© 1981 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

PERIODIC APPROACH TO SPACE CHARGE LIMIT IN PROTON LINEAR ACCELERATORS

S. Inagaki

National Laboratory for High Energy Physics
Oho-machi, Tsukuba-gun, Ibaraki-ken, 305, JAPAN

Summary

The analytical approach to space charge effect in drift tube linacs has been mainly divoted to estimating how the potential well is filled up by some given distribution. This paper describes a different analytical approach to this problem. Eventhough the structure of a drift tube linac is not uniform geometically, it is considered periodic from the rf point of view. Thus the equation of oscillation can be expressed in the same way as the AG focusing theory, longitudinally as well as transversely. This matrix treatment allows closer approximation of the physical gaps and drift spaces where the external field to the beam is so different. The results so obtained yields somewhat higher limits than those obtained by conventional method for drift tube linacs. The upper limits of the current for KEK proton linac and PIGMI APF structure predicted by this theory are discribed.

Introduction

In the conventional ways to treat the space charge effect in proton linear accelerators, the equation of the longitudinal oscillation

$$\frac{d}{dz}\left[\gamma_s^3\beta_s^3\frac{d}{dz}(\psi-\psi_s)\right]=-\frac{2\pi}{\lambda}\cdot\frac{e}{mc^2}\left[E_z(r,z,t)-E_z(r,z,t_s)\right] \qquad (1)$$

is rewritten as follows.

$$\frac{d}{dz}\left[\gamma_s^3\beta_s^3\frac{d}{dz}(\psi-\psi_s)\right]=\frac{2\pi}{\lambda}\cdot\frac{eE_0T}{m_0c^2}(1-S)\sin\psi_s\cdot(\psi-\psi_s), \qquad (2)$$

where the subscript s denotes the synchronous particle. E_0 is the average electric field in a cell, T is the transit time factor. Vlasov[1] expressed the space charge term S by

$$S=\frac{3\eta_0\lambda I M_z}{4\pi a_x a_y E_0 T\sin\psi_s}\cdot\frac{1}{(\psi-\psi_s)_{max}}, \qquad (3)$$

where

$$M_u=\frac{a_x a_y a_z}{2}\int_0^\infty\frac{ds}{(a_u^2+s)\sqrt{(a_x^2+s)(a_y^2+s)(a_z^2+s)}} \qquad (4)$$

$\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 120\,\pi\Omega$, a_u (u = x, y, z) is the half axis of the ellipsoid and I is the current. Eq. 2 means the decrease of the stable phase region by the factor of (1-S) and most papers[2,3,4] describe the value of S or the shift of stale phase due to space charge.

In the above formalism the space charge term is put into a single averaged equation. However, in the strong focusing theory it is essential that the periodical comfiguration of focusing and defocusing elements results in a net focusing over a period. In drift tube linacs the effect of external force to the space charge is so different in drift tubes and gaps. Eventhough the structure is not periodic geometrically, it is considered periodic from the rf point of view. Thus the equation of longitudinal oscillation can be expressed in the same way as the AG focusing theory.

Longitudinal Space Charge Effect

In order to treat the longitudinal motion periodically we define $d\ell$ to be the number of rf cycles needed by the synchronous particle to traverse the distance dz,

$$dz = v_s\,dt_s = \beta_s\lambda d\ell \quad . \qquad (5)$$

Instead of using a single equation for an averaged field in a cell, we use two separate equations, one in a gap and the other in a drift tube.

$$\frac{d^2\chi}{d\ell^2} = -\,n(\ell)\chi\ , \qquad (6)$$

$$n(\ell) = \begin{cases} n_1 = (1-S')F\ , & \text{in a gap} \\ n_2 = -S'F\ . & \text{in a drift space}\end{cases} \qquad (7)$$

F and S' are defined as follows

$$F = -\frac{2\pi e\lambda E_m}{m_0c^2\gamma_s^3\beta_s}\sin\psi_s\ , \qquad (8)$$

$$S' = \frac{E_0T}{E_m}S\quad , \qquad (9)$$

where E_m is the average field in a gap, $\chi = (\psi - \psi_s)$ and the approximation is made that the β_s is constant in Eq. 2.

If a linac consists of gaps with rf cylces ℓ_1 and drift tubes with rf cycles ℓ_2, the transform matrix $M = M_1M_2$ gives the stability condition as

$$-1 < 1/2\ \mathrm{Tr}M < 1\quad , \qquad (10)$$

where M_1 and M_2 are defined as follows.

$$M_1(\ell_1) = \begin{bmatrix}\cos\alpha & (n_1)^{-1/2}\sin\alpha \\ -n_1^{1/2}\sin\alpha & \cos\alpha\end{bmatrix}, \text{ in a gap}$$

$$M_2(\ell_2) = \begin{bmatrix}\cosh\beta & (-n_2)^{-1/2}\sinh\beta \\ (-n_2)^{1/2}\sinh\beta & \cosh\beta\end{bmatrix}, \qquad (11)$$

in a drift tube

where $\alpha = (n_1)^{1/2}\ell_1$ and $\beta = (-n_2)^{1/2}\ell_2$. (12)

For a conventional linac ($\ell_1 + \ell_2 = 1$), ℓ_1 changes from

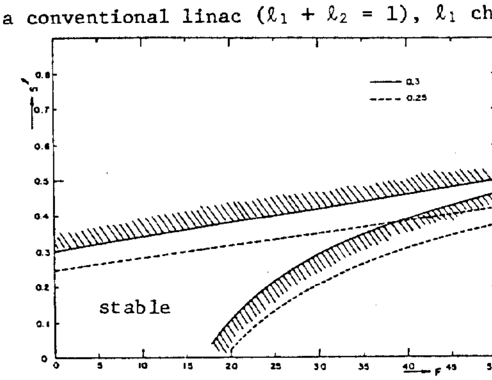

Fig. 1. Stability diagram for ℓ_1 = 0.3 or 0.25.

0018-9499/81/0600-3501$00.75© 1981 IEEE

0.25 to 0.3. Fig. 1 shows the stable region for ℓ_1 = 0.25 and 0.3. As is known from the figure, there exist a lower current limit as well as an upper one. In other word a certain amount of space charge is necessary to compensate for a too strong field. If we take the proton machine excited by 200 MHz and designed by ψ_s = 30° and E_m = 4 MV/m, then F = 0.501. If ax = ay = 5×10^{-3} m, and $(\psi - \psi_s)$max = 60°, the maximum current would be 349 mA.

Application to APF Structure

In the above argument, the space charge term S' is supposed to be constant through a cell. S' may change in the gap and the drift tube or there may be damping of the oscillation. Or one may say that at the condition where this theory is applied there may be other instabilities of the beam.

However above formalism has another application to an APF structure, in which the beam experiences the longitudinally focusing and defocusing region alternately. By the conventional way, it is impossible even to find the phase stable region.

Let us take the case where the superperiod consists of two gaps and two drift spaces ($\ell_1 + \ell_2 + \ell_3 + \ell_4 = 2$), with S' = 0. The four indices become as follows.

$$n(\ell) = \begin{cases} n_1 = F & \text{in the first gap } (\ell_1) \\ n_2 = 0 & \text{in the first drift space } (\ell_2) \\ n_3 = GF & \text{in the second gap } (\ell_3) \\ n_4 = 0 & \text{in the second drift space } (\ell_4) \end{cases} \quad (13)$$

and

$$G = \frac{E_{m3} \sin\psi_{s3}}{E_{m1} \sin\psi_{s1}} \quad . \quad (14)$$

Fig. 2 shows F vs. (-G) diagram for ℓ_1 = 0.35, ℓ_2 = 1.05, ℓ_3 = 0.35 and ℓ_4 = 0.25.

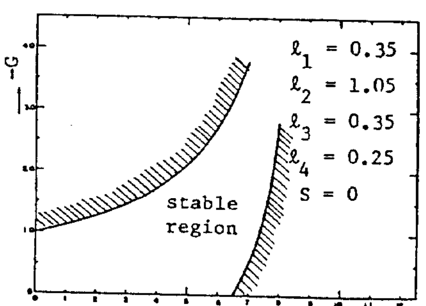

Fig.2 Stability diagram for F vs. (-G)

Next we include the space charge term for G = - 1 ($\psi_{s3} = -\psi_{s1}$). The indices are

$$n(\ell) = \begin{cases} n_1 = (1 - S')F & \text{in } \ell_1 \\ n_2 = -S'F & \text{in } \ell_2 \\ n_3 = (-1 - S')F & \text{in } \ell_3 \\ n_4 = -S'F & \text{in } \ell_4 \end{cases} \quad (15)$$

Fig. 3 shows F vs. S' stable region for ψ_s = - 30°, - 60° and - 80°. The upper line of S' gives a limitation on the space-charge effect.

If the injection energy is 250 keV, the frequency is 450 MHz, and the average electric field is 6 MV/m, then F = 0.5794, 1.0036 and 1.1452 for ψ_s = - 30°, - 60° and - 80° respectively. For ψ_s = - 80°, $a_x = a_y = 2.5 \times 10^{-3}$ m and Mz = 1/3, the upper current limit becomes 77.4 mA.

PIGMI APF[5] structure has seven supperperiod and each superperiod consists of four gaps and four drift tubes ($\Sigma \ell i = 4$). The synchronous phase of the first gap decreases by 2° in every superperiod (Table I).

C	CL	GL	DL	VS	PHI	T	$\bar{E}$	E_m	G	L
0				0.250						
			0.465							
1	1.853	0.342		0.258	-78.00	0.341	6.089	7.099	1.	0.339
			1.696							1.039
2	1.936	0.365		0.265	80.00	0.336	6.197	7.135	-1.012	0.353
			1.049							0.656
3	1.231	0.362		0.280	64.00	0.477	5.827	9.452	-1.223	0.224
			0.690							0.431
4	1.282	0.376		0.298	-62.00	0.491	6.091	10.197	1.297	0.235
			1.125							0.703
5	2.082	0.601		0.309	-76.00	0.382	5.317	7.491	1.	0.341
			1.838							1.042
6	2.110	0.607		0.320	78.00	0.387	6.479	8.716	-1.173	0.344
			1.168							0.662
7	1.361	0.396		0.334	63.60	0.510	5.432	10.223	-1.260	0.224
			0.763							0.432
8	1.417	0.411		0.359	-61.60	0.519	6.004	10.743	1.300	0.233
			1.251							0.709
9	2.260	0.646		0.375	-74.00	0.421	6.047	8.986	1.	0.332
			2.018							1.037
10	2.316	0.654		0.389	76.00	0.430	5.811	8.849	-0.994	0.336
			1.307							0.672
11	1.512	0.435		0.412	63.20	0.344	6.202	11.727	-1.212	0.224
			0.850							0.437
12	1.373	0.450		0.437	-61.20	0.557	5.913	11.532	1.170	0.231
			1.400							0.720
13	2.509	0.696		0.459	-72.00	0.470	6.037	10.229	1.	0.323
			2.228							1.034
14	2.555	0.705		0.479	74.00	0.478	5.961	10.292	-1.017	0.327
			1.472							0.683
15	1.688	0.477		0.506	62.80	0.581	6.023	12.383	-1.132	0.221
			0.933							0.442
16	1.754	0.492		0.536	-60.80	0.593	5.912	12.499	1.122	0.228
			1.574							0.730
17	2.771	0.750		0.566	-70.00	0.519	6.099	11.695	1.	0.323
			2.470							1.031
18	2.829	0.762		0.594	72.00	0.530	6.043	11.891	-1.029	0.318
			1.666							0.695
19	1.891	0.521		0.626	62.40	0.621	5.862	13.257	-1.069	0.217
			1.077							0.449
20	1.958	0.535		0.663	-60.40	0.630	6.073	14.001	1.108	0.223
			1.772							0.739
21	3.073	0.823		0.701	-68.00	0.556	5.937	12.326	1.0	0.308
			2.729							1.023
22	3.146	0.843		0.738	70.00	0.563	6.108	12.833	-1.055	0.316
			1.875							0.703
23	2.114	0.567		0.777	62.00	0.655	5.999	14.631	-1.132	0.213
			1.220							0.457
24	2.182	0.578		0.820	-60.00	0.663	5.945	14.879	1.127	0.217
			1.991							0.746
25	3.412	0.916		0.868	-66.00	0.582	5.943	12.883	1.0	0.309
			3.003							1.012
26	3.499	0.942		0.916	68.00	0.585	5.999	13.036	-1.027	0.313
			2.116							0.713
27	2.356	0.609		0.960	61.60	0.684	6.002	15.682	-1.187	0.205
			1.385							0.467
28	2.424	0.662		1.011	-59.40	0.692	6.008	16.303	1.187	0.210
			1.224							0.750

Table I PIGMI APF structure

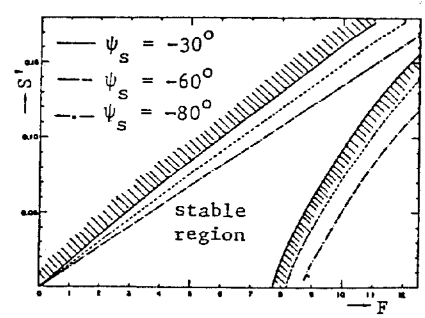

Fig.3 Stable region of FODO structure

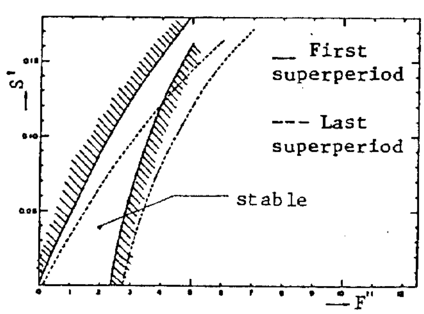

Fig.4 Stable region of FODODOFO structure

Thus the stability region should be located between the value of the first and the last superperiod (Fig. 4). Compared to a simple FODO structure in Fig. 3, the configurations of FODODOFO and a strong electric field increases the upper S' limit remarkably. In the last cell with F equal to 1.192, S' = 0.039 and the current is 220 mA for $(\psi - \psi_s)$max = 2 × 60°.

Transverse Space Charge Effect

The equation of transverse oscillation is given by

$$\beta\frac{d}{dz}(\gamma\beta\frac{dr}{dz}) = \frac{e}{m_0c^2}(E_r - c\beta B_\theta). \tag{16}$$

If we neglect the magnetic term and make approximation for $\partial Ez/\partial z$ and ρ by taking their values on the axis, Eq. 16 becomes

$$\beta\frac{d}{dz}(\gamma\beta\frac{dr}{dz}) = \frac{-er}{2m_0c^2}[\frac{\partial E_z(0,z,t)}{\partial z} - \frac{\rho(0,z,t)}{\varepsilon}], \tag{17}$$

where ρ is the charge density and ε is the dielectric constant. By assuming a constant β, the transverse equation corresponding to Eq. 6 is obtained as

$$\frac{d^2r}{d\ell^2} = -n_t(\ell)r , \tag{18}$$

and

$$n_t(\ell) = (G_t - S'_t)F_t , \tag{19}$$

where F_t, G_t and S_t' are defined by

$$F_t = \frac{e}{2m_0c^2}\cdot\frac{\beta_s{}^2\lambda^2}{\gamma\beta^2}(\overline{\frac{\partial E_z}{\partial z}\cos\psi})_1 , \tag{20}$$

$$G_t = (\overline{\frac{\partial E_z}{\partial Z}\cos\psi})/(\overline{\frac{\partial E_z}{\partial z}\cos\psi})_1 \tag{21}$$

and

$$S_t' = \frac{\rho}{\varepsilon} / (\overline{\frac{\partial Ez}{\partial z}\cos\psi})_1 . \tag{22}$$

In these expressions the field gradient along z is replaced by the average value with some effective length at the exit and the entrance of the drift tubes. The recovering force of the transverse oscillation is proportional to the electric field gradient while that of the longitudinal oscillation is proportinal to the electric field.

Let us consider a simple APF structure in which a superperiod consists of two gaps and two drift tubes in two rf lengths ($\Sigma\ \ell i = 2$). We assume that $\overline{\partial Ez/\partial z}$ is symetrical relative to the gap center, the gap rf length ℓ is 0.25 and the effective length of $\partial Ez/\partial z$ is 0.2 (Fig. 5). For the synchronous phase of − 60° in the first gap, Fig. 6 shows the stable phase region (P_{s2} vs. S_t') at various transverse fource F_t.

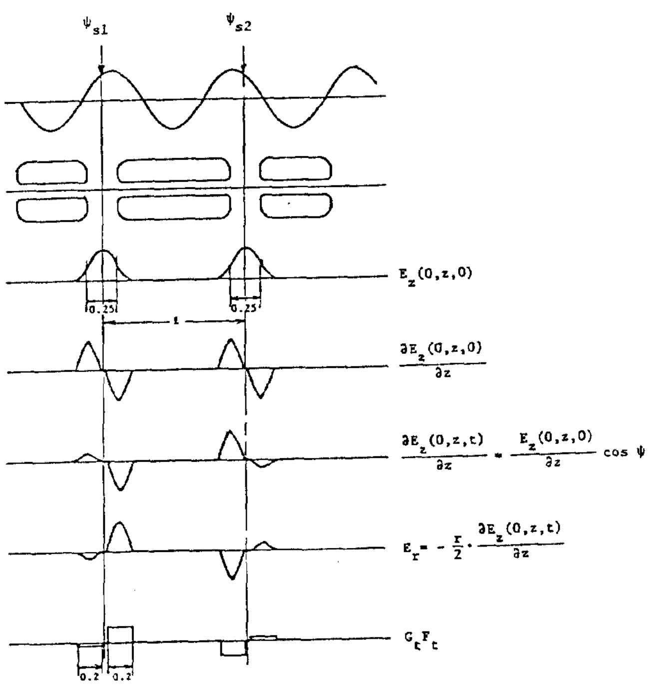

Fig.5 APF structure with two gaps

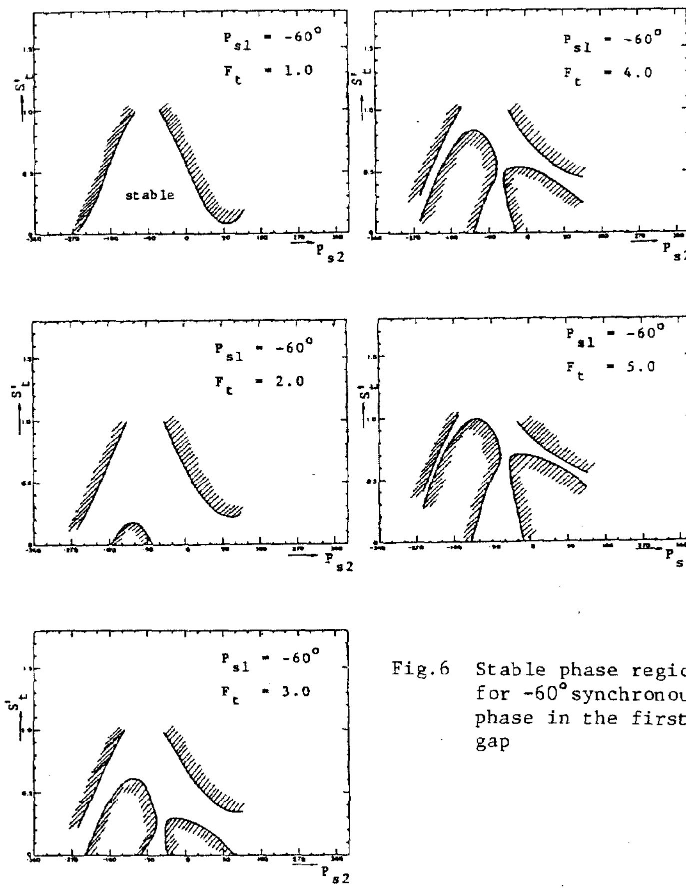

Fig.6 Stable phase region for -60° synchronous phase in the first gap

Conclusion

When the space charge term is supposed to be constant throughout a cell, it can be formulated as a longitudinal defocusing element. Eventhough the periodicity condition in a linac is not as stringent as that in a synchrotron, the above process gives upper and sometime lower current limit by stability condition. For the transverse motion the electric field drivatives at the exit and the entrance of the drift tube causes the focusing. It has also the applicability to an APF structure.

The current limit predicted by this theory has still higher value than the maximum current attaind in KEK.

Acknowledgements

The auther expresses his thanks to Prof. R. Gluckstern for the discussion and Dr. K. Crandall for reading the cnception in early stage.

References

1. A.D. Vlasov, NT2460-33 (1960).
2. R. Gluckstern, LA-3609, 237 (1966).
3. T. Nishikawa, SJC-A-67-1 (1967).
4. R. Gluckstern, Linear Accelerator, p.830 (1969).
5. D.A. Swenson, LASL, MP-14-2 (1977).