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A METHOD OF MEASURING AND CALCULATING THE LONGITUDINAL MICROWAVE COUPLING IMPEDANCE OF ISABELLE*

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Summary

Modes above the cutoff frequency of the ISABELLE vacuum chamber have been investigated to ascertain the microwave longitudinal coupling impedance. (The investigation is limited to those modes that have fields in the beam pipes.) Perturbation measurements of the electric fields were made between 2.6 and 2.8 GHz. A phenomenological method of calculating these impedances was developed. There was excellent agreement between the calculated and measured impedance. The theoretical results were then extrapolated to 5.5 GHz.

1. Definition of Coupling Impedance

The coupling impedance, Z, between the beam and cavities, formed by the vacuum chamber, is identical to the definition of shunt impedance, R_s, used for linear accelerators, and is defined as

$$Z = R_s = \left[\int_0^L E\,dz\right]^2 \Big/ \int_0^L P(z)\,dz. \qquad (1)$$

L is the length of the cavity, P(z) the power loss and $E = E_0F(z)f(z,t)$, where E_0 is the peak field, F(z) is the spatial field distribution as a function of z, and f(z,t) is dependent on the particle's velocity and position in relation to the phase of the rf field.

If we consider a cavity that has a periodic length L, then F(z) can be decomposed into a Fourier series having spatial harmonics:

$$F(z) = \frac{a_o}{2} + \sum_{k=1}^{\infty}\left(a_k \sin\beta_k z + b_k \cos\beta_k z\right), \qquad (2)$$

where the propagation constant $\beta_k = 2\pi k/L$.

We will only consider particles that have a constant velocity v_p; therefore, z and t are linearly related by the expression $z = v_pt$. The function f(z,t) depends on both the frequency of the cavity, ω_m, and the velocity of the beam. Using the definition that $\omega_m/v_p = \beta h = 2\pi h/L$, we can now write

$$f(z,t) = \sin\left(\omega_m t + \phi\right) = \sin\left(\frac{2\pi hz}{L} + \phi\right), \qquad (3)$$

where ϕ is the phase angle between the beam and cavity voltage.

We can now write Eq. (1) as

$$R_s = \frac{\left\{\int_0^L\left[\frac{a_o}{2} + \sum_{k=1}^{\infty}\left(a_k\sin\frac{2\pi k}{L}z + b_k\cos\frac{2\pi k}{L}z\right)\right]\sin\left(\frac{2\pi h}{L}z+\phi\right)dz\right\}^2}{\int_0^L P(z)dz} \qquad (4)$$

From the above equation we see that when $k \neq h$, then $R_s = 0$; and when $k = h$, we get $R_s > 0$. It should be emphasized that we are only dealing with synchronous particles, where h is an integer; therefore, the coupling impedance per cavity, of length L, is the same for all cavities.

For nonsyncronous particles, h is not an integer, and, therefore, the phase ϕ is no longer a constant but varies as a function of z and t. It is still possible to have $|R_s| > 0$, but the value of R_s will vary as a function of z and t. The integral of this impedance, over a period of time, will go through repeated zeros.

2. Perturbation Measurements

We would now like to find the coupling impedance, R_s, by measuring the field distribution along the beam line of a cavity and applying these results to Eq. (4). Figure 1A is a schematic representation of a typical section of the ISABELLE vacuum pipe where the dimensions shown are approximately those of the real machine. We now consider a cavity of length L by placing shorting planes as indicated in Fig. 1A. Of interest are the TM_{01m} modes which have an axial electric field distribution E_z, as shown in Fig. 1B; it is these fields that contribute to the longitudinal coupling impedance. (Note: this paper will only consider those modes that have fields in the beam pipes and no fields in the pump out boxes).

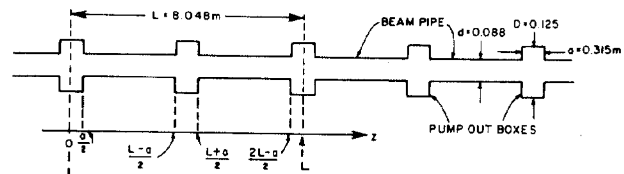

FIG. 1(A). ISA VACUUM CHAMBER MODEL.

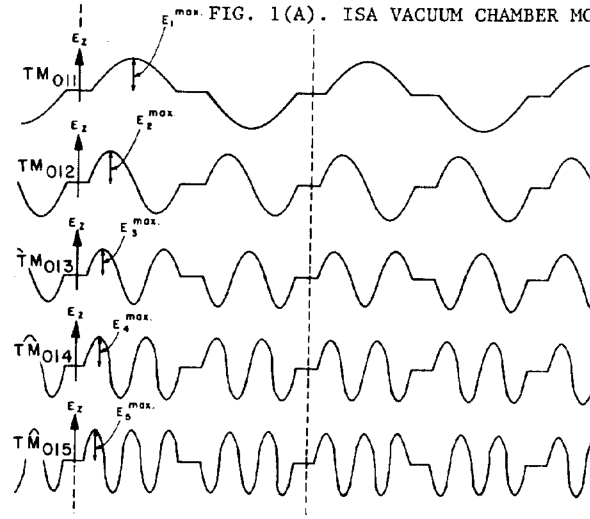

FIG. 1(B). SOME TYPICAL TM_{01m} FIELD DISTRIBUTIONS.

The fields are measured by pulling a small metal sphere, of radius r_o, along the axis of the cavity. We can now write

$$E_z = \left(\frac{\Delta f(z)/f \times W}{\varepsilon_o \pi r_o^3}\right)^{1/2} = E_oF(z), \qquad (5)$$

where E_z is the peak electric field in volts per meter at a point z, $\varepsilon_o = 1/(36 \times 10^9)$F/m, W is the stored energy in the cavity in joules, and f is the unperturbed frequency of the cavity. By substituting Eqs. (3) and (5) in Eq. (1), and remembering that

$$Q = 2\pi Wf \Big/ \int_0^L P(z)dz,$$

we get

$$R_s = \frac{2Q}{\omega^2\varepsilon_o r_o^3}\left[\int_0^L \sqrt{\Delta f(z)}\left(\sin\frac{2\pi h}{L}z + \phi\right)dz\right]^2. \qquad (6)$$

Equation (6) can now be evaluated for the different TM_{01m} modes by inserting the measured value of ω_m, Q_m, and $f_m(z)$. The actual data can be handled in two different ways: the information for $\Delta f_m(z)$ can be numerically inserted in Eq. (6); or as we were actually able to do, an analytic function can be fitted to the data. It was surprising that a relatively simple sine function matches the data to a high degree of accuracy; for example, the TM_{01m} modes, as depicted in Fig. 1B, when substituted in Eq. (6) becomes:

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$$R_s = \frac{2Q_m \Delta f_m^{max}}{\omega_m^2 \varepsilon_o r_o^3}\left\{\int_0^{a/2} 0 \times \sin\left(\frac{2\pi h}{L} z + \phi\right)dz \right.$$
$$+ \int_{a/2}^{(L-a)/2} \sin m\pi \frac{z - a/2}{L/2 - a} \sin\left(\frac{2\pi h}{L} z + \phi\right)dz$$
$$+ \int_{(L-a)/2}^{(L+a)/2} 0 \times \sin\left(\frac{2\pi h}{L} z + \phi\right)dz$$
$$+(-1)^m \int_{(L+a)/2}^{(2L-a)/2} \sin m\pi \frac{z-(L+a)/2}{L/2-a} \sin\left(\frac{2\pi h}{L} z+\phi\right)dz$$
$$\left. + \int_{(2L-a)/2}^{L} 0 \times \sin\left(\frac{2\pi h}{L} z + \phi\right)dz\right\}^2 , \qquad (7)$$

where

$$\Delta f_m^{max} = \frac{\varepsilon_o \pi r_o^3 f_m}{W_m} \times \left(E_m^{max}\right)^2 .$$

It is obvious that for our particular cavity, having the fields as shown in Fig. 1B, the only spatial harmonics that exist in Eq. (4) are the $a_k \sin 2\pi kz/L$ terms, where $k = 1, 3, 5, \ldots$. By comparing Eq. (7) with Eq. (4), it is apparent that for synchronous particles with $\phi = 0$, we get a maximum R_s for $h = k$ (an integer). Therefore, Eq. (7) reduces to

$$R_s = \frac{2Q_m \Delta f_m^{max} L^2}{\omega_m^2 \varepsilon_o r_o^3}\left\{\frac{\left[1 - 2a/L\right]\left[1 + (-1)^{m+h}\right]}{m\pi\left[1 - (h/m)^2(1 - 2a/L)^2\right]} \sin \frac{\pi h a}{L}\right\}^2 \qquad (8A)$$

for $a \neq L/2\ (1 - m/h)$, and

$$R_s = \frac{2Q_m \Delta_m^{max} L^2}{\omega_m^2 \varepsilon_o r_o^3}\left\{\frac{1}{8}\left[\frac{m}{h}\right]^2 \left[1 + (-1)^{m+h}\right]\right\}. \qquad (8B)$$

for $a = L/2\ (1 - m/h)$.

3. APPLICATION

Before attempting to apply the above method, it is essential to consider the dispersion curve of the cavity structure. The measured ω vs β diagram is shown in Fig. 2 by a solid line, where β is plotted in terms of the spatial harmonic number h. Also included on the diagram are the dispersion curves of the spatial harmonics, as indicated with dashed lines. Measurements were made on a single cavity of length L; with a longer cavity many more resonances would have been observed. The structure will no doubt have stop bands which have not been considered in this paper, but a new model is being built to study this problem.

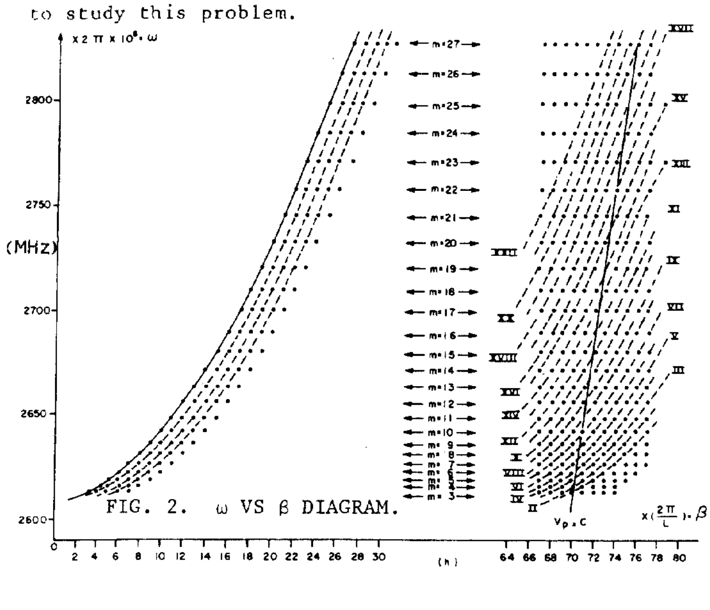

FIG. 2. ω VS β DIAGRAM.

Since we are interested in particles with velocity $v_p = c$, we must now find the spatial harmonics that match this particle velocity, that is $v_p = v_\phi = c$. By plotting the line $v_p = c$ in Fig. 2, we can immediately determine where this occurs. Using Eq. (8), it is now possible to calculate the shunt impedance as a function of ω_m for the different spatial harmonics.

To illustrate how Eq. (8) is used to calculate R_s for the different modes, let us consider Fig. (3) which is an expanded section of Fig. (2). For those curves identified by odd Roman numerals, $R_s = 0$. This results from the fact that for these curves m + h equals an odd number. For the curves indicated by even Roman numerals, the R_s must be calculated at the point where the curve intersects the line $v_p = v_\phi = c$.

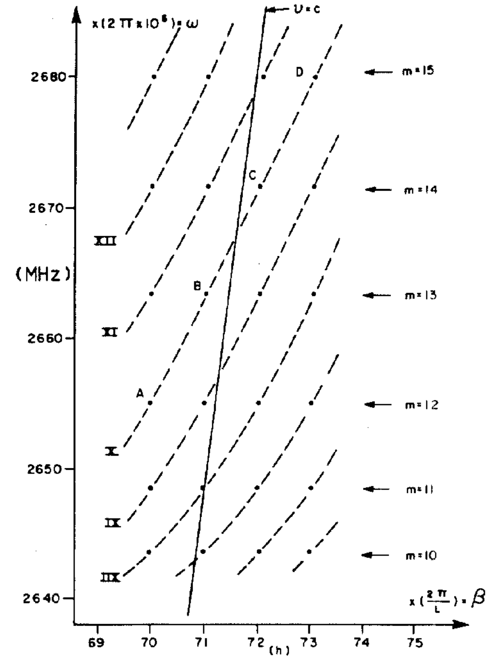

FIG. 3. EXPANDED ω VS β DIAGRAM.

Let us first consider the case when a mode lies on the $v_p = c$ line as indicated in Fig. 3 for m = 11 and h = 71. It is a simple matter to calculate R_s of this spatial harmonic by substituting in Eq. (8) the values of m = 11, k = 71, and the measured Q_{11}, ω_{11}, Δf_{11} max.

The second case is when a mode does not lie on the $v_p = c$ line. For the curve X, in Fig. (3), we calculate R_s for the points A, B, C, and D as was done in the previous example. Each of these points has its own set of parameters for m, h, Q_m and also its own phase velocity $v_\phi = \omega_m/\beta = f_m L/h$. The results are plotted in Fig. 4, showing R_s vs. v_ϕ/c. The condition of interest is where $v_\phi/c = v_p/c = 1$, which corresponds to the interaction of a spatial harmonic wave and particle wave both traveling with a velocity equal to c, and the resulting $R_s = 685\Omega$.

4. RESULTS OF MEASUREMENTS

Of interest is the total impedance around the ring as seen by the beam and expressed as a ratio of Z/n, where $Z = R_s \times L_o/L$ (where L_o is the total length of the structure around the ring and L, as before, is the length of a single cavity), and $n = f_m/f_o$ (where f_o = 78kHz, the fundamental rotational frequency of the ISA). For the results given in this paper we used the approximate relation that $Z = 400 \times R_s$. Figure 5 is a plot of the measured Z/n for different modes between 2.6 and 2.8 GHz. It should be noted that some of the modes have a $Z_n = 0$. These modes all have a value of n + h equal to an odd number, and from Eq. (8) we see that the

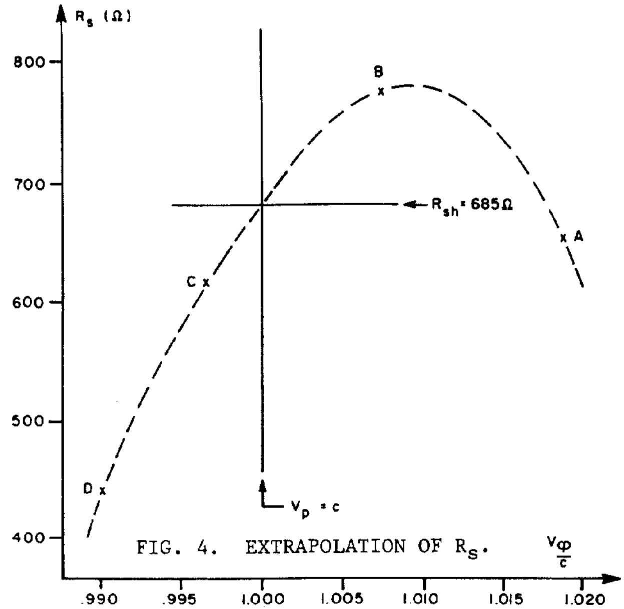

FIG. 4. EXTRAPOLATION OF R_s.

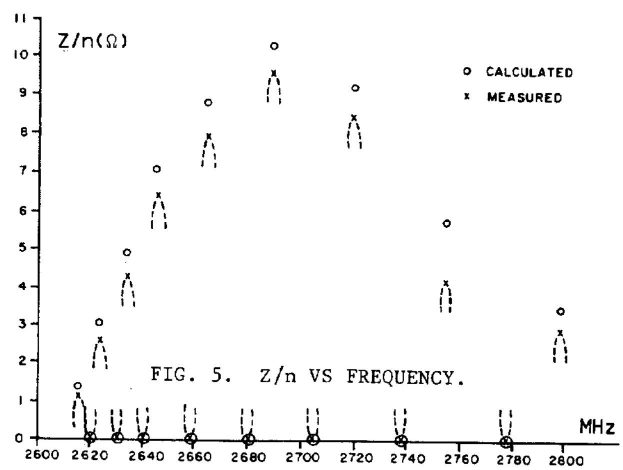

FIG. 5. Z/n VS FREQUENCY.

corresponding $R_s = 0$. What this implies, for the particular structure being considered, is that the symmetry of the structure does not allow certain spatial harmonics to exist. The remaining modes shown have relatively narrow resonances with Q_m values between 4100 and 4600.

5. CALCULATIONS

As was previously stated we are only considering those modes that have electric fields in the beam pipe and no field in the pump out boxes. As was shown in the reference, the stored energy in the pump out boxes is considerably less than 1% of the stored energy in the beam pipes; therefore, the beam pipes, Fig. 1A, can now be treated independent of the pump out boxes by assuming a magnetic boundary at a/2, (L-a)/2, (L+a)/2 and (2L-a)/2. Since we are considering the TM_{01} type modes that have a radial $J_o(k_c r)$ dependence, where $k_c = 2j_{01}/d$, it is a straightforward matter to write the field equation and calculate the stored energy and average power loss in the beam pipes for the various modes. By making the appropriate substitutions in Eq. (8A and B) we get

$$R_S = \frac{2Z_o^2 f_c^2 L}{\pi J_1^2(j_{01}) R_{\square} f_m^2 d} \left\{ \frac{[1-2a/L][1+(-1)^{m+h}]}{m\pi[1-(h/m)^2(1-2a/L)^2]^2} \sin\frac{\pi h a}{L} \right\}^2 \quad (9A)$$

for $a \neq L/2(a-m/h)$, and

$$R_S = \frac{2Z_o^2 f_c^2 L}{\pi J_1^2(j_{01}) R_{\square} f_m^2 d} \left\{ \frac{1}{8}\left[\frac{m}{h}\right]^2 \left[1+(-1)^{m+h}\right] \right\} \quad (9B)$$

for $a = L/2(1-m/h)$,

where $R_{\square}$ is the ac surface resistivity and $f_m = c\left[(j_{01}/\ d)^2 + (m/2\ )^2\right]^{1/2}$, the frequency of the mth mode, and f_c the cut off frequency.

6. RESULTS OF CALCULATIONS

As was done in section 4, Z/n is expressed in terms of R_s. Using Eq. (9A and 9B), Fig. 5 shows a comparison between the measured and calculated values of Z/n. As shown, there is an excellent agreement between the measured and calculated values between 2.6 and 2.8 GHz.

In principle one can now calculate the Z/n above 2.8 GHz where measurements become difficult to perform because of the mixing of the TM_{01m} modes with other modes. The calculated values of Z/n up to 5.5 GHz are shown in Fig. 6.

At first, the finding of such high Z/n values above 2.8 GHz created some doubt about the results but a simple argument can be made to account for these high values. As the frequency increases some of the modes contain a large spatial harmoic component having a phase velocity $v_\phi = c$. It is these components which couple strongly to the beam and are responsible for the high values of Z/n. A further check was made by making some spot measurements between 3.0 and 3.6 GHz. The measurements clearly showed that in this frequency

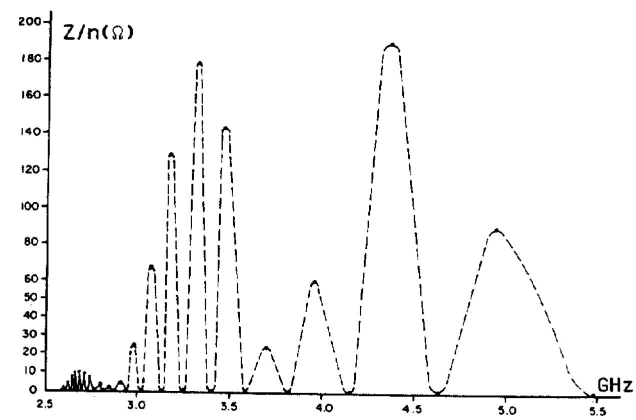

FIG. 6. CALCULATION Z/n VS FREQUENCY.

range the measured values agreed with those shown in Fig. 4 to within ±30%. The large measurement errors were due to the lack of proper equipment. Even considering the most optimistic results, the values of Z/n are high enough to be of concern.

If the impedances as measured prove to be troublesome, it would be necessary to change the design of the structure. Some thought has already been given to design changes which would lower the Q_m values and thereby reduces these impedances. Any major change must be carefully considered for its possible impact on other machine design parameters.

We would like to thank Louis Mazarakis for making the measurements.

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