IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

RADIATION LOSSES DUE TO THE INTERACTION OF CHARGED PARTICLES WITH CONDUCTING BODIES.

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Abstract

When particle bunches travel near perfectly conducting bodies, an electromagnetic field is induced which modifies the energy of the particles. In this paper we discuss the interaction of cylindrical bunches and toroïdal perfectly conducting bodies. This problem is treated in terms of electromagnetic wave scattering As a matter of fact, when the electromagnetic waves that travels together with the bunch, illuminates the obstacle, a scattered electromagnetic field appears, which modifies the energy of the bunch particles. This scattering problem is treated directly in the time domain, using space-time integral equations and solved by a time stepping approach. From the knowledge of the scattered field on the bunch axis, a transient potential seen by every particle is determined and the total energy loss of the bunch can be deduced. A numerical application for the case of toroïdal obstacles with circular cross-section is presented. The transient potential obtained by beam-obstacle interaction is relevant to the determination of the bunch longitudinal equilibrium distribution. Particularly in high energy storage rings this potential (as well as the beam cavity interaction potential) can modify the bunch shape and involve current limitations.

Introduction

When a particle bunch travels near perfectely conducting bodies, an electromagnetic field is induced which modifies the energy of the particles.

The purpose of this paper is to present the interaction between a cylindrical bunch of particles and a toroïdal perfectely conducting obstacle. The bunch and the obstacle are located in the free space and have the same axis zz'.

The interaction problem is treated in terms of electromagnetic wave scattering and gives (as beam cavities interaction studies {1} , {2}) a transient potential seen by every particle. This interaction potential modifies the energy distribution and the total energy of the bunch.

When bunches are expected to be short enough to ignore the presence of the vacuum chamber, this approach can be applied to evaluate electromagnetic losses in particles accelerators and storage rings.

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Scattering problem formulation

Bunch-obstacle interactions can be deduced from the scattering of the E.M. wave (which travels together with the bunch) by the toroïdal obstacle. As a matter of fact, when the bunch travels in free space (or on the axis of a perfectely conducting vacuum chamber), it generates an electromagnetic field (E^i, H^i). This field is a TM field (but TEM when particles velocity equals light velocity : $\vec{v} = c$). Because of circular symmetry, the magnetic field $\vec{H}^i$ has only the azimuthal component H^i_θ .

When this incident wave $(\vec{E}^i, \vec{H}^i)$ illuminates the perfectely conducting toroïdal obstacle, it induces, on the scatterer surface S, a current with density vector $\vec{J}$. Then $\vec{J}$ generates in all the space the scattered field $(\vec{E}^s, \vec{H}^s)$ which is related to the total field $(\vec{E},\vec{H})$ by :

$$\vec{E} = \vec{E}^i = \vec{E}^s \qquad \vec{H} = \vec{H}^i + \vec{H}^s$$

Using Green function {6 }and the boundary condition on the surface S, we obtain {3} , {4} , {5} a space time integral equation verified by the azimuthal component H_θ (M,t) of the total magnetic field on the surface S

$$(1)\quad H_\theta(M,t) = 2\, H^i_\theta(M,t) + 2PV\int_{(C)} FP\int_0 H_\theta(M_o,t')\,\frac{\partial K}{\partial n_o}(M,M_o,t-t')\,dC_o\,dt'$$

with :

$$- K(M,M_o,t) = \begin{cases} 0 & \text{if } c\,t \notin \{D_1, D_2\} \\ \dfrac{c}{4\pi r}\sqrt{\dfrac{D^2-c^2t^2}{c^2t^2-D_1^2}} - \sqrt{\dfrac{{}^2t^2-D_1^2}{D_2^2-c^2t^2}} & \text{if } ct \notin \{D_1, D_2\} \end{cases}$$

- (C) scatterer cross-section contour
- $D_1 = M\,M_o$, $D_2 = M\,M'_o$; M'_o is the miror image with respect to the ring axis (Fig. 1)
- r is the distance form M to the axis
- PV : Principal value of space integral {6} .
- PF : Finite part of time integral {6} .

The total magnetic field component H_θ (P,t) at any point P of space can be deduced from H_θ (M,t) by :

$$(2)\; H_\theta(p,t) = H^i_\theta(P,t) + \int_{(C)} FP\int_0^\infty H_\theta(M_o,t')\frac{\partial K}{\partial n_o}(P,M_o,t-t')\,dC_o\,dt'$$

Using Maxwell equations, the total E.M. field $(\vec{E},\vec{H})$ is obtained everywhere from the knowledge of $H_\theta(P,t)$.

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Evaluation of the bunch-obstacle interactions potential

From the knowledge of $H_\theta(P,t)$ we obtain, using Maxwell equations, the total electric field $\vec{E}(P,t)$ at any point P of the space and, particularly, the $E_z(P,t)$ longitudinal component near the bunch axis.

Then, the potential acting on a particle located at a distance r of the axis is defined by :

$$V(r,t_o) = \int_{-\infty}^{+\infty} E_z(r,z,z+t_o v)\, dz$$

where t_o is the time delay from the first particle of the bunch.

The transient potential can also be obtained directely from the $H_\theta(M,t)$ component {8} :

$$V(r,\tau) = -\int_0^{t_o} \int_{-\infty}^{+\infty} \int_{(C)} \frac{2\Gamma(r,z,M_o)}{c\ \varepsilon\ r} H_\theta\,(M_o,z+x-D)\ dz\ dx\ dC_o$$

$$\text{with :}\ -\Gamma(P,M_o)=FP\int_{D_1}^{D_2} \frac{\partial K}{\partial n_o}\,(P,M_o,T)dT,\quad D=\frac{D_1+D_2}{2}$$

The energy loss $<V(r,t)>$ for the whole bunch with an infinitely thin cross section is given by :

$$<V(0,t_o)> \ = \int_{-\infty}^{+\infty} V(0,t_o)\ \ I(t_o)\ dt_o$$

where $I(t_o)$ is the current intensity.

Numerical application

This method is applied to a particular bunch obstacle interaction :

- Particles Bunch : we consider a cylindrical bunch with radius a and length l which travels, with light velocity c, on the obstacle axis (z',z). The total charge of the bunch Q is distributed on the cylinder surface. The azimuthal distribution is uniform. The longitudinal distribution is given by the charge density :

$$\rho\ (r,z-ct) = \frac{Q\,\delta(r-a)}{\pi a\ \ l}\ \sin^2 \frac{\pi(z-ct)}{l}\,\{\,Y(z-ct+l)-Y(z-ct)\}$$

Y is the unit step function and δ the Dirac function.

- The obstacle : We have choosen a toroïdal perfectly conduction obstacle with circular cross-section {fig.1} .

- Results :

a) First the magnetic field $H_\theta(M,t)$ is obtained on the scatterer surface by solving the integral equation (2). Time evolution of $H_\theta(M,t)$ is given (fig.2) for differents points of the contour (C). The incident field which would have been present O' in case of no obstacle gives the reference in time and amplitude.

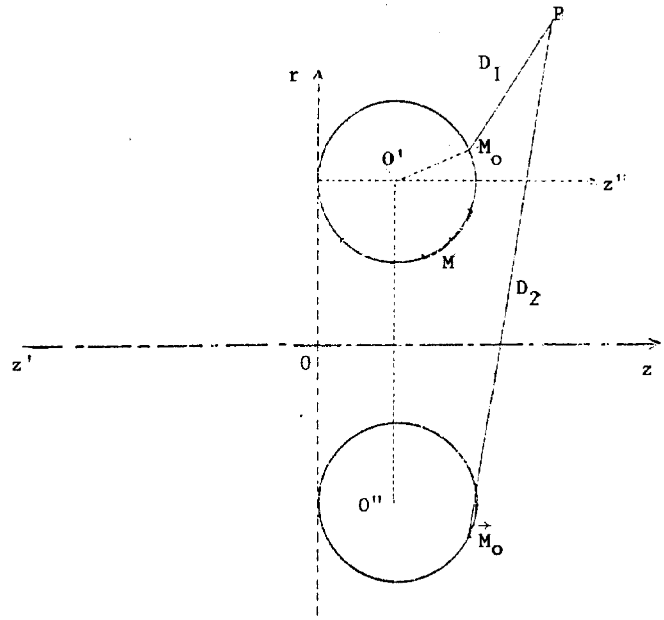

Figure 1 : Toroïdal obstacle cross-section

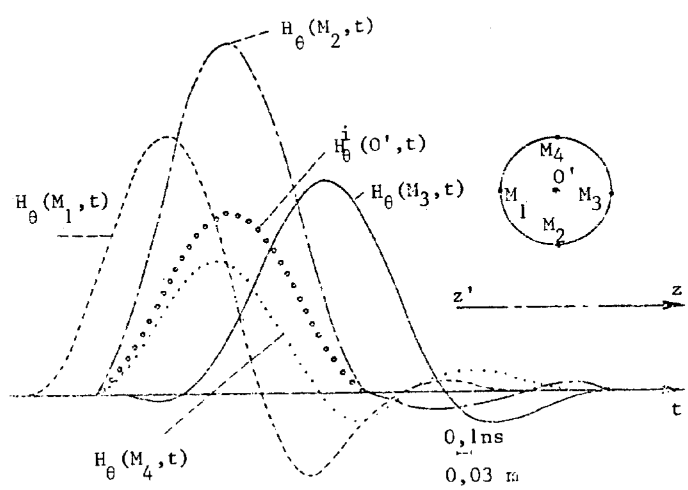

Figure 2 : Time variations of the total magnetic field on the surface S.

b) From these results the electric field is deduced everywhere particularly on the bunch axis. The figure (3) presents the time evolution of the electrical longitudinal component $E_z(P,t)$ at a point P of the z'z axis. Because of causality principle {8} the field at the point P appears with the time delay $\frac{AM_TP}{c}$

The bunch shape is also presented on this figure to identify the field which acts on every particle of the bunch.

c) The transient potential or the energy loss as a function of the particle position inside the bunch, is presented on the figure(4)for a bunch of 10^{12} particles with lenght l = 0,2 m.

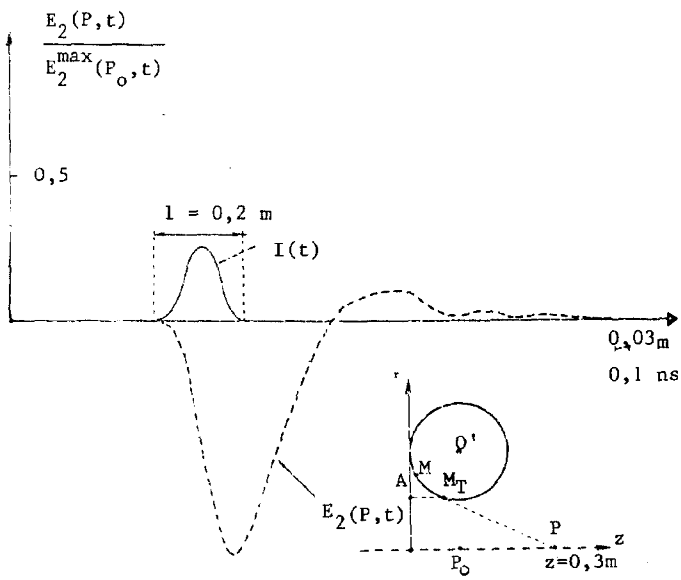

Figure 3 : Time variation of the total electric field on the axis.

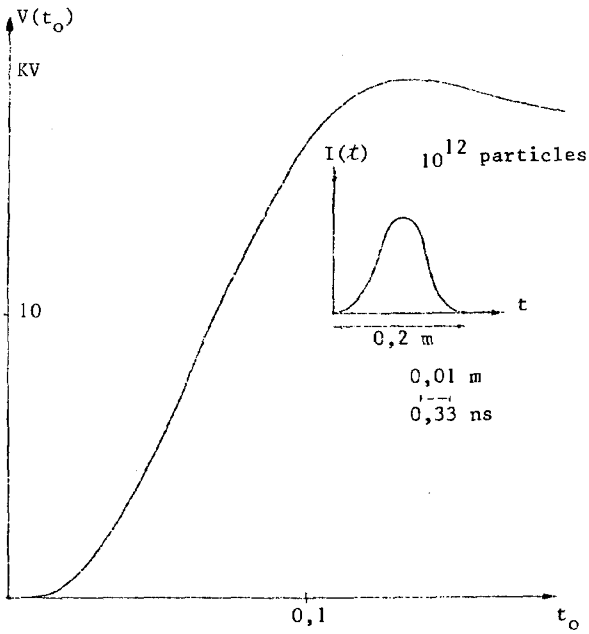

Figure 4 : Transient potential as a function of the particle position.

Conclusion

Bunch-obstacle interaction study has been regarded as a transient scattering problem in the particular case of circular symmetry. This method can also be applied to bunch obstacle interactions without symmetry {5} but prohibitively large computer times do not permit to consider bunches and obstacles with very different sizes.

The transient potential or the energy lost by every particle of the bunch is easily deduced from the scattering problem. The comparison with bunch cavity interaction effects {2} , {8} shows that the bunch interactions with obstacles (or with large discontinuities of the vacuum chamber) induces losses and bunch shape modifications. This feature is particularly important in high energy storage rings where bunch obstacles interaction contribute like beam-cavities interactions to generate bunch lengthening.

Références

{1} PAPIERNIK,A ; CHATARD-MOULIN,M and JECKO, B : "Fast beam-cavity interaction and its effect on bunch shape in storage rings" Proc. 9th Intern. Conf. on High Energy Accelerators". STANDFORD California p. 375 (1974).

{2} PAPIERNIK,A ; JECKO,B and CHATARD-MOULIN, M : "Radiation losses in electron ring cavities". Nucl. Instr. and Meth. vol. 130 p.315-324 (1975).

{3} JECKO, B ; PAPIERNIK, A : "A new space-time integral equation for two-dimensional transient scattering" Electr. Letters vol.14 n°16 (Août 1978)

{4} JECKO, B ; and PAPIERNIK, A : "Diffraction en régime transitoire par des obstacles conducteurs ou diélectriques". 1980 International Symposium Digest ANTENNAS and PROP. 80 CH 1567-8-AP QUEBEC (Canada).

{5} JECKO, B ; and PAPIERNIK, A : "Time-domain transient scattering by conducting and dielectric obstacles using a new integral equation approach" Int. URSI Symp. (1980) on E.M. waves 132-81 MUNICH - Germany -.

{6} SCHWARTZ, L : "Théorie des distributions" Hermann PARIS VIè (1966).

{7} HARRINGTON, R.F. : "Field computation by moment methods". The Mac-Millan Company NEW-YORK (1968).

{8} B. JECKO : Thèse de Doctorat d'Etat (1979) Université de LIMOGES - France -.

{9} BENNETT, C.L. and WEEKS, L. : "Transient scattering from conducting cylinders". IEEE Trans. Ant. and Propagation vol. AP-18 n°5 (sept. 1970).