IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

# ELECTRON ACCELERATION IN A HOMOGENIOUS DISC LOADED GUIDE WITH BEAM LOADING EFFECT

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## Abstract

The problem of steady state beam loading in electron linac guides is considered. Obtained solutions differ drastically from the results published in the other papers and books. The threshold currents which can be accelerated in electron linacs were defined. It is shown that the efficiency of an accelerating guide in steady state operation can't exceed 0.7 at the relative guide length $\xi = l/\lambda = 5$ and 0.83 at $\xi = 10$. The limiting charge of single bunch is obtained. It can't exceed 40 nC or $2.4 \times 10^{11}$ electrons/bunch for the 30 MW power generator at 2GHz and the disc loaded guide (DLG) with the parameter $a/\lambda = 0.20$.


## I. Introduction

The theory of electron acceleration in a disc loaded guide takes into account the beam loading effect was developed earlier.[1,2,3,4] But unfortunately it's not full and can't explain the following practical phenomena:

1. As a rule the calculations according to this theory differ from experimental results.
2. The theory[1] predicts the possibility of obtaining the large values of efficiency (up to about 100%) and accelerating currents. But experimental values of efficiency are small particularly in multy-section linacs. As for the currents they don't exceed several ampers even at high values of RF power.
3. The existing theoretical analysis and sofisticated mathematical models of beam unstability in high current linacs can't fully explain the beam break-up phenomena[5].

These factors stimulated the author to undertake the following investigations.

## II. Basic equations and assumptions

We consider the problem of electron in linac section with assumption that the disc-loaded guide has constant dimensions, electrons interact with the operating mode only, longitudinal motion of electrons is taken into account, beam bunches is supposed to be infinitesimal, only relativistic case is considered, there is no slip between electrons and the wave, the bunches are placed in the crest of the wave.

The power equilibrium equation can be written down in the following form[1]

$$\frac{dP}{dz} = -2\alpha P - \frac{1}{2} I_1 E \;, \qquad (1)$$

where P is the generator power; E is the accelerating field amplitude on the axis of the disc loaded guide; $\alpha$ is the attenuation constant; z is a longitudinal coordinate; $I_1$ is the amplitude of the first current harmonique. $I_1 = 2q/T$; q is the bunch charge; T is the RF oscilation period. The disc loaded series impedence can be represented in the following form[4]

$$R_s = E^2/2P \qquad (2)$$

Determine the solution of the equation (1) taking into account (2).

$$E = E_{in} e^{-\alpha z} - (I_1 R_s/2\alpha)(1 - e^{-\alpha z}). \qquad (3)$$

In the above mentioned works it was assumed that at the section input only the generator created field acted on the bunches and the radiation field of bunches is equal zero

$$E_{in} = E_{go} \,. \qquad (4)$$

In general case the last assumption

is not valid even for the first bunch entering the section.

## III. Initial conditions for a steady state operation.

We suppose that the disc-loaded guide is matched to the feeding rectangular guide by means of a coupler. We consider the case, when $E_{go}=0$. Determine the momentary power radiated by bunch with the charge q and velocity v

$$P_{mom}= qvE_r \qquad (5)$$

The average power P is equal one half the momentary power $P_{mom}$. Substituting P from (2) in (5) we get the radiation field

$$E_{roDLG}= (1/4)\ I_1 R_s \lambda . \qquad (6)$$

If the coupler of DLG is the cavity with the length $L_c=L_{DLG}$ the radiation field of the coupler at the entrance of DLG is equal

$$E_{roc}=(1/4)\ M_1 I_1 R_s \lambda \quad , \qquad (7)$$

where $M_1=\sin(\omega\, t_c/2)/(\omega\, t_c/2) \cong 1$, (8) $t_c$ - the time of bunch motion in the coupler. We can write that

$$E_{ro}= E_{roDLG}+ E_{roc}= \tfrac{1}{2} I_1 R_s \lambda \qquad (9)$$

and

$$E_{in}= E_{go}- E_{ro}= E_{go}- \tfrac{1}{2} I_1 R_s \lambda \ . \qquad (10)$$

## IV. Accelerating field, energy and efficiency

Using (3) and (10) one can express the field which acts on the accelerated bunches

$$E=E_{go} e^{-\alpha z}-I_1 R_s \lambda\, \frac{1-(1-\alpha\lambda)e^{-\alpha z}}{2\alpha\lambda} \qquad (11)$$

or

$$E=E_{go}\ e^{-\alpha z}-m_1 \frac{1-(1-\alpha\lambda)e^{-\alpha z}}{\alpha\lambda} \quad , \qquad (12)$$

where

$$m_1=\frac{I_1 R_s \lambda}{2E_{go}} = \frac{I\ E_{go}\lambda}{2\ P_{go}} \ . \qquad (13)$$

The $m_1$ is the beam loading parameter and the I is the beam current, $I=I_1/2=q/T$. The energy gain in a DLG with the length $z=l$ is expressed as

$$V=E_{go}\lambda\left[\frac{1-e^{-\alpha l}}{\alpha\lambda} - m_1\frac{\alpha l-(1-\alpha\lambda)(1-e^{-\alpha l})}{(\alpha\lambda)^2}\right]. \qquad (14)$$

The efficiency of a homogenious DLG

$$\eta=2m_1\left[\frac{1-e^{-\alpha l}}{\alpha\lambda} - m_1\frac{\alpha l-(1-\alpha\lambda)(1-e^{-\alpha l})}{(\alpha\lambda)^2}\right]. \qquad (15)$$

We can determine $m_1$ for the maximal length $z=l_m$ at which $E=0$, $V=V_{max}$, $\eta=\eta_{max}$. These conditions correspond to the value

$$m_1=\frac{e^{-\alpha l_m}}{1-(1-\alpha\lambda)e^{-\alpha l_m}} \qquad (16)$$

For steady state regime $m_1 \leq 1$. The case $m_1=1$ corresponds to such regime when acceleration is absent even at the input of a DLG. In other words it corresponds to the threshold current of the bunches at z=0 when the radiation field is equal to the field created by the microwave generator. Determine the $V_{max}$ and $\eta_{max}$ from the parameters $\alpha$, $\lambda$ and $l_m$ using (14), (15) and (16).

The maximal energy is

$$V_{max}=E_{go}\lambda\, \frac{1-(1+\alpha l_m)\ e^{-\alpha l_m}}{\left[1-(1-\alpha\lambda)\ e^{-\alpha l_m}\right](\alpha\lambda)} \ . \qquad (17)$$

The maximal efficiency is

$$\eta_{max}= 2e^{-\alpha l_m}\frac{\left[1-(1+\alpha l_m)\ e^{-\alpha l_m}\right]}{\left[1-(1-\alpha\lambda)\ e^{-\alpha l_m}\right]^2} \ . \qquad (18)$$

## V. Discussion

From the expressions (13) and (16) we determine the limit current $i_L$ at the input cross-section of a DLG for $m_1=1$

$$i_L= \frac{I_L}{\sqrt{P_{go}}} = 2\left(\frac{E_{go}\lambda}{\sqrt{P_{go}}}\right)^{-1} = F(a/\lambda\ ,\ t/\lambda\ ,\theta), \qquad (19)$$

where $a$ - the radius of a disc diaphragm, $t$ - the disc thickness, $\theta$ - the phase shift per cell.

In Fig.1 the current $i_L$ for the case $\theta=\pi/2$ and $t/\lambda=0.038$ versus the parameter $a/\lambda$ is shown. It's calculated using the data of the DLG Handbook[6].

In Fig.2 the curves of the maximal efficiency versus $\xi = l/\lambda$ at fixed $\alpha\lambda$ are shown.

In Fig.3 the curves of $m_1$ corresponding the maximal energy and maximal efficiency versus $\alpha\lambda$ at fixed $\xi$ are drawn. They are calculated using (16) and (18) at z=1, when the accelerating field is equal to zero.

Now we estimate the limiting charge of a bunch being accelerated in a linac guide. Here we take into account only the lower passband of a DLG. The limiting charge could be determined from the condition that the radiation field at the input of DLG is equal to the generator field.

$$q_L = 2\frac{\sqrt{P_{go}}}{f}\left(\frac{E_{go}\lambda}{\sqrt{P_{go}}}\right)^{-1}. \qquad (20)$$

For $t/\lambda = 0.038$, $\theta = \pi/2$ and $\beta_W = 1$ we have that $E_{go}\lambda/\sqrt{P_{go}} = 5.6(a/\lambda)^{-2}$. The expression for $q_L$ can be written in the following form

$$q_L = \frac{\sqrt{P_{go}}}{2.8\ f}\left(\frac{a}{\lambda}\right)^2 \cdot 10^{-6}, \qquad (21)$$

where $q_L$ - the limiting charge of the bunch, C; f - the operating frequency, MHz; $P_{go}$ - the generator power, MW.

Using the expression (21) bunch charges in terms of an electron charge ($q_e$) were calculated for different loading parameters $a/\lambda$. The results are summarised in the table for f=2GHz and $P_{go}$=30MW.

Table

| $a/\lambda$ | 0.10 | 0.141 | 0.173 | 0.20 |
|---|---|---|---|---|
| $N = q/q_e$ | $6\times10^{10}$ | $1.2\times10^{11}$ | $1.8\times10^{11}$ | $2.4\times10^{11}$ |

If we consider these results in connection with the RF oscillations period we can see that the limiting "current" of a single bunch lies in the range from 20 to 70 A.

## VI. Conclusion

The obtained results could be of interest in desingning of the accelerating guides for storage ring, for superconducting electron linacs and linear colliding electron-positron beams installations. The conventional electron linacs operate in the pulsed regime, so one have to take into consideration the transients duration of which is about $10^3$ periods of RF oscillations. This problem is considered in the Burshtein-Voskresensky book[5].

The results of this paper can be applied to other cases of linac DLG constructions.


## Acknowledgements

The author would like to thank professor O.Valdner for the discussions of this work. He gratefully acknowledges the helpful discussion and critical remarks by professor A.Shalnov and doctors N.Nechaev, Iu.Mayorov, V.Volodin and V.Vikulov.


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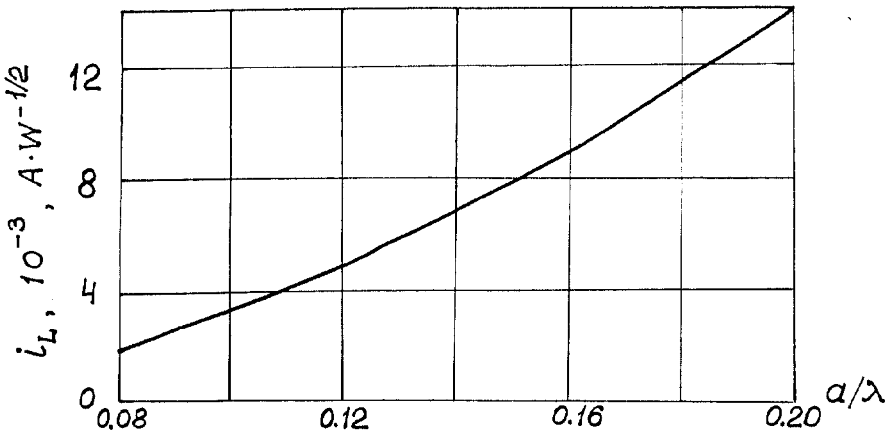


Fig.1. Limiting current $i_L$ vs $a/\lambda$.

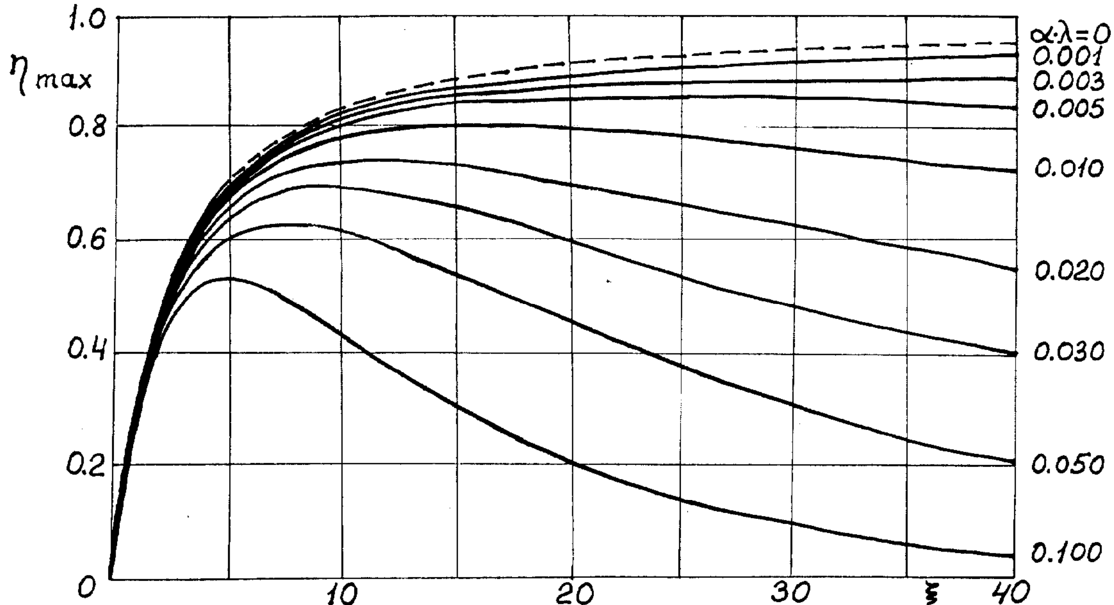


Fig.2. Maximal efficiency $\eta_{max}$ vs $\xi$ at fixed $\alpha\lambda$.

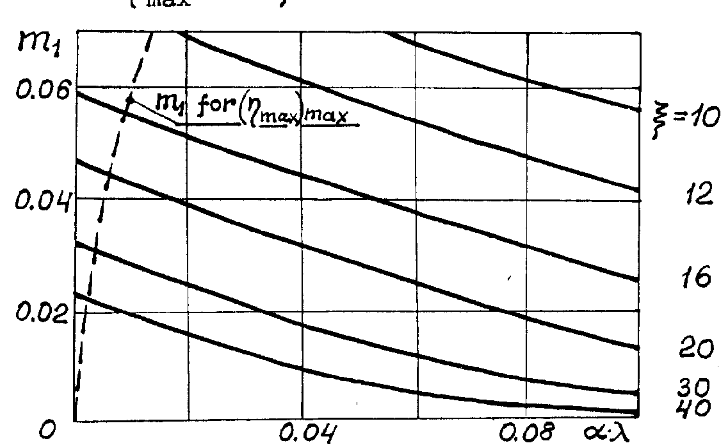


Fig.3. Parameter $m_1$ vs $\alpha\lambda$ at fixed $\eta_{max}$ and $\xi$.