2\pi}\sin(n\varphi)\,d\varphi_o\right)^2\right]^{\frac{1}{2}} \tag{2}$$

and

$$\theta_n = \tan^{-1}\left[\int_o^{2\pi}\cos(n\varphi)\,d\varphi_o \Big/ \int_o^{2\pi}\sin(n\varphi)\,d\varphi_o\right] , \tag{3}$$

where I_o, β_o and φ_o are the dc current, the relativistic factor and the phase at the start of simulation respectively.

We assume that the cavity impedances at higher harmonic frequencies ($n \geq 2$) are negligible. Then the voltage induced on the gap is

$$V_{ind} = - Z_{c1}\, i_1 , \tag{4}$$

where Z_{c1} (= $|Z_{c1}|\exp(j\theta_{c1})$) is the cavity impedance at the external frequency. The total cavity voltage given by the sum of the external voltage V and V_{ind} is expressed as[1]

$$V_t = |V_t| \exp[j(\varphi + \theta_t)] , \tag{5}$$

where

$$|V_t| = |V| \sqrt{1 + 2\epsilon\cos\theta_i + \epsilon^2} , \tag{6}$$

$$\theta_t = \tan^{-1}\left(\frac{\epsilon\sin\theta_i}{1+\epsilon\cos\theta_i}\right) , \tag{7}$$

$$\epsilon = |V_{ind}|/|V| \quad \text{and} \tag{8}$$

$$\theta_i = \theta_1 + \theta_{c1} + \pi . \tag{9}$$

The synchrotron equations are then given by[1]

$$\frac{dw}{dt} = \frac{e|V_t|}{2\pi}\left[\sin(\varphi + \theta_t) - \sin(\varphi + \theta_t)_s\right] \tag{10}$$

$$\frac{d\varphi}{dt} = \frac{h\eta_s\Omega_s^2}{\beta_s^2 E_s}\, w , \tag{11}$$

where h is the harmonic number, η is the frequency slip factor, E is the total energy, Ω is the angular revolution frequency of particles and w is the canonical momentum difference given by

$$w = \int_{E_s}^{E} \frac{dE}{\Omega(E)} . \tag{12}$$

The subscript s means that the corresponding symbol refers to the synchronous particles.

Methods of Simulation

For particle simulation we examined two methods; one is the turn-by-turn integration of the difference equations[2] and the other is the numerical integration of the differential equation. The former is much easier in programming and can in principle give more accurate solution than the latter. A disadvantage of this method is to take much more computer time in our case. One method which reduces the computer time is to scale appropriately the ratio of the revolution frequency to the synchrotron oscillation frequency.[3] In this method, however, the matching between the computer problem and the real problem may be troublesome.

The differential equation method can save the computer time with all actual parameters retained. It is important, however, to notice that we should be very careful in choosing a numerical method. H.G. Hereward[4] already pointed out that the popular Runge-Kutta method does not precisely conserve phase space when integrating equations which are Liouvillian. The predictor-corrector methods have the advantage that a tight control over error is possible at each step by repeated application of the corrector.

We wrote two simulation programs; one is based on the turn-by-turn method and the other is based on the Adams-Bashforth method which is one of the predictor-corrector methods. The former was used to check the accuracy of the latter method. The results obtained by both methods turned out to be in perfect agreement,

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if the time step of the latter method is taken to be smaller than around one four hundredth of the period of the synchrotron oscillation. With the time step of 1 ms, which is about one five hundredth of the shortest period of synchrotron oscillation in the capture process, the computer time can be reduced to about one twentieth of that consumed by the use of the turn-by-turn method.

Expression for Capture Efficiency

The momentum distribution of the stack will be controlled, taking such factors into acount as a space allowed for the stack, beam stability and capture efficiency. Here we want to estimate capture efficiencies for various possible distributions. In order to save the computer time, we use the following expression for capture efficiency[1]

$$E = S\frac{N_c}{N}\iint f_c(w,\varphi)\, f_i(w,\varphi)\, dw\, d\varphi \quad , \tag{13}$$

where

fc: normalized distribution function of captured particles mapped back to their initial positions, obtained by a particle simulation with uniform initial distribution,

fi: Normalized initial distribution function for which capture efficiency is to be estimated,

S: phase-space area occupied by the uniform distribution used in the simulation,

N: total number of initial particles in the simulation,

Nc: total number of captured particles obtained by the simulation.

The functions fc and fi are normalized as

$$\iint f(w,\varphi)\, dw\, d\varphi = 1 \quad . \tag{14}$$

Once a result of the particle simulation with uniform inital distribution is obtained, we can get capture efficiencies for arbitrary initial distributions by using (13).

When the azimuthal structure of initial distribution can be neglected, as is usually the case with a stacked beam, (13) reduces to

$$E = \Delta w \frac{N_c}{N}\int g_c(w)\, g_i(w)\, dw \quad , \tag{15}$$

where $\Delta w = S/2\pi$ and g_c and g_i are the normalized distribution functions. For actual calculations it is convenient to write (15) in the form of summation. We divide Δw into m channels. Let N_k be the number of captured particles initially located at channel k and G_k be the discrete initial distribution function which is normalized as

$$\sum_{k=1}^{m} G_k = 1 \quad . \tag{16}$$

Then (15) becomes

$$E = \frac{m}{N}\sum_{k=1}^{m} N_k G_k \quad . \tag{17}$$

Captured Efficiencies for Assumed Initial Distributions

The phase space density of the injected bunch is assumed to be 2.12 x 10^{11} particles/ev.sec and the stacked current is taken to be 9.6A which is 1.2 times the design current of 8A.[5] On the basis of these figures we assumed the simple momentum distributions of the stack, which are shown in Fig. 1 by the stacking efficiency $\eta(p)$ as a function of momentum.[6] Three groups are shown in the figure, the distributions in (a) having the maximum stacking efficiency η_m of 1.0, those in (b) having η_m of 0.909 and those in (c) having η_m of 0.833. From the figure we got the normalized initial distribution functions G_k.

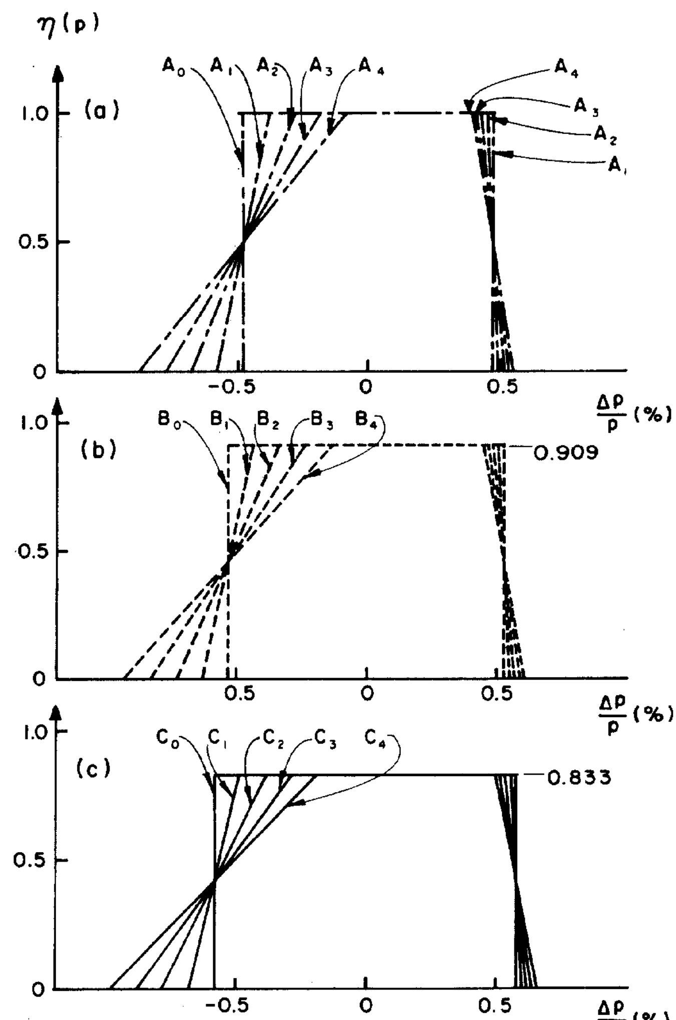

Fig. 1. Assumed momentum distributions of the stack, represented by the stacking efficiency as a function of momentum. These are made on the assumption that the phase-space density of the injected bunch is 2.12 x10^{11} protons/ev.sec and the stacked current is 9.6A.

In order to get high capture efficiency and less phase-space dilution, the iso-adiabatic voltage rise[7] will be used. The final voltage V_2 of the capture process will be around 20 kV. We take the initial voltage V_1 as 2 kV and the parameter α specifying degree of adiabaticity as 0.5. For $\alpha = 0.5$ it takes 2.34 sec for the voltage to rise from 2kV to

20 kV. The particle simulation program was made to run with the final voltages of 18, 20 and 22 kV. The beam induced voltage was neglected in these runs. In all runs 1,600 particles are initially spaced uniformly in φ from 0 to 2π and in w from -160 ev.sec to 160 ev.sec (corresponding to $\Delta p/p$ from -0.8% to 0.8%). From the results of simulations we got the values of N_k.

Capture efficiencies for the initial distributions given in Fig. 1 were calculated from (17) with the values of G_k and N_k, and the results are shown in Fig. 2. As expected, the capture efficiency is more dependent on the final voltage when the momentum spread of the stack is larger. In order to check the results, one run of the particle simulation was made with the nonuniform initial distribution corresponding to A_3 in Fig. 1. The voltage program with α = 0.5, V_1 = 2kV and V_2 = 20 kV was used to rebunch the 1000 particles. The capture efficiency obtained is 95.5%, which is in very good agreement with the figure of 95.4% given in Fig. 2 for the distribution A_3.

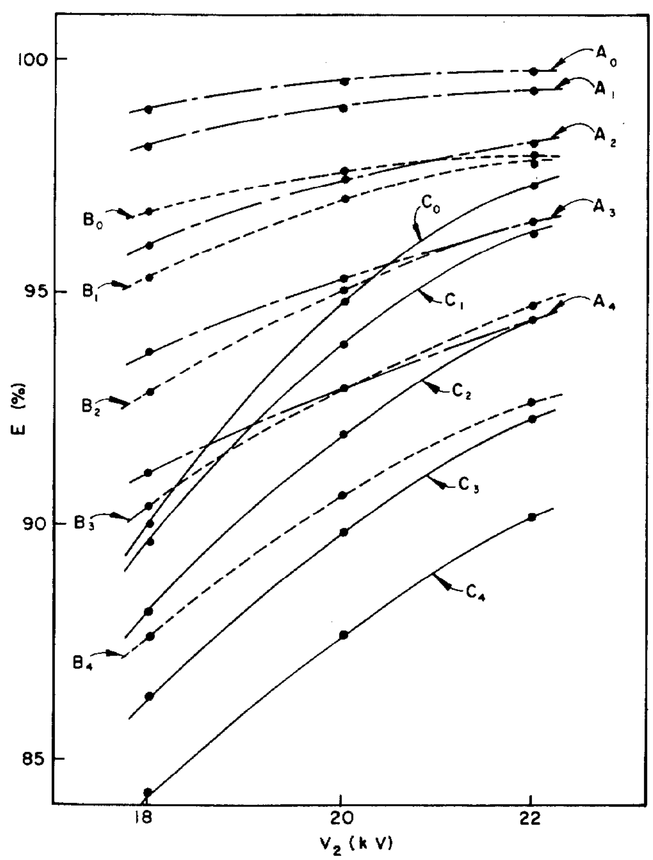

Fig. 2. Capture Efficiency E versus capture voltage V_2 for the initial distributions shown in Fig. 1, calculated from (17) with the values of N_k obtained from the particle simulations with uniform initial distribution and G_k obtained from Fig. 1.

Discussion

We assumed here the simple momentum distributions of the stack to get a rough idea of the capture efficiency. To get more realistic figures we must use distributions obtained from the computer simulations of the stacking process. Another modification of distribution comes from the fact that the space allowed for the stack is limited. Beam stability will be given the highest priority in controlling the actual distribution, though this is of course not in favor of the capture efficiency. At any rate, by the method presented here, we can easily estimate capture efficiencies for more realistic distributions which we are now simulating.

The bucket area will be almost filled up with particles at the end of the capture process, because the phase-space area of the stack will be comparable with the final bucket area. The rf voltage at the start of acceleration will be limited to 20 ~ 22 kV from the requirement of the momentum spread of the beam. To make some free area around the bunch, it may be better to rebunch the stack with the voltage lower than 18 kV and then increase it adiabatically to around 20 kV before acceleration. Unavoidable reduction of the capture efficiency will have to be compensated by adjusting the amount of the injection current and/or the momentum distribution of the stack.

Effects of the beam-induced cavity voltage are now being studied. The results obtained so far from the simulation with only the impedance at external rf frequency included shows that the induced voltage and its effect on the particle motion are small.

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