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LASER BEAT WAVE ELECTRON ACCELERATOR*

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Summary

A static periodic magnetic field (wiggler) together with a radiation field, can induce a beat wave in the presence of an injected electron beam. This beat wave, if properly phased, can trap and continuously energize the electron beam. To optimize the transfer of energy from the photons to electrons, both the wiggler amplitude and wavelength are spatially increased as a function of acceleration distance. The acceleration process is self-consistently analyzed and includes radiation depletion and space charge effects. Two numerical illustrations are given using different radiation sources.

Introduction

A number of authors have proposed methods for accelerating electrons utilizing the electromagnetic energy of a radiation field in the presence of a periodic static magnetic field.[1-3] Electric fields of typical lasers, in a pulsed, focused mode, can be $> 10^9$ eV/cm. These vacuum fields are luminous, rapidly oscillating both in space and time and, hence, impart an insignificant amount of energy to the particles. It is possible, however, to utilize a small fraction of the radiation field in the presence of a periodic magnetic field (wiggler) to continuously energize a beam of electrons.

The combined action of the radiation field and wiggler magnetic field on the electron beam results in a beat wave or ponderomotive wave. The magnetic wiggler field periodically changes the momentum of the electrons in such a way as to give the electrons a velocity component in the direction of the radiation electric field. The electrons, if properly phased with respect to the beat wave, can be trapped and continuously energized. As the electrons are accelerated the necessary resonance condition can be maintained by gradually increasing the wavelength of the wiggler field as a function of distance z along the acceleration region. Furthermore, to optimize the transfer of energy from the photons to electrons it will be shown that the vector potential associated with the wiggler field should also be increased as a function of z. Optimizing the transfer of energy from the radiation field to the trapped electrons requires increasing both the transverse as well as the axial energy of the electrons in a controlled way. After the trapped electrons have been energized, the transverse coherent motion can be converted into axial motion by adiabatically removing the wiggler field.

Since the energized electron beam may be sufficiently intense to deplete the radiation field as well as to induce self space charge fields, the acceleration process is analyzed self-consistently. Both the amplitude and phase of the radiation field are self-consistently obtained as functions of z. The effects of induced space charge field due to particle trapping is also handled self-consistently. Finally two detailed numerical examples of a radiation beat wave accelerator are given.

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Self-Consistent 1-D Formulation

In our analysis, we assume a cold electron beam, and linearly polarized radiation field and wiggler field. We will present a self-consistent 1-D nonlinear formulation including space charge effects.[4,5] The physical principle of the laser accelerator is identical to the inverse mechanism of the free electron laser (FEL). The vector potential associated with the generalized linearly polarized wiggler and temporal steady state radiation field are

$$\underset{\sim}{A}_w(z) = A_w(z)\cos\left(\int_0^z k_w(z')dz'\right)\hat{e}_x , \qquad (1)$$

$$\underset{\sim}{A}_R(z) = A_R(z)\sin((\omega/c)z - \omega t + \varphi(z))\hat{e}_x \qquad (2)$$

where A_R, A_w, φ, and $k_w = 2\pi/\ell_w$ are assumed to be slowly varying functions of z, ℓ_w is the wiggler wavelength, and ω is the constant frequency of the radiation field. In all cases of interest $|A_w| >> |A_R|$ by many orders of magnitude. Furthermore it is necessary to have $k_w r_b << 1$, where r_b is the radius of the electron beam, in order to neglect spatial gradients in the wiggler.

The ponderomotive wave is the result of the beating of the wiggler field and the electromagnetic radiation field and arises from the $c^{-1}(\underset{\sim}{v} \times \underset{\sim}{B})\cdot\hat{e}_z$ term in the axial particle momentum equation. Since the generalized momentum in the x direction is constant, the x component of the particle momentum is $P_x = |e|/c(\underset{\sim}{A}_w(z) + \underset{\sim}{A}_R(z,t))\cdot\hat{e}_x$ and $c^{-1}(\underset{\sim}{v}\times\underset{\sim}{B})\cdot\hat{e}_z = (|e|/m_oc^2)(2\gamma)^{-1}(dA_w^2/dz - (2\gamma m_oc^2/|e|)\partial\phi_{pond}/\partial z)$, where $\phi_{pond} = -|e|/(2\gamma m_oc^2))A_wA_R \sin\psi$ is the ponderomotive potential, $\psi(z,\psi_o) = \int_0^z(k_w(z') + \omega/c - \omega/v_z(z',\psi_o))dz' + \varphi(z) + \psi_o$ is the phase between the electrons and the ponderomotive wave, ψ_o is the initial phase at the entrance to the interaction region, γ is the total relativistic gamma factor, and $v_z = \omega/(\omega/c + k_w - \partial\psi/\partial z + d\varphi/dz)$ is the axial electron velocity. In order for the electrons to couple to the ponderomotive wave, we require $v_z \simeq v_{ph} = \omega/(\omega/c + k_w)$. Thus the resonance condition is $\omega \simeq 2\gamma_z^2 v_z k_w$, where $\gamma = \gamma_z\gamma_\perp$, $\gamma_\perp = (1 + (|e|A_w/(m_oc^2))^2/2)^{\frac{1}{2}}$ and $\gamma_z = (1 - (v_z/c)^2)^{-1/2}$ is the axial gamma factor.

Self-consistent non-linear steady state equations for the laser accelerator are given below. Details of the analysis can be found in Refs. 4-5. The equation governing the relative phase between the electrons and the ponderomotive wave is given by a generalized pendulum-like equation,[5]

$$\frac{\partial^2\psi(z,\psi_o)}{\partial z^2} = \frac{d^2\varphi}{dz^2} + \frac{dk_w}{dz} - \frac{\omega/c}{4\gamma^2}\left(\frac{|e|}{m_oc^2}\right)^2\frac{\partial A_w^2}{\partial z} - \frac{\omega/c}{\gamma^2}\left(\frac{|e|}{m_oc^2}\right)^2 k_wA_wA_R\cos\psi + \frac{2\omega_b^2/c^2}{\gamma\gamma_z^2}\left(\langle\cos\psi\rangle_{\psi_o}\sin\psi - \langle\sin\psi\rangle_{\psi_o}\cos\psi\right), \qquad (3)$$

where $\omega_b = (4\pi|e|^2 n_o/m_o)^{\frac{1}{2}}$ is the plasma frequency, n_o is the electron beam density, and $\langle(\ )\rangle_{\psi_o} = (2\pi)^{-1}\int_0^{2\pi}(\ )d\psi_o$ is the ensemble average over initial phases. The first three terms of Eq. (3) includes the effect on the phase due to the variation of the radiation phase, wiggler wavenumber and wiggler amplitude, respectively. The fourth term represents the ponderomotive wave, and the last term denotes the effect of space charge (collective) waves on the phase. The amplitude and the phase of the radiation is governed by

$$\left(\frac{\omega}{c} - k\right) A_R = \frac{\omega_b^2}{c^2}\frac{c}{2\omega} A_w \left\langle \frac{\sin\psi}{\gamma}\right\rangle_{\psi_o} \quad (4)$$

$$k^{\frac{1}{2}} \frac{d}{dz}(A_R k^{\frac{1}{2}}) = \frac{1}{2}\frac{\omega_b^2}{c^2} A_w \left\langle \frac{\cos\psi}{\gamma}\right\rangle_{\psi_o} \quad (5)$$

where $k = d\varphi/dz + \omega/c$.

The rate of change of energy of the electron is

$$\frac{\partial}{\partial z}(\gamma(z,\psi_o)m_o c^2) = -\frac{\omega/c}{2\gamma(z,\psi_o)}\frac{|e|^2}{m_o^2 c^4} A_w(z)A_R(z)\cos\psi(z,\psi_o) \quad (6)$$

In the ponderomotive wave, the particle with constant phase is denoted as the resonant particle. For the resonant particle, $\gamma_R = \gamma_\perp \gamma_{zR}$, where $\gamma_{zR} = (k/(k_w + d\varphi/dz)/2)^{\frac{1}{2}}$, The energy of the electrons associated with the perpendicular and axial motion can be increased independently.

To maintain a resonant particle, either $k_w(z)$ or $A_w(z)$ can be prescribed, but not both. The relation is

$$\frac{dk_w}{dz} + \frac{d^2\varphi}{dz^2} - \frac{\omega/c}{2\gamma_R}\frac{|e|^2}{m_o^2 c^4}\left(\frac{\partial}{\partial z}\frac{A_w^2}{2} + 2k_w A_w A_R \cos\psi_R\right) = 0 \quad (7)$$

Assuming that the electron energy, at the entrance to the interaction region $z = 0$, is matched to the phase velocity of the ponderomotive wave, the fraction of particles that will be trapped depends on the amplitude of the trapping potential, $|e|\phi_{trap}$, as well as the axial electron velocity spread, Δv_z. The trapping condition is $\Delta\gamma_z/\gamma < |e|\phi_{trap}/(\gamma m_o c^2) = 2\sqrt{2}\,(\gamma_z/\gamma)(|e|/m_o c^2)(A_w A_R)^{\frac{1}{2}}$, where $\Delta\gamma_z = \gamma\,\gamma_z^2 \Delta v_z/c$ is the spread in axial γ. The various contributions to $\Delta\gamma_z$ are wiggler gradients, emittance, self-field and intrinsic energy fluctuations at the cathode.

Example 1

As an example of a 10.6 μm laser accelerator we choose a CO_2 laser with an energy of 5 kJ, pulse duration of 1 ns and laser beam waist r_o=0.5 cm. The amplitude of the radiation vector potential is $A_R(0) = 52$ statvolts ($E_R(0) = 10^8$ eV/cm). The Rayleigh length $z_o = r_o^2\,\omega/c$ is 7.8 m. The tapered linearly polarized wiggler initially has a magnetic field $B_w(0) = 7$ wavelength of $\ell_w(0) = 2.8$ cm, amplitude of the vector potential $A_w(0) = 2.25 \times 10^3$ statvolts and an interaction length of 12 m. The parameters of the electron beam, injected into the accelerator region at $z = 0$, are beam current $I = 1$ kA, beam radius $r_b = 0.1$ cm, beam energy $\gamma(0) = 43.2$ ($\gamma_\perp(0) = 1.4$, $\gamma_z(0) = 31.6$), particle density $n_o = 3.32 \times 10^{12}$ cm^{-3} and plasma frequency $\omega_b = 1.05 \times 10^{11}$ sec^{-1}.

Chosing the resonant phase $\psi_R = -2.5$ such that $\cos\psi_R = -0.8$, and tapering the wiggler field as shown in Fig. 1, the energy of the accelerated electrons increase as shown in Fig. 2. The rate of increase of energy averages to about .7 MeV/cm. The final parameters are $B_w = 50$ kG, $\ell_w = 8$ cm, $\gamma_z = 63$, $\gamma_\perp = 26.1$, $\gamma = 1640$. In this example, the radiation has been depleted by about half (Fig. 1), and about 30% of the electrons in the injected beam have been accelerated.

Example II

In this example, we will utilize the radiation from a FEL to accelerate the electrons. A schematic diagram is shown in Fig. (3). The optical cavity system is composed of four mirrors, which if properly shaped could contain the radiation field. One cavity is used for producing the radiation using the FEL mechanism. Another cavity is used to accelerate the electrons using the beat wave mechanism. The temporal sequence of the acceleration process is to build the radiation up to the desirable value first, and then to inject a second electron beam with the appropriate energy into the acceleration region.

The radiation power in the cavity should be about a factor of two or more of the final desirable accelerated electron beam power. We will consider a radiation beam with wavelength $\lambda = 0.025$ cm, beam waist $r_o = 0.5$ cm, beam power $P_R = 10^{11}$ watts, Rayleigh length $z_o = 31.4$ cm, and peak electric field $E_R = 4.4 \times 10^4$ statvolts/cm (13.2 MeV/cm). Since the damage threshold of the electric field on the copper mirror is 0.5 MeV/cm for 1 μsec at $\lambda = 0.025$ cm, the mirrors must be separated by $L = 50\,z_o$ or 15 m. The Q of the cavity will then be approximately 10^8.

For the numerical calculation of the electron acceleration, we will make the resonant particle, constant phase approximation. In this approximation all particles as assumed to have the same constant phase, ψ_R. The electron beam in this approximation consists of a pulse train of macro particles separated in distance by $2\pi v_z/\omega$ Furthermore, we will chose the radiation field to be the fundamental TEM_{00} free space field. Radiation depletion will not be considered in this example. The energy of the accelerated macro particles are calculated by Eqs. (6) and (7).

The tapered wiggler initially has a magnetic field $B_w(0) = 3$ kG, wavelength of $\ell_w(0) = 2.5$ cm, amplitude of the vector potential $A_w(0) = 1.2 \times 10^3$ statvolts, and an interaction length of 2 m. Parameters of the electron beam, injected into the acceleration region at $z = 0$, are beam energy $\gamma(0) = 7.9$, $\gamma_\perp(0) = 1.1$ and $\gamma_z(0) = 7.1$

Chosing $\cos\psi = -0.8$, locating the minimum waist of the Gaussian radiation beam at $z = 2z_o = 60$ cm, and tapering the wiggler field as shown in Fig. (4), the energy of the accelerated electrons increase as shown in Fig. (5). The fastest rate of the energy increase (Fig. 5) is 0.7 MeV/cm at the point where the Gaussian radiation field (Fig. 4) is the largest. The final parameters are B_w = 29 kG, ℓ_w = 7.5 cm, γ_z = 11.9, $\gamma_\perp$ = 13.9, and γ = 164.

For a circularly polarized wiggler, the rate of acceleration is changed by adding an axial guide field, B_z, to the expression.

$$\frac{d(\gamma_R m_o c^2)}{dz} = \frac{-\omega/c}{2\gamma}\,\frac{|e|^2}{m_o^2 c^4}\,\frac{A_w A_R}{(\Omega_o/\gamma - k_w v_z)}\cos\psi$$

where $\Omega_o = |e|B_z/m_o c$ is the cyclotron frequency. One notices that if $\Omega_o \simeq \gamma k_w v_z$, the rate of acceleration can be substantially increased.

References

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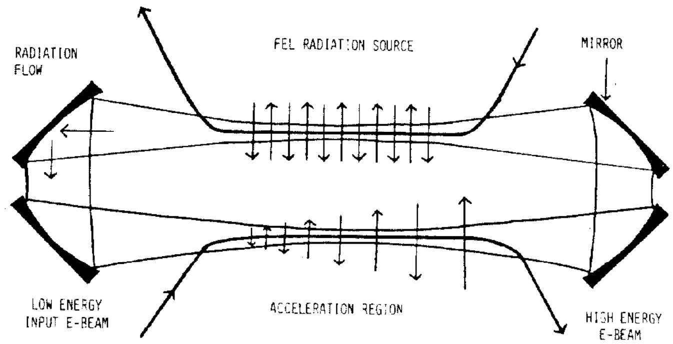

Fig. 3. Schematic diagram of the acceleration process utilizing the radiation field obtained from an FEL.

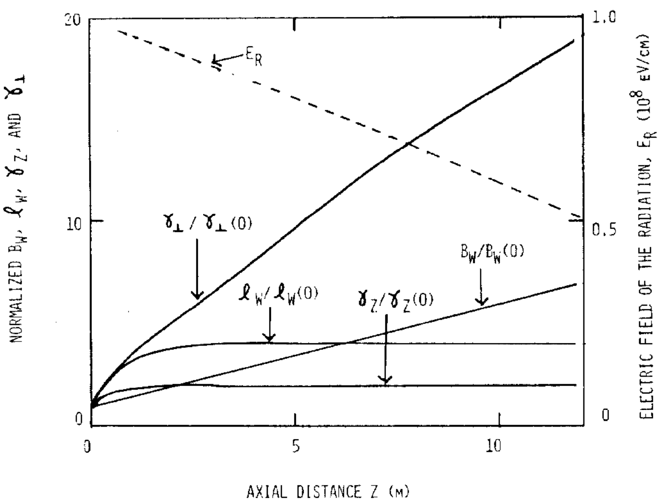

Fig. 1. Plots of the radiation electric field amplitude E_R, and normalized B_w, ℓ_w, γ_z and $\gamma_\perp$ as a function of axial distance z.

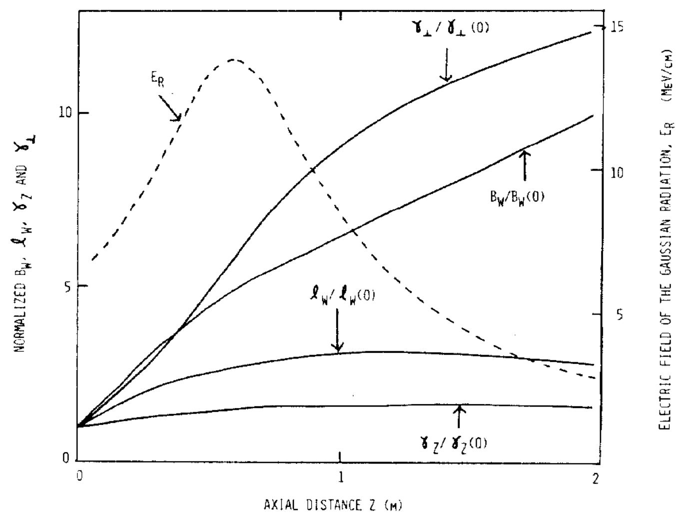

Fig. 4. Plots of the fundamental Gaussian free space diffracted radiation beam, and normalized B_w, ℓ_w, γ_w and $\gamma_\perp$ as a function of axial distance z.

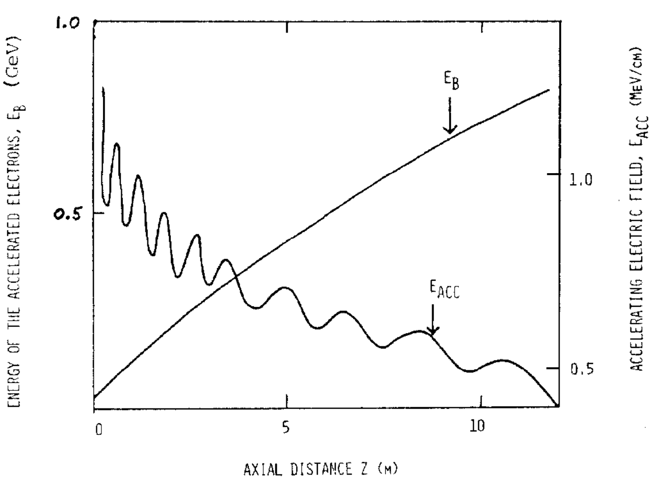

Fig. 2. Plots of the effective accelerating electric field, E_{acc}, and the energy of the accelerated electrons, E_b, as a function of axial distance z.

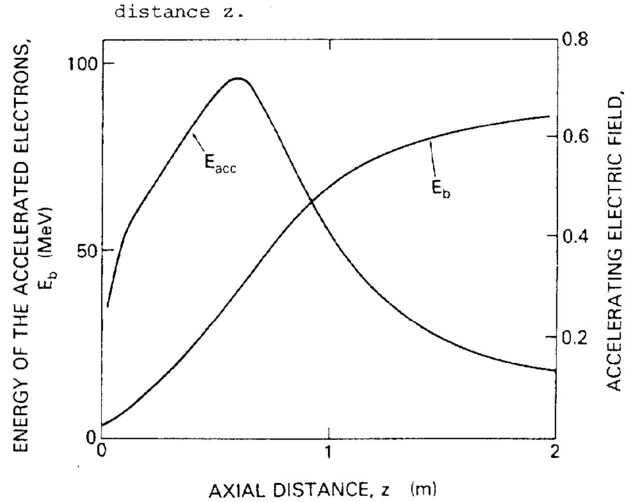

Fig. 5. Plots of the effective accelerating electric field, E_{acc}, and the energy of the accelerated electrons, E_b, as a function of axial distance z.