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STABILITY OF BOUNDED ELECTRON BEAMS NEUTRALIZED BY CO-MOVING IONS

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Summary

It has been claimed[1] that bounded ion beams neutralized by co-moving electrons are inherently unstable for currents above the threshold for the Pierce instability.[2] In this paper, it is noted that the Pierce instability can be suppressed for certain conditions satisfied by the Occidental neutralized light ion beam driver for inertial confinement fusion.[3]

A model is presented for the analysis of guided wave propagation and instability along a cylindrical waveguide that is partially filled with a cold drifting electron beam. The beam is assumed to be fully neutralized by a positive ion background of infinite mass that is drifting at the same velocity as the electron beam -- thereby removing streaming instabilities from consideration.[4] Under these conditions, the electron-ion beam is found to be stable for both zero and infinite applied longitudinal magnetic fields, except for $\omega = 0$ waves satisfying certain axial boundary conditions first derived by Pierce in 1944.[2] It is demonstrated for pulsed beams, whose downstream boundary is just the beam head, that the excited waves never catch the beam head and hence cannot induce the Pierce instability. It is also shown for zero applied magnetic field that the guided waves are excited within a boundary layer on the beam surface whose skin depth becomes negligible for sufficiently high current density beams of a given acceleration voltage.

Infinite Magnetic Field

Pierce in 1944[2] and many subsequent authors[5,6,7,8] have investigated instabilities involving axially displaced electrodes in electron beam driven microwave tubes. As illustrated in Fig. 1, the geometry most commonly treated consists of an evacuated cylindrical waveguide capped on each end with grids that are immersed in a strong (effectively infinite) longitudinal magnetic field and partially filled with a cylindrical longitudinally drifting electron beam.

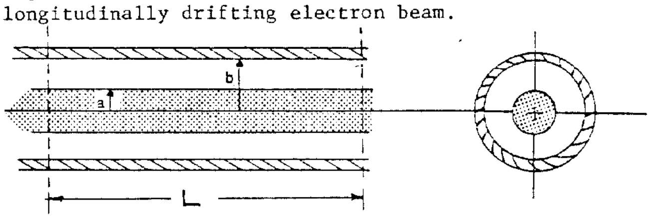

Fig. 1: Illustration of a cylindrical waveguide partially filled with an electron-ion beam.

The symmetric waveguide modes that interact with the beam are the well known transverse magnetic (TM) modes which obey the wave equation

$$(\nabla_T^{\,2} + T^2)E_z = 0 \qquad (1)$$

where E_z is the longitudinal electric field within the beam

$$E_z(r,z,t) = A\, J_o(Tr)\, e^{i(kz-\omega t)} \;;\; r < a \qquad (2)$$

and T is the radial eigenvalue which connects the boundary conditions

$$\left(1-\frac{\omega_{be}^2}{(\omega-kv_{be})^2}\right)\frac{\Gamma}{T}\,\frac{J_1(Ta)}{J_o(Ta)} = \frac{J_1(\Gamma a)N_o(\Gamma b)+J_o(\Gamma b)N_1(\Gamma a)}{J_o(\Gamma a)N_o(\Gamma b)-J_o(\Gamma a)N_o(\Gamma b)} \qquad (3)$$

with the beam-plasma parameters

$$T^2 = \left(\frac{\omega^2}{c^2} - k^2\right)\left(1 - \frac{\omega_{be}^2}{(\omega - kv_{be})^2}\right) \qquad (4)$$

which is valid for non-relativistic beams ($v_{be}/c << 1$), where v_{be} and c are the beam and light velocities, ω_{be} is the beam plasma frequency, and a neutralizing background of ions of infinite mass is assumed. The Bessel functions are regular of the first (J) and second (N) kind.

A typical $\omega - k$ dispersion diagram for these modes is exhibited in Fig. 2 where the normal TM waveguide waves with $\omega/k > c$ and the beam supported slow TM waves with $\omega/k < c$ are identified. Since the electric field peaks on-axis, the slow waves have become known as body waves.[6]

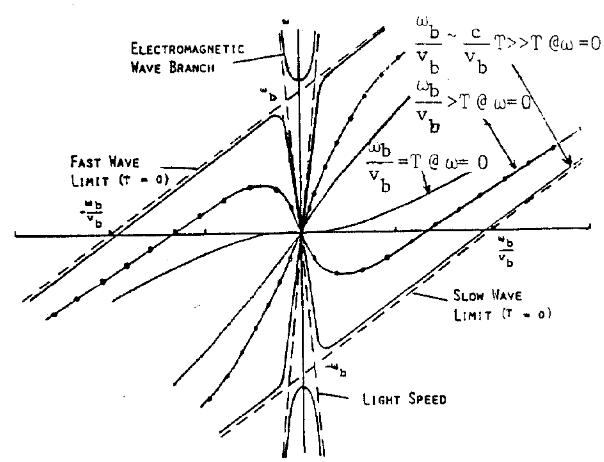

Fig. 2: Dispersion diagram for guided fast and slow plasma waves for the case of an infinite longitudinal magnetic field.

Pierce[2] first noted that the beam induced Doppler shifted dispersion relation is asymmetric about the ordinate axis for all $\omega \neq 0$, so that cavity resonances involving two equal but oppositely directed wave numbers are effectively spoiled. He thus considered resonances among the four waves that exist when $\omega = 0$, with wave numbers

$$k = \pm\, 0,\ \pm\left(\frac{\omega_{be}^2}{v_{be}^2} - T^2\right)^{1/2} \qquad (5)$$

and found that an absolute instability (involving a complex ω with the real part equal to zero) occurred whenever the threshold condition

$$\left(\frac{\omega_{be}^2}{v_{be}^2} - T^2\right)^{1/2} = \frac{\pi}{L} \qquad (6)$$

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was exceeded. Frey and Birdsall[7] performed more detailed calculations on the Pierce instability, finding instability within the following regions

$$\frac{(2n-1)\pi}{L} < \left(\frac{\omega_{be}^2}{v_{be}^2} - T^2\right)^{1/2} < \frac{2n\pi}{L}, \; n = 1,2,3,\ldots \tag{7}$$

They also concluded for mobile ions (i.e. finite ion mass), for which

$$T^2 = \left(\frac{\omega^2}{c^2} - k^2\right)\left(1 - \frac{\omega_{bi}^2}{\omega^2} - \frac{\omega_{be}^2}{(\omega - kv_{be})^2}\right) \tag{8}$$

that the instability was suppressed. However, Faulkner and Ware[8] claimed that Frey and Birdsall did not properly track the roots of the dispersion relation, and that in fact, finite ion mass did not suppress the Pierce instability.

As documented in a review by Lawson,[5] and in a more recent treatment by Poukey et.al., the Pierce instability is a potential problem for microwave tubes involving steady state electron beams.

However, Poukey et.al. also claimed that pulsed beams were subject to the Pierce instability and that the instability could occur even in the absence of end effects when a threshold dependent only on transverse boundary conditions was exceeded. That the latter result is in error can be proven by separately investigating Eq.(4) for ω in the quasi-static limit ($\omega/k \ll c$, as treated by Pierce[2])

$$\omega = kv_{be} \pm \frac{\omega_{be}^2}{(1 + T^2/k^2)^{1/2}} \tag{9}$$

and in the quasi-dynamic limit ($v_{be} \ll \omega/k \lesssim c$)

$$\frac{\omega}{k} = c\,\frac{\omega_{be}}{(\omega_{be}^2 + T^2\,c^2)^{1/2}} \tag{10}$$

Since T, ω_{be}, and v_{be} must be real, it is obvious that ω is real for all real k -- a sufficient condition for no instability as in fact deduced by Pierce in 1954[9] and subsequently confirmed by many others.[5,10] Consequently, axial (but not radial) boundaries are necessary for exciting the Pierce instability among zero-frequency waves. Radial boundaries only reduce the wave number of the unbounded fast and slow space charge waves and introduce a band of evanescent waves for $\omega_{be}/v_{be} < T$.

It is also not correct that all pulsed beams are unstable, especially beams used in inertial confinement fusion (ICF) systems. An ICF driver (such as two opposing electron beams focused on a target in the center of a magnetic hill) characteristically has the moving beam head as the downstream boundary. Since the beam head always travels faster than the slow space charge wave (as indicated in Fig. 2) reflections from the downstream boundary -- a necessary condition for establishing the Pierce instability -- never occur.

Zero Magnetic Field

The work of Poukey et.al.[1] represents the first attempt to identify the Pierce instability as a potential problem for neutralized ion beam ICF drivers. They investigated the same model exhibited in Fig. 1 but without an applied longitudinal magnetic field. Their one-dimensional analysis (with no transverse boundaries) incorporated the same zero frequency wave numbers originally analyzed by Pierce[2]

$$k = \pm\, 0,\ \pm \frac{\omega_{be}}{v_{be}} \tag{11}$$

and later by Frey et.al.[7] and Faulkner et.al..[8] Consequently, and correctly, they arrived at the same conclusions as their predecessors -- the Pierce instability can excite fast and slow electrostatic waves on the electron beam for certain axial boundary conditions. The computation was made between two fixed boundaries, as required for the Pierce instability, and also was significantly higher than the threshold for the Pierce instability.

Poukey et.al. did not identify or discuss the waves involved in their bounded beam simulation, thereby overlooking an important physical mechanism in isotropic neutralized electron beams -- the collisionless plasma skin effect. As first noted by Trivelpiece and Gould,[6] the TM waves on an isotropic bounded beam satisfy a different wave equation than Eq.(1)

$$(\nabla_T^2 - T^2)E_z = 0 \tag{12}$$

where the radial dependence of the longitudinal electric field within the beam varies according to a modified Bessel function of the first kind

$$E_z(r,z,t) = A\, I_o(\tau r)e^{i(kz - \omega t)} \tag{13}$$

the determinental Eq.(3) is similar except that $T = i\tau$, and the beam-plasma parameters are connected to τ in a different manner than Eq.(4)

$$\tau^2 = k^2 - \frac{(\omega - k\,v_{be})^2}{c^2} + \frac{\omega_{be}^2}{c^2} \tag{14}$$

Since the modified Bessel function peaks on the beam boundary, the wave energy and plasma interaction predominantly occurs there, hence the slow waves are called surface waves[6] (as opposed to the body waves treated by Pierce). They ripple the surface of the plasma column just as observed in the simulation of Poukey et.al. The penetration depth of surface waves is characteristically τ^{-1} and hence not greater than the collisionless plasma skin depth, c/ω_{be}. This is apparently ~3cm, as inferred from the Poukey et.al. simulation. However, for the ORC neutralized lithium ion beam, typically V = 5MV and J = 10 A/cm^2 so that

$$\omega_{pe} = \left(\frac{J}{\varepsilon_o \frac{m}{e} v_o}\right)^{1/2} = 1.3 \times 10^{10}\ \text{rad/sec} \tag{15}$$

and c/ω_{pe} ~ 2.3cm << typical module radii of 12cm. As the beam is bunched and ballistically focused during propagation, the penetration depth of the guided waves becomes progressively smaller.

The ω - k dispersion diagram for guided waves with zero magnetic field are exhibited in Fig. 3. Note that the surface wave branches approach the asymptotic limits

$$\omega = kv_{be} \pm \frac{\omega_{be}}{\sqrt{2}} \tag{16}$$

which are characteristically different than the body wave limit

$$\omega = kv_{be} \pm \omega_{be} \tag{17}$$

There is consequently a modification to the analytical results of Pierce,[2] Frey et.al.,[7] and Ware et.al.[8] which is currently being investigated in more detail.[11]

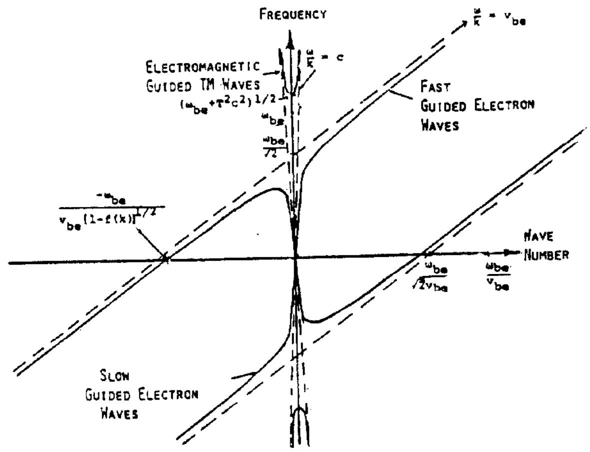

Fig. 3: Dispersion diagram for guided fast and slow plasma waves for the case of no applied magnetic field.

Conclusion

A model has been presented for the analysis of guided wave propagation and instability along a cylindrical waveguide that is partially filled with a cold drifting electron beam and neutralized by a co-moving, but otherwise immobile ion beam. Beam stability in the absence of axial boundaries has been established. If the downstream boundary is the beam head, then prior to impact with a target the waves which can be excited never reflect from the head and hence do not induce the Pierce instability. In a zero magnetic field, the waves are surface waves which can occupy a negligibly thin layer on the beam radial boundary for sufficiently high current density -- there is no upper limit to the current density achievable with isotropic neutralized electron-ion beams.

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