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BEAM DYNAMICS IN THE RADIO FREQUENCY QUADRUPOLE

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SUMMARY

A study of the beam dynamics in the Radio Frequency Quadrupole (RFQ) proposed by Kapchinskii[1] and Taployakov has been carried out. Different schemes to design the RFQ for a better transmission efficiency and lower emittance growth for an ion beam are explored. A three-dimensional particle code which includes the effect of the space charge force has been employed to follow the dynamics of a beam in the RFQ. Our calculations show that the RFQ is able to bunch a DC beam with good efficiency and accelerate it to high energies suitable for the injection into conventional linacs. The transmission efficiency of a beam has been studied as the current or the final energy of the beam increases. A comparison between our calculational results and those from recent experiments at Los Alamos are presented.

I. INTRODUCTION

In the conventional Linac, the radial focusing force for the beam is provided by periodically spaced magnetic quadrupoles. The magnetic force, which depends on the velocity of the particles, usually is quite weak at the initial stage of the accelerating scheme where the particles were gaining energy from 750 keV to 2 MeV. Yet it is recognized generally that the quality of the beam, for example its emittance is determined in this initial stage, and preserved through the energy range from 2 - 10 MeV. Therefore, it is desirable to employ an alternative focusing mechanism which will exert stronger focusing force to produce a beam of better quality.

The RFQ was first proposed by Kapchinskii[1] and Taplyakov in 1969. In the past two years or so, considerable effort has been put into the study of the RFQ in Los Alamos[2-6]. It is basically an electric quadrupole structure which provides continuous radial focusing and longitudinal accelerating forces for the beam. A potential field $V_o \mathrm{Sin}\,(wt+\phi)$ is applied on a four-vane resonator to produce the quadrupole electric field. The longitudinal field is obtained by periodic variations of the radius of the pole tip.

II. BEAM DYNAMICS

To study the beam dynamics in the RFQ, we have invoked the electric field given by Kapchiniskii and Taplyakov.[1] The potential function, to the lowest order, is

$$\phi = \frac{AV}{2}\cos(Kz)\, I_o\left(K\sqrt{x^2+y^2}\right) + \frac{BV}{2a^2}\left(x^2-y^2\right) \qquad (1)$$

$$\text{where } A = \frac{m^2-1}{m^2 I_o(Ka)+I_o(Kma)}, \quad B = 1 - A\, I_o(Ka) \qquad (2)$$

$$K = \frac{2\pi}{\beta_s \lambda}, \quad \lambda = 2\pi c/w \qquad (3)$$

$$V = V_o \sin(wt+\phi), \qquad (4)$$

with c denoting the speed of light, $\beta_s c$ the synchronizing velocity, w the R-F frequency, V_o the maximum potential amplitude across the vane, ϕ the phase angle, I_o the modified Bessel function, m and a parameters related to the vane structure.

The conventional procedure for particle simulation for linacs is to write the equations of motion in terms of the independent spatial variable z. However, in our application the current density is high enough that the space charge force needs to be included. We have chosen Z_s, the z coordinate of the synchronized particle, as the independent variable. It serves the purpose of giving us the positions of all the particles in the beam at the same instant, which is necessary for calculating the space charge force. In the mean time, it also enables us to keep track of the position of the beam relative to the structure of the accelerator.

The equations of motion are

$$\frac{d^2x}{dZ_s^2} + \frac{1}{\beta_s}\frac{d\beta_s}{dZ_s}\frac{dx}{dZ_s} = \frac{qx}{Mc^2\beta_s^2}\left[-\frac{AK^2}{4}\cos(Kz) - \frac{B}{a^2}\right]V_o \sin(wt+\phi) + \frac{qE_x}{Mc^2\beta_s^2}, \qquad (5)$$

$$\frac{d^2y}{dZ_s^2} + \frac{1}{\beta_s}\frac{d\beta_s}{dZ_s}\frac{dy}{dZ_s} = \frac{qy}{Mc^2\beta_s^2}\left[-\frac{AK^2}{4}\cos(Kz) + \frac{B}{a^2}\right]V_o \sin(wt+\phi) + \frac{qE_y}{Mc^2\beta_s^2}, \qquad (6)$$

$$\frac{d\beta_z}{dZ_s} = \frac{qAK}{2Mc^2\beta_s}\sin(Kz)\left[1 + \frac{K^2(x^2+y^2)}{4}\right]V_o \sin(wt+\phi) + \frac{qE_z}{Mc^2\beta_s}, \qquad (7)$$

where x-y-z denotes the coordinates of the particle in the Cartesian frame of reference, β_z is V_z/c, β_s is β_z for the synchronized particle, and $\vec{E}$ denotes the electric field due to space charge. Note that the Bessel functions in the potential field have been simplified for our particular applications by assuming $K\sqrt{(x^2+y^2)} \ll 1$.

III. DESIGN OF THE RFQ STRUCTURE

In order to minimize the radial emittance growth, it is important to have a matched beam in the RFQ. Because the energy of the beam increases along the z-axis, both the accelerating strength A and the focusing strength B as shown in Eq. (2) can be functions of z as well. Therefore the question of how to match the beam along the whole machine is quite complex. One way to simplify the problem is the following.

Defining $R_1 = \frac{qAK^2V_o}{4M}$ and $R_2 = \frac{qBV_o}{Ma^2}$, Eq. (5) then becomes

$$\frac{d^2x}{dZ_s^2} + \frac{1}{\beta_s}\frac{d\beta_s}{dZ_s}\frac{dx}{dZ_s} = \frac{1}{c^2\beta_s^2}\left[-R_1\cos(Kz) - R_2\right]\sin(wt+\phi) + \frac{qE_x}{Mc^2\beta_s^2}. \qquad (8)$$

One may require the machine designed in the parameter range such that $R_2 \gg R_1$ and R_2 = constant. So the transformation matrix will essentially be defined by R_2 only, and the same matched beam can be found for different cells along z.

Another factor that determines the efficiency of the machine is the particle loss. To minimize the

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particle loss due to the longitudinal phase mixing, one possible solution has been proposed by Kapchinskii and Taplyakov (K-T). In this method, the zero current longitudinal oscillation frequency and the length of the separatrix are kept as constants.

Los Alamos Scientific Laboratory (LASL)[5] has designed the RFQ structure for the proof-of-principle experiment in a somewhat different way. They have separated the structure into four sections, and only in one of the sections has the K-T method been invoked. The other sections are designed by keeping one or several of the variables such as R_2, A, ϕ, a, m as constants while varying the others linearly. We have examined both the proposed method by K-T and the modified method by LASL. We set up beams passing through the two schemes with exactly the same input condition. Both are matched for the same R_2 value, yet the emittance growth of the beam in LASL design is smaller than that by the K-T method.

IV. SIMULATION RESULTS

A. No Space Charge Case

In our simulation we first investigate the case where the space charge force was neglected. The parameters which define the structure of the RFQ are as follows: $B/a^2 = 25.4$, A: $0 \rightarrow 0.58$ m: $1 \rightarrow 2$, a: 0.2 cm $\rightarrow$ 0.12 cm, $\phi = -90^\circ \rightarrow -34^\circ$. The RFQ accelerates a proton beam of 100 KeV up to 2 MeV. The total length is 2.75 meters. The other parameters that were used in the simulation are V_o = 44 kV, and W = (425 MHz) $\cdot$ 2π. The emittance of the input beam is $\varepsilon_{xi} \sim 0.012$ marad-cm, $\varepsilon_{yi} \sim 0.011$ mrad-cm. The beam is assumed to have zero spread in V_z initially, thus $\varepsilon_{zi} \sim 0$. Throughout the whole RFQ, the emittance in the radial direction stays constant. The capture efficiency is about 95%.

B. With Space Charge

1. Calculation of Space Charge Force: Before presenting the results for the case that current $I \neq 0$, we need to discuss the method by which the space charge force is calculated. The electric field is calculated from the point-point summation method

$$\vec{E}_j = \sum_{i=1}^{N} \frac{(\vec{r}_j - \vec{r}_j)}{\left[|(\vec{r}_i-\vec{r}_j)|^2 + \Delta^2\right]^{3/2}} \tag{9}$$

where j stands for the j^{th} particle, N the total number of particles, and Δ, the cut-off distance between particles which serves the purpose of avoiding any divergence in the calculation. Since both the total charge and the volume of the beam are finite, N and Δ are interrelated. It has been found that the calculated emittance of a beam is larger when either Δ or N is smaller. Depending on the choice of N and Δ, the discrepancy in ε can be more than 40%. Therefore, it is essential to employ the correct method in the evaluation of the space charge force. Our understanding in this area is still not complete and more effort will be required on this problem.

In the calculation for the RFQ, the beam is started with a diameter $\sim$0.2 cm, and length of 1 cm. In the later stage of the acceleration, the beam diameter stays about the same, whereas the beam length due to bunching has been shrunk down to about 0.4 cm. Therefore, the field values in the beam should be somewhere between that obtained analytically from a infinite cylinder and a sphere.

We have used the analytical results from the infinite cylinder and the sphere as a guideline to determine the values of N and their corresponding Δ. Two calculations are carried out. One is for a beam assumed to be of length Z_{max} = 1 cm and radius R = 0.1 cm. Let the electric field for the i^{th} particle calculated from Eq. (9) be $\vec{E}_i$ and that from an infinite cylinder be $\vec{E}_i^{(c)}$. The deviation of the two results is measured by

$$\Delta\bar{E}_j = \frac{\sum_{i=1}^{N} \left(E_{ij} - E_{ij}^{(c)}\right)^2}{\sum_{i=1}^{N} \left(E_{ij}^{(c)}\right)^2} \tag{10}$$

where j = (x,y,z) stands for the three components of the field. The other calculation carried out is for a short beam with Z_{max} = 0.2 cm and radius R = 0.1 cm. This case is compared to the results gained from a spherical beam with radius 0.1 cm. Let Δ_1 be the value of Δ which gives the best results for the long beam with Z_{max} = 1 cm, such that $\Delta\bar{E}_x$ and $\Delta\bar{E}_y$ are minimum, and Δ_2 be the best value for the short beam case where Z_{max} = 0.2 cm. Since these two cases considered are the extreme conditions for our beam in the RFQ, we can take Δ_1 and Δ_2 as the upper and lower bound of the Δ to be used.

2. Calculation of Emittance: Using the same RFQ design mentioned in the previous section, we have also simulated the cases for both I = 35 mA and I = 100 mA from 100 KeV to 2 MeV. The distribution of the mean square of x and phase spread in the z direction $\Delta\phi$ are plotted in Figs. 1 and 2, respectively. The capture efficiency and emittance growth are listed in Table I.

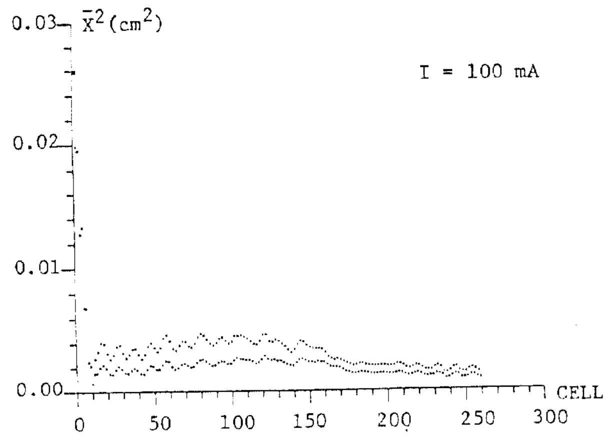

Figure 1. Mean square of the beam spread in the x direction for a beam with I = 100 mA.

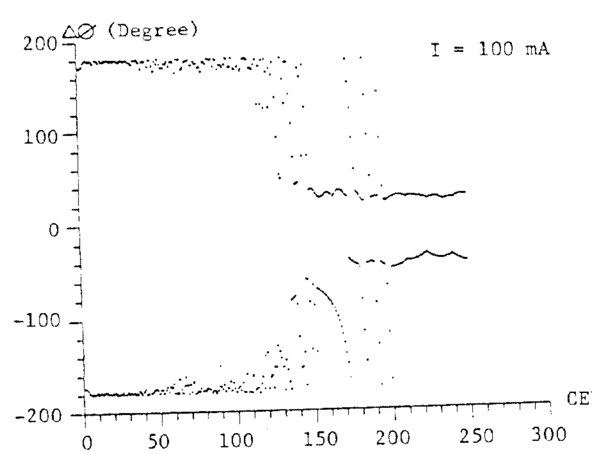

Figure 2. Upper and lower bounds of the phase spread in the z direction for a beam with I = 100 mA.

TABLE I

Simulation results for the RFQ. The decreases in the emittance are due to the loss of particles in the beam.

Current	Input Radial RMS Emittance (90% Contour)	Output Energy	Capture Efficiency	Emittance Growth Rate
35 mA	0.014 cm-MR	2 MeV	72 %	0.68
35 mA	0.014 cm-MR	640 KeV	75 %	0.80
100 mA	0.017 cm-MR	2 MeV	40 %	0.47
100 mA	0.017 cm-MR	640 KeV	45 %	0.70

Experiments and theoretical calculations have been carried out for a "proof-of-principle" test stand for the RFQ by the LASL group. We have used our particle code to simulate the beam transmission for a similar setup. The input energy of the beam is at 100 keV. At the end of 172 cells, where the machine length being 110.3 cm, the output energy of the beam is $\sim$ 640 keV. The input rms normalized emittance is $\varepsilon_{in} \sim 0.023$ mrad-cm. According to our simulation, the capture efficiency is $\sim$ 70%. The output emittance ε_{out} of the 90% contour is decreased from that of ε_{in} by 24%. This decrease of the emittance is caused by the loss of the particles. According to the experimental results reported by the LASL group,[5] their capture efficiency is $\sim$ 80%, and $\varepsilon_{out}/\varepsilon_{in} \sim 1.2$.

The discrepancy between our results and that from the experiment may be due to the following reasons. First, it may be due to the inaccuracy in the calculation of the space charge force in the beam. As we mentioned before, the understanding in this area is still quite limited, and a better way of dealing with this problem may very well improve the results. Second, the discrepancy may be caused by the larger number of cells used in our design than that of the experimental device. Using fewer number of cells means shorter time for the space charge force to disperse the beam against the wall of the RFQ. So it may be possible to improve the capture efficiency by decreasing the total number of cells in the RFQ device.

V. DISCUSSION

It has been found that the RFQ with four sections devised by the LASL group provides the best capture efficiency and emittance for the beam. However, it is difficult to put down definite guidelines to optimize the performance of the RFQ in its design. This area needs further exploration.

In our simulation, we have neglected the image field[7] induced by the beam in the RFQ. The image field should decrease the longitudinal diverging field experienced by the particles, which in turn may improve the calculated capture efficiency of the beam. As mentioned above, we also need to put more effort into the investigation of the method of calculating the space charge force for a non-neutral beam in the finite-size-particle simulation model. For the application to a higher current beam, it is essential that we fully understand this area.

VI. REFERENCES

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