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BEAM-BEAM SIMULATION FOR LEP

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Summary

A multi-particle two-beam (strong-strong) simulation program has been written for investigation of the beam-beam effect in 'LEP'. The motion of the superparticles is treated in six-dimensional phase space and the effects of quantum excitation and radiation damping are included. The effects of perturbations to the superperiodicity (errors) are also included. Non-zero dispersion at the RF cavities allows computation of single beam synchro-betatron resonances. These resonances may also be driven by the beam-beam interaction either by non-zero dispersion at the interaction point or by the longitudinal motion of the interaction 'point' between a particle and the centre of the 'other' beam due to the synchrotron motion of the particle. After each revolution the parameters influencing the beam-beam force (e.g. the beam dimensions and the beam current) are re-evaluated in order to simulate a real situation. For the beam-beam force an elliptical beam with Gaussian charge distribution has been assumed. The computation of this force is speeded up by using tabulated values of the complex error function and a fast interpolation procedure. The 'beam-beam limit' is shown to be a function of many machine parameters but is always significantly below the assumed maximum value of 0.06.

1. Introduction

An important limitation of electron-positron storage rings results from the electromagnetic forces which the bunches of one beam exert on the particles of the other beam during collision. An analytical description of the limitations imposed by this beam-beam interaction does not yet exist, although several attempts have been made at computer simulations[1,2,3] of the effect of a strong beam on a weak particle. Here the emphasis has been on the simulation of a real machine (strong-strong) with coupling mechanisms between longitudinal and transverse motion, aperture limitations, and the inclusion of perturbations to the superperiodicity.

A faster technique has been developed for the calculation of the beam-beam 'kick'. This involves interpolation between tabulated values of the complex error function and allows re-evaluation of the beam-beam strength after each collision.

2. Simulation Technique

The initial distributions of a large number of particles (typically 200) in the three phase planes are random with pre-specified variances. Each particle in each beam is 'tracked' through (i) an RF cavity, (ii) a beam-beam interaction and (iii) a traversal of a machine arc. This procedure is repeated until each beam has completed one turn. The position of each particle is then compared with aperture limitations (typically 10 σ) and those particles which fall outside are excluded from further tracking. The remaining particles are then used to recalculate the beam current, the specific luminosity, the beam variances and hence the new beam-beam kick parameters. This cycle is repeated until the 'beam' has been circulating for about 1.5 damping times.

2.1 RF Cavity

The energy gain in traversing an RF cavity is

$$\Delta E_{RF} = \frac{eV(\phi,\phi_s)}{n_{RF}} \tag{1}$$

where $V(\phi,\phi_s)$ is the voltage seen by a particle at ϕ; for the normal case

$$V(\phi,\phi_s) = \hat{V}\left(\sin(\phi+\phi_s) - \sin\phi_s\right) \tag{2}$$

Each RF station may have a different $V(\phi, \phi_s)$. The change in betatron amplitude (horizontal or vertical) is

$$\Delta y_{RF} = - D_{yRF}\frac{\Delta E_{RF}}{E_s} \tag{3}$$

where D_{RF} is the dispersion and may be different in each station,
E_s is the energy of the synchronous particle.

2.2 Beam-beam

The integrated kick given to a particle from a three-dimensional Gaussian beam is (here the vertical equations are given, the horizontal are similar):

$$\Delta Z'_{bb} = \frac{2Nr_e}{\gamma}\frac{Z}{\sqrt{\pi}}\int_{-\infty}^{\infty}\int_{0}^{\infty}\frac{\exp\left\{-\frac{x^2}{2\sigma_x^2+t}-\frac{z^2}{2\sigma_z^2+t}-\frac{s^2}{2\sigma_s^2+t}\right\}ds\,dt}{(2\sigma_x^2+t)^{\frac{1}{2}}\cdot(2\sigma_z^2+t)^{\frac{3}{2}}\cdot(2\sigma_s^2+t)^{\frac{1}{2}}} \tag{4}$$

Equation (4) was evaluated numerically and compared with the two-dimensional equation[4]

$$\Delta Z'_{bb} = \frac{Nr_e}{\gamma}\sqrt{\frac{2\pi}{(\sigma_x^2-\sigma_z^2)}}\,\mathcal{R}\left\{w\left(\frac{x+iz}{\sqrt{2(\sigma_x^2-\sigma_z^2)}}\right) - \exp\left(-\frac{x^2}{2\sigma_x^2}-\frac{z^2}{2\sigma_z^2}\right) \cdot w\left(\frac{x\frac{\sigma_z}{\sigma_x}+iz\frac{\sigma_x}{\sigma_z}}{\sqrt{2(\sigma_x^2-\sigma_z^2)}}\right)\right\} \tag{5}$$

where $w(x+iz)$ is the complex error function.

It was found that for all reasonable accelerator parameters equation (5) was accurate to around .2%. In order to speed up the calculation of the beam-beam kick, a table of values of $w(x+iz)$ is computed using an accurate algorithm[5]. During the simulation the $w(x+iz)$ is evaluated by fast interpolation[6] between the tabulated values. The accuracy of the interpolation procedure is better than 10^{-3}. For particle positions which fall outside the range of tabulated values the accurate algorithm[5] is used.

In equation (5) x and z are the displacements between the particle and the centre of gravity of the other beam and are given by

$$y = y_\beta \pm y_s + D_{yi}\frac{\delta E}{E_s} \tag{6}$$

where y_β is the betatron displacement
y_s is the separation between the centres of the beams
and D_{yi} is the dispersion at the interaction point.

Both y_s and D_{yi} may be different in all interactions.

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2.3 Transition between Collisions

In the absence of synchrotron motion the transition matrix is given by

$$e^{-\frac{t_s}{\tau_y}} \begin{pmatrix} \cos\mu_y & \beta^*_y \sin\mu_y \\ -\frac{\sin\mu_y}{\beta^*_y} & \cos\mu_y \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \qquad (7)$$

where t_s is the time between collisions and τ_y is the transverse damping time, (μ_y may be different between each interaction point).

In the presence of synchrotron motion the μ_y is modulated at synchrotron frequency by a non-zero chromaticity and the point of interaction between a bunch and an off-energy particle is displaced by

$$\Delta\ell_n = \Delta\ell_{n-1} - \frac{C}{4\gamma_t{}^2 k_b}\frac{\delta E}{E_s} \qquad (8)$$

where subscript n denotes the number of the collision. The transition matrix between collisions is then

$$\begin{pmatrix} T_{11}+\Delta\ell_n T_{21} & T_{12}-\Delta\ell_{n-1} T_{11}+\Delta\ell_n \left(T_{22}-\Delta\ell_{n-1} T_{21}\right) \\ T_{21} & T_{22}-\Delta\ell_{n-1} T_{21} \end{pmatrix} \qquad (9)$$

Longitudinally the transition of a machine arc is accompanied by an energy loss and a phase change given by

$$\Delta E = a_1 \delta E + a_2 \delta E^2 + a_3 \delta E^3 + \ldots \qquad (10)$$

$$\Delta\phi = \frac{\pi h}{\gamma_t{}^2 k_b}\frac{\delta E}{E_s} \qquad (11)$$

In the absence of non-linear losses all coefficients except a_1 in equation (10) are zero. For the study of higher order wigglers[7] these coefficients may be evaluated and 'switched on'.

2.3.1 Quantum excitation. Quantum excitation is simulated by adding a random number (Gaussian distribution with a mean of zero and a given variance) to the y co-ordinates once per traversal of an arc. The rms values for the distributions are

$$\sigma^2_{yrad} = \frac{4.\sigma^2_{yss}\, t_s}{\tau_y} \qquad (12)$$

where σ_{yss} is the steady state (unperturbed) beam size in x, z or δE and calculated from the radiation integrals of the machine lattice.

3. Discussion of Results

Based on survey errors[8] and other considerations a set of superperiod perturbations was calculated (see Table 1). Unless otherwise stated, other perturbations were set to zero. The perturbations for each interaction were generated randomly with a mean of zero and the variances specified in Table 1. Aperture limits were arbitrarily set at ten rms beam radii in all three phase planes. Vertically the beam radius was taken to be the value obtained from 'optimum' coupling; in reality the real aperture will be somewhat larger. Other parameters were taken from the LEP design[9] at design energy with the high luminosity insertions. Many simulation runs have been performed for LEP; in this section

Table 1 : Perturbations

Parameter	σ horiz.	σ vert.
phase advance/2π	.047	.047
beam separations	$\frac{\sigma_{xo}}{200}$	$\frac{\sigma_{zo}}{20}$
dispersion in interactions (m)	.08	.005

due to space limitations it is only possible to illustrate a few of the most interesting results.

In Fig. 1 the beam-beam was simulated with super-

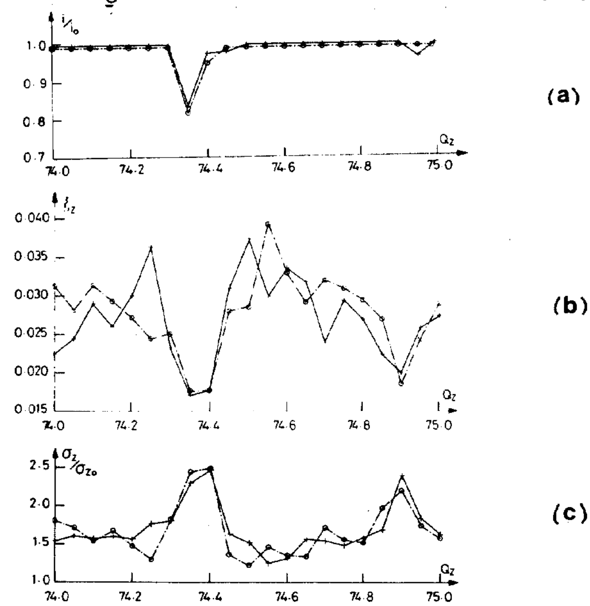

Fig. 1 : Dependence on Q_{zo}; errors in; long. modulation out. (Q_{xo} = 70.31)

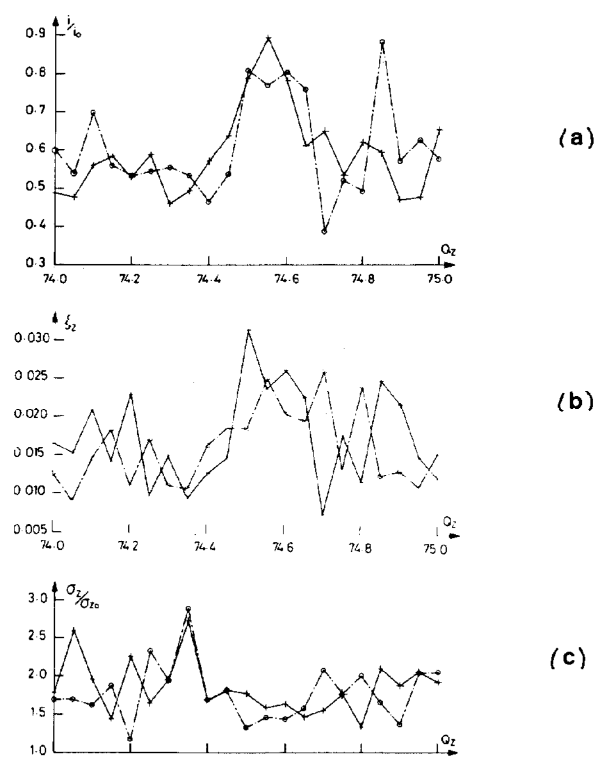

Fig. 2 : Dependence on Q_{zo}; errors in; long. mod. in.

period perturbations included but without longitudinal modulation of the collision point. Each trace represents one of the two beams. A loss of particles is apparent at a Q value of 74.35 corresponding to a coupling resonance[10] driven by the beam-beam. The maximum obtained value of the beam-beam strength parameter ξ_z is around .03 (Fig. 1b), whereas the unperturbed value is .06. The vertical beam size σ_z (Fig. 1c) is seen to be increased a factor of $\simeq 1.5$ in general and $\simeq 2.5$ at the beam-beam coupling resonance. Fig. 2 shows a run with realistic parameters for LEP. The calculated 'errors' and the longitudinal modulation motion are included. It can be seen (Fig. 2a) that there is loss of particles at all tune values in the integer range. The losses in this case occurred against the vertical aperture limitation. Fig. 2b shows that the maximum ξ_z is around .025. Fig. 2c shows an increase in σ_z of about a factor of 2. This increase would of course be greater and the particle losses less had the vertical aperture limitations been retracted to say 20 σ_z.

In Fig. 3 the proposed higher harmonic cavity[11] was switched on. It is clear that the situation is not improved. In fact the non-linear synchrotron motion seems to increase the effect due to beam-beam synchrobetatron resonances.

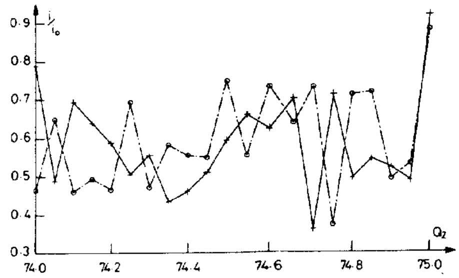

Fig. 3 : Dependence on Q_{zo}; bunch lengthening cavity on; errors in; long. mod. in

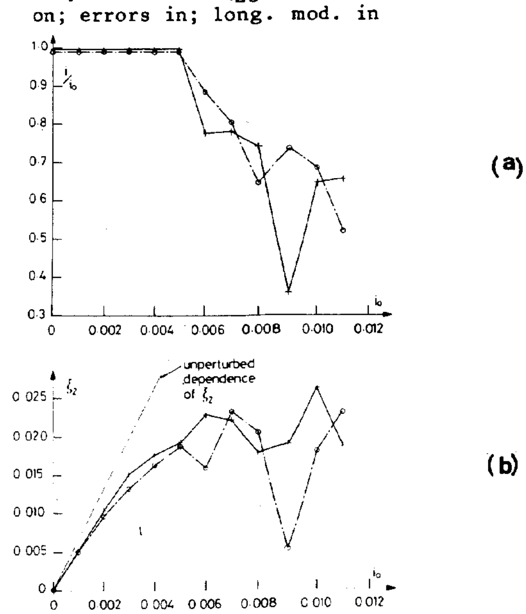

Fig. 4 : Dependence on beam current; errors in; long. mod. in (Q_{xo} = 70.31 Q_{zo} = 74.54)

For the next series of runs the vertical tune value was fixed and the intensity per beam was increased.

Fig. 4 shows the simulation results for a realistic LEP. It is clear from Fig. 4a that above about one half of the design current (9.1 mA), beam losses occur. Fig. 4b shows that the ξ_z does not increase linearly and above a certain value even tends to saturate. For this particular tune value (which from previous runs was the best tune value in the integer range) the beam-beam limit sets in at around .020.

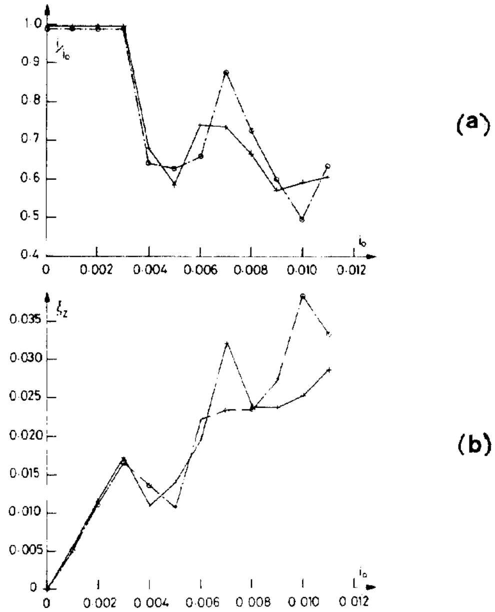

Fig. 5 : Dependence on beam current; errors out; long. mod in (Q_{xo} = 70.31 Q_{zo} = 74.54)

In Fig. 5 all machine errors were switched off, otherwise the conditions are identical to Fig. 4 conditions. It is clear that even for this 'perfect' machine the situation is not greatly improved, in fact from the current loss viewpoint there is a deterioration. In Fig. 5b it can be seen that ξ_z increases more linearly until .017 and then decreases due to current losses. A retraction of the vertical aperture limitation would certainly allow ξ_z to increase somewhat more.

4. Conclusions

Realistic results seem to be produced by simulation of the beam-beam effect provided all known phenomena are included.

For the range of parameters tested until now the simulation predicts a maximum beam-beam tune shift of around .025 for the LEP machine. This is significantly below the .060 design value. The simulation technique must now be used to search for regions which allow an increased ξ_z and for ways of suppressing the elements contributing to a reduction in the beam-beam limit.

5. References

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