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SELF-CONSISTENT ANALYSIS OF THE SELF-RESONANT ELECTRON-CYCLOTRON LINEAR ACCELERATOR

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The self-resonant acceleration mechanism uses plane electromagnetic waves to increase the energy of the electrons in cyclotron resonance with the wave. The self-consistent analysis shows that complete energy conversion is possible over a finite interaction length. A possible realization of this mechanism would use a 25 TW/cm^2 CO_2 laser beam to accelerate a 25 kA/cm^2 electron beam in a 50 kG magnetic field from 50 MeV to 1 GeV in 20 m with 65% conversion efficiency.

Introduction

The motion of charged particles (to be concrete, electrons) of rest mass m and charge -e under the joint action of a magnetostatic field B_0 and an electromagnetic wave with frequency ω and propagation vector (wavenumber) k parallel to B_0 depends on the frequency difference $\Delta\omega = \omega-\Omega/\gamma-k\beta_{\parallel}c$, where γmc^2 is the total electron energy, $\beta_{\parallel}c$ is the electron velocity along B_0, c is the speed of light and $\Omega_0 = eB_0/mc$ is the nonrelativistic electron gyrofrequency. In general, the motion is oscillatory. However, for the case of plane waves ($n=kc/\omega=1$), the quantity of $\gamma(1-\beta_{\parallel})$ is conserved, since the electric and magnetic components of the wave are equal to each other, and therefore the energy and axial momentum of the electrons change at the same rate. (Gaussian units are used in all equations, with the exception of the equations with practical interest, where clearly noted MKSA units are used.) A consequence of this is that in the case of exact synchronism, i.e. $\Delta\omega=0$, this synchronism is maintained throughout the motion, regardless of the relative orientation of the electron transverse momentum $\gamma\beta_{\perp}mc$ and the wave fields, and unlimited energy gain is expected for all electrons.

This phenomenon [1,2] (called self-resonance in Ref. 1) has also been confirmed experimentally [3-5]. Additional investigations have shown that a non-monochromatic pulse can still result in substantial acceleration [6], that an additional invariant exists even for a time dependent wave amplitude [7] or a wavepacket [8], and that compensation of $n\neq1$ by a spatial variation of the magnetostatic field is possible but not readily realizable[9]. Additional literature items include a covariant formulation and integration of the equations of motion [10], the study of trapping and the suggestion of inertial acceleration if the diffractive index n is a function of the axial coordinate z [11], and the calculation of the appropriate variation n(z) in the adiabatic approximation [12]. All these studies have treated the electrons as test particles and have omitted their feed-back on the wave.

To the knowledge of the author, no further investigations into this problem have been done. The apparent reason for this sudden lack of interest is associated with the difficulties of realizing a practical high power and high energy accelerator based on this mechanism. The microwaves, originally considered to drive the acceleration, do not have the power necessary to produce significant acceleration within reasonable distances and in addition have to be contained in a waveguide, which destroys the self-resonance (since n<1). However, the present-day high-power laser systems appear able to provide the necessary radiation source, and this mechanism has been rediscovered (Ref. 13 and independently by this author). As will be shown below, a 25 TW/cm^2 CO_2 laser beam can in principle accelerate a 25 kA/cm^2 electron beam from 50 MeV to 1 GeV in 20 m with 65% energy conversion (or to 1.2 GeV in 30 m with 80% conversion).

This paper is a continuation of the earlier research on this subject, with the important added feature of self-consistency in the evolution of the particles and the wave. First, these self-consistent equations are presented for both the wave and the electrons and the integrals of the motion are obtained. It is seen that $\gamma(1-\beta_{\parallel})$, although not an exact invariant, is conserved with very high accuracy for typical values of the parameters. The problem is reduced to a quadrature and the relation $\gamma(z)$ is obtained for the most interesting case of synchronism and initial axial injection of the particles. Complete energy conversion is possible over a finite length, while extremely high conversion efficiencies occur at much smaller distances (e.g. 83% at half the maximum length). In addition, relations of practical interest are obtained, e.g. relating the interaction length to the beam current and the laser power density, and the potential of the mechanism is demonstated by an example. Finally, the paper concludes with a discussion of peripheral issues, such as the effects of beam temperature, of inaccuracies in the external field, and the diffraction effects of the radiation beam.

Self-consistent Analysis

In this section the self-consistent equations are obtained for the steady state behavior of an electron beam under the action of a uniform magnetic field $B_0\hat{e}_z$ and a plane wave propagating along $\hat{e}_z$ with vector potential amplitude A(z) and phase $\phi(z,t)$. The frequency and wavenumber are given by $\omega = \partial\phi/\partial t$ and $k = -\partial\phi/\partial z$. Because of the helical configuration of the interaction, it is convenient to introduce the unit vectors $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$, where $\hat{e}_1$ and $\hat{e}_2$ are obtained by rotating $\hat{e}_x$ and $\hat{e}_y$ by the angle ϕ. In terms of these unit vectors, the vector potential of the wave is given by $A\hat{e}_2$, the electric field by $(\omega/c)A\hat{e}_1$ and the magnetic field by $kA\hat{e}_2-(dA/dz)\hat{e}_1$. The field evolution is obtained from Maxwell's equations. Faraday's law requires that ω=const. Introducing the refractive index $n(\zeta)=k(\zeta)c/\omega$, the wave quiver velocity $A(\zeta)=eA(\zeta)/mc^2$, and the normalized distance $\zeta=\omega z/c$, and neglecting the electrostatic and magnetostatic self-fields of the beam, one obtains from Ampere's law the equations for A and n,

$$\frac{d}{d\zeta}(nA^2) = \varepsilon^2 A\beta_1 , \qquad (1)$$

$$\frac{d^2}{d\zeta^2}A = (n^2-1)A + \varepsilon^2\beta_2 , \qquad (2)$$

where $\varepsilon(\zeta)=\omega_p/\omega$ is the ratio of the plasma frequency to the wave frequency, the former given in terms of the density $N(\zeta)$ by $\omega_p^2(\zeta)=4\pi e^2N(\zeta)/m$, and β_1c, β_2c, β_3c are the electron velocity components along the three unit vectors.

The evolution of $\varepsilon^2(\zeta)$, i.e. of the density, is given by the continuity equation in steady state,

$$\frac{d}{d\zeta}(\varepsilon^2\beta_3) = 0 , \qquad (3)$$

and the evolution of the velocity components β_1, β_2, β_3 is given by

$$\frac{d}{d\zeta}(\gamma\beta_1) = (1-n\beta_3)\frac{\gamma\beta_2-A}{\beta_3} - \frac{\Omega_0}{\omega}\frac{\beta_2}{\beta_3} , \qquad (4)$$

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$$\frac{d}{d\zeta}(\gamma\beta_2) = -(1-n\beta_3)\frac{\gamma\beta_1}{\beta_3} + \frac{\Omega_0}{\omega}\frac{\beta_1}{\beta_3} + \frac{dA}{d\zeta}\,, \tag{5}$$

$$\frac{d}{d\zeta}(\gamma\beta_3) = -nA\frac{\beta_1}{\beta_3} - \frac{dA}{d\zeta}\frac{\beta_2}{\beta_3}\,, \tag{6}$$

where $\Omega_0=eB_0/mc$ is the nonrelativistic electron gyrofrequency in B_0 and $\gamma(\zeta)=(1-\beta_1^2-\beta_2^2-\beta_3^2)^{-1/2}$ is the electron energy in units of the rest energy mc^2. Eqs. (1) - (6) form a complete set of equations that specifies A, n, ε, β_1, β_2 and β_3 as functions of ζ in terms of the assigned injection conditions at $\zeta=0$. It is convenient to supplement these equations with the equation for γ,

$$\frac{d\gamma}{d\zeta} = -A\frac{\beta_1}{\beta_3}\,. \tag{7}$$

Due to the high nonlinearities involved, it does not appear feasible to integrate these equations analytically, and recourse to either approximate integrations or to numerical solutions appears necessary. In either case, it is useful to know that four first integrals exist. They are given by

$$\begin{aligned} I_1 &= \varepsilon^2\beta_3\,, \\ I_2 &= \varepsilon^2\gamma\beta_3 + nA^2, \\ I_3 &= \tfrac{1}{2}\left[(n^2+1)A^2 + \left(\frac{dA}{d\zeta}\right)^2\right] + \varepsilon^2\gamma\beta_3^2\,, \\ I_4 &= \tfrac{1}{2}\left[(\gamma\beta_1)^2 + (\gamma\beta_2-A)^2\right] - \frac{\Omega_0}{\omega}\gamma\,. \end{aligned} \tag{8}$$

One can recognize I_1, I_2, and I_3 as proportional to the conserved fluxes of particles, energy and momentum, while I_4 is the spatial analog of the invariant in Ref. (7), where a time (rather than position) dependent field amplitude was considered.

Conspiciously absent from the above list of integrals of the motion is the quantity $R_1=\gamma(n-\beta_3)$. This quantity would have been an invariant if either both A and n were constant or they happened to vary in the relation $dA/dn=-\gamma\beta_3/\beta_2$. In addition, one is interested in the quantity $R_2=\gamma(1-n\beta_3)$, since the value $R_2=\alpha$, where $\alpha=\Omega_0/\omega$, corresponds to exact synchronism. If in particular $n=1$, then R_1 and R_2 are equal to $R_3=\gamma(1-\beta_3)$, and the condition $R_3=\alpha$ leads to the self-resonant acceleration. As will be shown below, it is a fortunate coincidence that for the parameter values of interest, all these three quantities remain very close to the value α.

In practical applications, one is expected to use the most powerful radiation source available, i.e the CO_2 laser with wavelength $\lambda=1.06 \times 10^{-3}$ cm. Then for a magnetostatic field as strong as $B_0=100$ kG, the value of Ω_0/ω is equal to $\alpha=10^{-2}$. It can be seen that the smallest value of γ consistent with $R_3=\alpha$ is obtained for $\beta_1=\beta_2=0$. Accordingly, to minimize the input electron energy, it is desirable to inject the electron beam parallel to the magnetostatic field. In addition, this choice minimizes the thermal beam spread and also simplifies the analysis. Hence, it will be assumed from here on that the initial values of the electrons are given by $\beta_{10}=\beta_{20}=0$, $\beta_{30}=(1-\alpha^2)/(1+\alpha^2)$, and $\gamma_0=(1+\alpha^2)/(2\alpha)$, where a subscript "$_0$" denotes the values at $\zeta=0$. (If ions were to be accelerated, the value of α would have been several orders of magnitude smaller and would correspond to a prohibitively high value of γ_0, hence the choice of electrons as the particles to be accelerated.)

Of interest is also a feeling of the numerical values of two more parameters. These are I_1 and A_0. It is trivial to relate them to the beam current density J and to the radiation energy flux S. One obtains

$$\begin{aligned} I_1 &= 1.87 \times 10^{-5}\,(J\lambda^2), \\ A_0^2 &= 3.66 \times 10^{-11}\,(S\lambda^2), \end{aligned} \tag{9}$$

where $\lambda=2\pi/k$ is the radiation wavelength and $J\lambda^2$ and $S\lambda^2$ are expressed in Amperes and Watts, respectively. Obviously, for $\lambda=1.06 \times 10^{-3}$cm and any realistic choice of beam current, the quantity I_1 is many orders of magnitude smaller than unity, while A_0 may be as large as 0.1.

First it will be shown that R_3 remains very close to the value α. From the definition of I_4 one can obtain the exact expression

$$R_3-\alpha = \frac{A_0^2-A^2+2A\gamma\beta_2}{\gamma(1+\beta_3)-\alpha}\,. \tag{10}$$

The denominator is essentially equal to 2γ, while from Eq. (5) one can obtain $\gamma\beta_2=A-A_0$, if $R_2-\alpha=0$. Then, using also the definitions of I_1 and I_2, it can be seen that $R_3-\alpha$ increases very slowly, reaching the extremely small value $I_1/2<<\alpha$, when $A=0$. Accordingly, R_3 may be taken to be constant to very high accuracy.

To show that R_1 and R_2 are also essentially constant, it is observed that combinations of I_2 and I_3 give the relations

$$\begin{aligned} R_1-\alpha &= \frac{1}{2I_1}\left[-(n^2-1)A^2+\left(\frac{dA}{d\zeta}\right)^2\right]\,, \\ R_2-\alpha &= \frac{n}{2I_1}\left[(n^2-1)A^2+\left(\frac{dA}{d\zeta}\right)^2\right]\,, \\ R_3-\alpha &= \frac{1}{2I_1}\left[(n-1)^2A^2+\left(\frac{dA}{d\zeta}\right)^2\right]\,, \end{aligned} \tag{11}$$

Since all electrons are assumed to start with $\beta_{10}=\beta_{20}=0$, the terms proportional to $(n-1)$ are not expected to be significant. Such terms originate from the component of the electron current across the electric field of the wave. This component is small, since under the assumed synchronism only the component driven by the electric field is expected to be significant. Accordingly, the dominant term is the one with the derivative $dA/d\zeta$. Hence, all three quantities in Eqs. (11) are bounded by $I_1/2<<\alpha$.

The above discussion justifies setting $n=1$ and $R_1=R_2=R_3=\alpha$ in the equations of motion. Then, with $\gamma\beta_2=A-A_0$, one can obtain expressions relating $\gamma\beta_1$, $\gamma\beta_3$ and A to γ, so that Eq. (7) can be integrated. A rather compact expression of the result is

$$\frac{\pi}{2}\frac{z}{z_{max}} = \left(1+2\frac{\gamma_0-\alpha}{\gamma_{max}-\gamma_0}\right)\arcsin\sqrt{\eta} - \sqrt{\eta(1-\eta)}\,, \tag{12}$$

where $\gamma_{max}=\gamma_0+A_0^2/I_1$ is the maximum energy (in units of the rest energy) attainable by the beam and $\eta=(\gamma-\gamma_0)/(\gamma_{max}-\gamma_0)$ is the efficiency with which electromagnetic energy is converted to kinetic energy. The value $\eta=1$ corresponds to $\gamma=\gamma_{max}$, i.e. complete energy conversion. This occurs at $z=z_{max}=\frac{1}{4}\lambda A_0^2(2\alpha)^{-\frac{1}{2}} I_1^{-3/2}$.

Fig. (1) shows the dependence of η on z/z_{max} for the most interesting case $\gamma_{max}>>\gamma_0$. At $\eta<<1$ this curve reproduces the standard proportionality $\gamma\propto z^{2/3}$. For larger values of η, the energy gain per unit length decreases somewhat, total energy conversion is nevertheless possible for a finite length of interaction. High conversion efficiency is possible for a substantially shorter

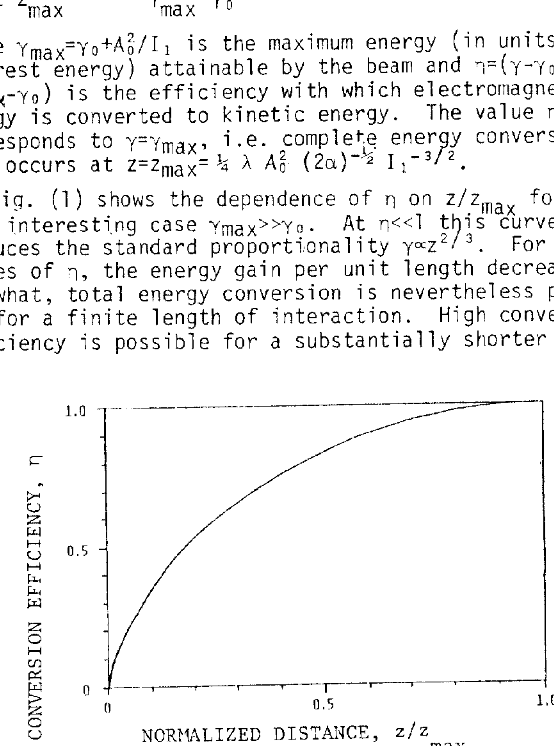

Fig. 1 - Conversion efficiency vs. normalized distance

length. For example, at the length $z/z_{max}=\frac{1}{2}$ the efficiency is $\eta=0.83$, while the length $z/z_{max}=2/3$ corresponds to $\eta=0.93$.

For practical applications one can conveniently use Fig. (1), since in general it is expected that $\gamma_{max}>>\gamma_0$. Given $\alpha=\Omega_0/\omega$, the maximum interaction length with a CO_2 beam is given by

$$z_{max}[m] = \frac{800}{\sqrt{\alpha}} \frac{S[GW/cm^2]}{J^{3/2}[A/cm^2]} \quad , \tag{13}$$

in terms of the radiation beam energy flux and the electron beam current density. Of course, the highest electron energy attainable under complete conversion ($\eta=1$) is given by $E_{max}[GeV] = S[GW/cm^2]/J[A/cm^2]$, plus the small amount of initial energy.

Discussion

To illustrate the potential of this acceleration mechanism, it is convenient to consider the CO_2 radiation energy flux S and the electron beam current density J as functions of the initial and final electron energies, E_0 and $E<E_{max}$, the interaction length $z<z_{max}$, the magnetostatic field amplitude B_0 and the corresponding conversion efficiency η. These functions are shown in Fig. (2) for the particular choices E_0=25 MeV, E=1 GeV, z= 10 m and B_0=100 kG. Conversion to different choices is straightforward in view of the scaling relations

$$\begin{aligned} S &\propto E^3 B_0 z^{-2} \quad , \\ J &\propto E^2 B_0 z^{-2} \quad , \end{aligned} \tag{14}$$

which are exact in the limit $E/\eta>>E_0>>0.5$ MeV. The constraint $E_0 \propto B_0^{-1}$ supplements the scaling laws.

As can be seen in the Figure, a 130 kA/cm^2 electron beam in a 10 m long 100 kG strong magnetic field can be accelerated from 25 MeV to 1 GeV by a 200 TW/cm^2 radiation beam at 65% conversion efficiency. If the interaction length is assumed equal to one Rayleigh length, then the radiation cross-section is 1 cm^2, giving a radiation power of 200 TW, about 10 times more than what is readily available at the present time. This limitation can be overcome at the expense of the interaction length and the initial beam energy. Thus, using the scaling laws, it can be seen that increasing the interaction length to 20 m and the initial beam energy to 50 MeV, while reducing the magnetostatic field to 50 kG gives an acceleration to 1 GeV of a 25 kA/cm^2 electron beam by a 25 TW/cm^2 CO_2 radiation pulse. The conversion efficiency is again 65%.

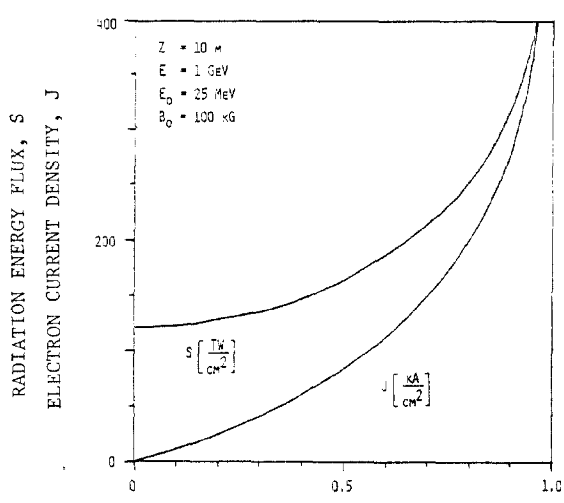

Fig. 2 - Radiation energy flux S and electron beam current density J vs. conversion efficiency η for acceleration of electrons to 1 GeV from 25 MeV in a 100 kG field in 10 m. For different input parameters, the scaling laws can be used, see Eqs. (14).

The acceleration process is not expected to have any serious limitations associated with temperature in the beam. The quantity of interest is $R_3=\gamma(1-\beta_3)$ and it is required that $\delta R_3<<R_3=\alpha$. It can be seen that this requirement is satisfied by simply $\delta\gamma<<(\gamma^2-1)^{\frac{1}{2}}\simeq\gamma$, while deviations from the assumed axial injection can be tolerated, provided $(\gamma\beta_\perp)^2<<\frac{1}{2}$ initially. Similarly, the external magnetic field must be uniform enough so that $\delta\alpha<<\alpha=\Omega_0/\omega$, which gives $\delta B_0<<B_0$. Also, inhomogeneities in the radiation pulse amplitude are no problem either. Those associated with the modulation of the input pulse are insignificant due to the minute relative lag of the electrons relative to the pulse (a few picoseconds). On the other hand, the dispersion of the beam is not particularly important, since the synchronism does not depend on the radiation pulse amplitude. Actually, the dispersion can be taken advantage of, in order to compensate for the increasing Larmor radius of the particles, as they acquire transverse momentum in the acceleration process. Finally, the extraction of the accelerated beam can be accomplished simply by adiabatically reducing the magnetostatic field to zero, thus converting to exclusively axial motion of the beam. In view of the above and the highly impressive expected performance of this acceleration process, it appears to be a viable candidate for a laser driven linear electron accelerator.

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In equations (14), the quantities S and J are proportional to B_0^{-1} (and not to B_0). Accordingly, the example cited in the abstract and above Fig. 2 should read: S=22 TW/cm^2 CO_2, J=14.5 kA/cm^2, E_0=25 MeV, E=1 GeV, z=30 m, B_0=100 kG, η=65%.