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I. Introduction

The magnetic field in superconducting dipoles and quadrupoles have undesired higher multipoles present. These higher multipoles are caused by deviations from the ideal configuration of current and iron that produces a pure dipole or a pure quadrupole field. End effects are a source of higher multipoles. Correction coils may be provided which can be used to correct the effects due to some of the undesired multipoles, particularly the lower multipoles. The higher undesired multipoles, for which there are no effective correction coils, may limit the good field aperture of the magnet. The higher multipoles will curve the working line, the dependence of the  $v$ -value on the momentum. The effects, that may limit the good field aperture, will depend on how the accelerator is operated. One effect that may limit the good field aperture is that the curvature of the working line due to the higher multipoles may make it difficult to control the transverse collective instability. For the case of the ISABELLE accelerator, one preliminary magnet design<sup>1</sup> would lead to higher multipoles being present that could limit the good field aperture to a momentum acceptance of  $\Delta p/p = 2\%$ . It is found that the 20-pole multipole in the quadrupole plays a major role in limiting the aperture, and a modification of the quadrupole design could lead to a good field aperture of about  $\Delta p/p = 3\%$ , due to this higher multipole effect.

II. Curvature of the Working Line

The presence of the higher multipoles will curve the working line  $v_x(p)$ ,  $v_y(p)$ . Let us write the median plane field, including the higher multipoles as,

$$B_y(x, s, 0) = - \left\{ B_0 + G x + B_2 x^2 + B_3 x^3 + \dots \right\}. \quad (2.1)$$

The dependence of  $v_x$  on momentum can be computed from the result

$$\Delta v_x(p) = \frac{1}{4\pi} \int ds \beta_x \frac{1}{B_0(1 + \frac{\Delta p}{p})} \left[ -(\Delta p/p)G + 2B_2 X_p \Delta p/p + 3B_3 X_p^2 (\Delta p/p)^2 + \dots \right]. \quad (2.2)$$

This result is derived in Section V. It is a perturbation theory result valid to first order in the perturbing field gradient,  $-(\Delta p/p)G + 2B_2 X_p (\Delta p/p) + 3B_3 X_p^2 (\Delta p/p)^2 + \dots$ . Higher order terms due to  $G$  and  $B_2$  probably vary slowly enough with momentum, for proton storage accelerators, so that they can be corrected with the correction coils and are then not important in this calculation. The higher order terms due to  $B_2$ ,  $B_4$ , etc. have not been included in what follows, and the conclusions stated in this paper need to be checked with calculations using a tracking program that include the higher order effects of higher multipoles.

The dependence of  $v_y$  on momentum can be computed from the result

$$\Delta v_y(p) = - \frac{1}{4\pi} \int ds \beta_y \frac{1}{B_0(1 + \frac{\Delta p}{p})} \left[ -(\Delta p/p)G + 2B_2 X_p \Delta p/p + 3B_3 X_p^2 (\Delta p/p)^2 + \dots \right]. \quad (2.3)$$

In Eqs. (2.2) and (2.3),  $\beta_x$  and  $\beta_y$  are the  $\beta$ -functions for  $\Delta p/p = 0$ . However  $X_p$  is a function of momentum. The variation of  $X_p$  with momentum was not included in the computed results given below. For ISABELLE, the variation of  $X_p$  with momentum is about 2% from  $\Delta p/p = 0$  to  $\Delta p/p = 0.01$ .

III. Effect Of High Field Multipoles On The Good Field Aperture

The effect of the high field multipoles on the good field aperture may be illustrated by considering the effect in the ISABELLE accelerator. The higher multipoles that are present in one preliminary design of the ISABELLE magnets are given in Table 1. The field due to a multipole is given by  $B_0 b_n x^n$ , where the multipole coefficient  $b_n$  is given in Table 1.  $B_0$  is the field in the main dipoles. The higher multipoles in Table 1 are part of the design.<sup>1</sup> They are present in all magnets of this design and are not to be confused with the random error multipoles.

Table 1.

Higher field multipoles in one preliminary design of the ISABELLE magnets. The multipole field is given by  $B_0 b_n x^n$  where  $B_0$  is the main dipole field.

Dipoles		Quadrupoles	
n	$b_n$	n	$b_n$
2	----	1	----
4	----	5	----
6	1.72 E-9	9	-3.01 E10
8	7.60 E-10	13	-3.20 E-13
10	-3.48 E-11	17	-3.04 E-17
12	-2.70 E-12	21	9.69 E-20
14	2.29 E-14		
16	1.25 E-15		

Using the multipoles given in Table 1, and Eqs. (2.2) and (2.3),  $v_x(p)$  and  $v_y(p)$  can be computed. The result for  $v_x(p)$  is shown by the solid line curve in Fig. 1. In computing  $v_x(p)$ , the lower multipoles  $b_2$ ,  $b_4$  and  $b_5$ , for which there are correction coils, have been omitted. Figure 1 shows the curvature of the working line due to the presence of higher field multipoles for which there are not correction coils.

In order to determine the good field aperture, a criterion is needed for deciding how much uncorrectable curvature in the working line can be allowed. In ISABELLE an important effect is the transverse instability, particularly during the stacking period. The presence of curvature in the working line may make it difficult to control the transverse instability. The criteria, adopted in this paper, is that the curvature in the working due to the higher multipoles will be considered as limiting the good field aperture when it changes the design of the working line by 50%. The dashed straight lines in Fig. 1 show this limit on the slope of the

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working line due to the higher multipoles. This criterion is based on some work by C. Pellegrini for the case of a quadratic perturbation of the working line, where a 50% change in the slope of the working line was found to appreciably affect the transverse instability.

Using the above criterion one finds from Fig. 1, the acceptable momentum aperture of  $\Delta p/p = 2.1\%$ . The dashed curve in Fig. 1 shows the curvature of the working line introduced by just the dipole, omitting the contribution of the higher multipoles in the quadrupoles. One finds a good field aperture of  $\Delta p/p = 2.3\%$  due to the dipoles only. The  $b_6, b_8, b_{10}$  and  $b_{12}$  multipoles compete with each other so as to give the larger aperture of  $\Delta p/p = 2.7\%$ . It appears that the good field aperture of  $\Delta p/p = 2.1\%$  is largely due to the higher multipoles in the quadrupoles,  $b_9$  and  $b_{13}$ , in particular the large size of the  $b_9$  and the lack of competition between  $b_9$  and  $b_{13}$  which both have the same sign.

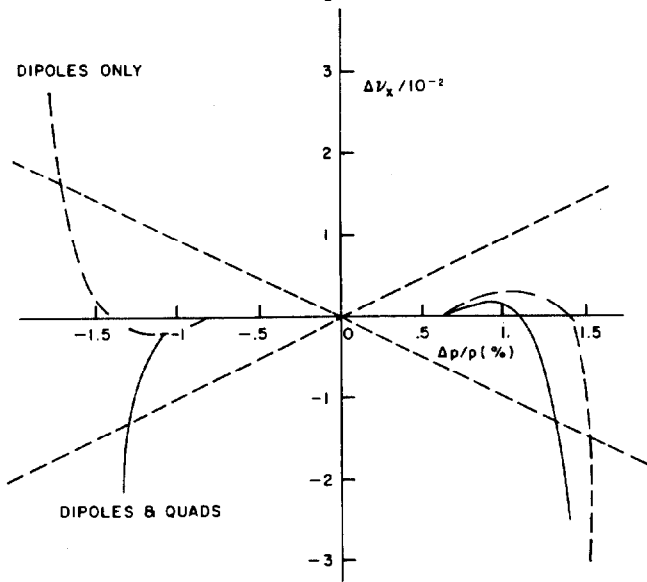


Fig. 1.  $v_x(p)$  versus  $\Delta p/p$  resulting from presence of higher field multipoles.

In Fig. 2, the effect of changing the  $b_9$  multipole in the quadrupoles on the  $v_x$  versus  $\Delta p/p$  working line is shown. In these runs,  $b_9$  is varied from the original value of  $b_9 = -3 \times 10^{-10}/\text{cm}^{-9}$  to  $b_9 = 1.75 \times 10^{-10}/\text{cm}^{-9}$ , while  $b_{13}$  is held constant. It is found that the good field aperture due to the higher multipoles can be increased from  $\Delta p/p = 2\%$  to  $\Delta p/p = 3\%$  by varying  $b_9$ . In ISABELLE not all of the increased aperture may be useable because of limitations due to the vacuum chamber dimensions and the closed orbit errors. However, as the closed orbit correction is improved, more of the increased aperture would become useable.

In Figs. 1 and 2,  $v_y$  vs.  $\Delta p/p$  is not shown. Usually  $v_y$  is effected less than  $v_x$  by the higher multipoles.

#### IV. Comments On Sagitta And End Effects

If the dipole magnets are straight, and not curved to follow the central orbit of the particles, than it may be important in computing  $v_x(p)$  and  $v_y(p)$  to compute the integrals in Eqs. (2.2) and (2.3) along the path of the particle rather than along the axis of the magnet. In ISABELLE, the dipole magnets are straight and the sagitta is about  $\pm 6$  mm. For the higher multipoles, being 6 mm further away from the dipoles axis can appreciably change the field gradient due to

the higher multipoles. The effect of the sagitta was included in this paper.

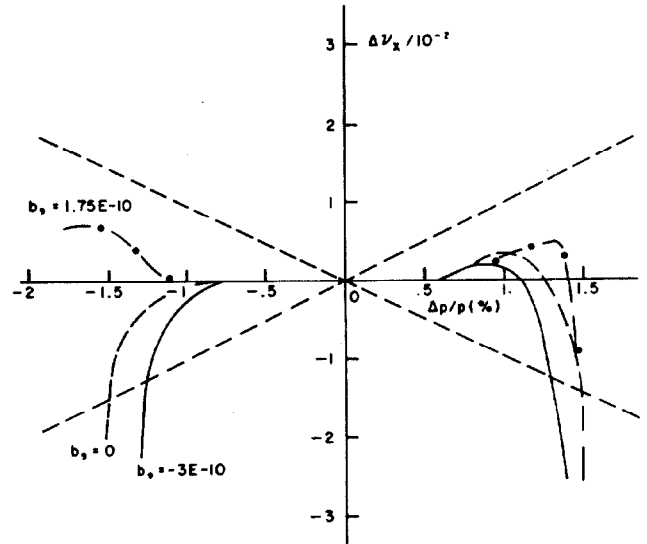


Fig. 2. Effect of varying  $b_9$  on  $v_x(p)$  versus  $\Delta p/p$ .

The higher multipoles due to end effects can also appreciably affect the good field aperture.<sup>2</sup> In this paper, the effects of the end multipoles were not included in the computations of  $v_x(p)$  and  $v_y(p)$ . The effects of the end multipoles will be treated in a future paper.

#### V. Derivation of $v_x(p)$ , $v_y(p)$ Results

The linearized equations of motions are:

$$\frac{d^2x}{ds^2} + \frac{1}{\rho^2} x = \frac{1}{B\rho(1+\Delta p/p)} B_y(x,s,y) - \frac{1}{B\rho} B_y(o,s,o), \quad (5.1a)$$

$$\frac{d^2y}{ds^2} = -\frac{1}{B\rho(1+\Delta p/p)} B_x(x,s,y), \quad (5.1b)$$

where  $x,y$  are the transverse coordinates relative to the central orbit for  $\Delta p/p = 0$ .

First,  $\bar{X}_p(s) = X_p \Delta p/p$ , the location of the off momentum orbits is found. It is assumed that the vertical field is given in the medium plane by

$$B_y(x,s,o) = -\{B_0 + Gx + B_2x^2 + B_3x^3 + \dots\}. \quad (5.2)$$

where  $B_0, G, B_2, B_3 \dots$  can be functions of  $s$ .

From Eq. (5.1a), one finds

$$\left\{ \frac{d^2}{ds^2} + \frac{1}{\rho^2} \right\} \bar{X}_p = \frac{-1}{B\rho(1+\Delta p/p)} \left\{ -B_0 \Delta p/p + Gx_p + B_2x_p^2 + B_3x_p^3 + \dots \right\}. \quad (5.3)$$

or

$$\left\{ \frac{d^2}{ds^2} + \frac{1}{\rho^2} + \frac{G}{B\rho(1+\Delta p/p)} \right\} \bar{X}_p = \frac{-1}{B\rho(1+\Delta p/p)} \left\{ -B_0 \Delta p/p + B_2 \bar{X}_p^2 + B_3 \bar{X}_p^3 + \dots \right\}. \quad (5.4)$$

Note it is not assumed that  $\bar{X}_p$  is linear in  $\Delta p/p$ .

Now the equations for the betatron motion are found. We write the off momentum central orbit  $x = X_p(s) + u$ ,  $u$  being the linear betatron motion

around the off momentum orbit  $x = \bar{X}_p(s) = X(s)\Delta p/p$ , and put this in Eq. (5.1a) for  $x$ , and find using Eq. (5.4) for  $\bar{X}_p$

$$\frac{d^2}{ds^2} u + \left( \frac{1}{\rho} + \frac{G}{B\rho} \right) u + \frac{u}{B\rho(1+\Delta p/p)} \left\{ -G\Delta p/p + 2B_2\bar{X}_p + 3B_3\bar{X}_p^2 + \dots \right\} \quad (5.5)$$

where higher order terms in  $u$  have been dropped. Equation (2.1) for the  $v$ -shift  $v_x(p)$  follows from Eq. (5.5).

The vertical betatron oscillation are now treated. From  $B_x = (\partial B_y / \partial B_x) y$  one finds:

$$B_x = - \left\{ G + 2B_2 x + 3 B_3 x^2 + \dots \right\} y.$$

From Eq. (5.1b) one finds

$$\left( \frac{d^2}{ds^2} - \frac{G}{B\rho} \right) y - \frac{y}{B\rho(1+\Delta p/p)} \left\{ -G\Delta p/p + 2B_2\bar{X}_p + 3B_3\bar{X}_p^2 + \dots \right\} = 0. \quad (5.6)$$

Equation (2.3) for  $\Delta v_y(p)$  follows Eq. (5.6).

#### Acknowledgments

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#### References

1. P.F. Dahl and H. Hahn, BNL ISABELLE Project Tech. Notes 154 and 161 (1980). (Unpublished).
2. G. Parzen, BNL ISABELLE Project Tech. Note 265 (1981). (Unpublished).