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BEAM QUALITY LIMITATIONS TO THE SINGLE PASS FEL DYNAMICS

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Abstract

The limitations on the FEL performances due to the electron beam length, energy spread and emittance are examined.

1. Introduction

During the last years ingrowing interest has been devoted to the single pass machines as electron beam (e.b.) sources for the Free Electron Laser (FEL) operation.[1,2]

Although the qualities of a superconducting Linac, such as the Stanford one,[1] are the ideally suitable ones for the study of the most important FEL operating characteristics, a number of practical considerations (e. g. the high cost of construction, the limitation on the current and the criticity of operation) has suggested to look for e.b. sources such as conventional linacs or microtrons. Many theoretical papers have been, therefore, aimed to understand the main features of the FELs operating with single pass machines.[3-5]

This note is devoted to the understanding of the limitations in the FEL operation due to the e.b. quality, in particular we shall investigate how energy spread and emittance affect characteristics of laser parameters such as the gain and the criticity of operation.

The analysis will be carried out in terms of the theoretical picture developed in[4,5]; thus before going further we recall the main features of the model

a) We assume to work in the low gain regime relevant to the Stanford experiment as well as to the other proposed FEL Compton experiments (for further comments see[5]).

b) The second assumption is the multimode[6] one, owing to the bunched structure of the e.b. which generates a kind of mode locking in the laser beam (l.b.).

c) The third pivoting point is the introduction of the so called "Super-Mode" picture[4] to overcome the difficulties of the longitudinal modes analysis.

2. Theoretical Picture

In dealing with the FEL theory we came across, as already stressed, with two main problems. Namely the necessity of a multimode analysis, carried out by means of an expansion of the laser field in terms of longitudinal modes of the optical cavity[6]; and, subsequently, with the obvious necessity of overcoming the difficulties arising in dealing with several thousands of longitudinal modes.

We found, therefore, expedient to lock for a new kind of "modal" expansion, different from the conventional one in the fact that each "mode", actually recognized as a Super Mode (SM), is a "coherent" superposition of a collection of longitudinal modes.[4]

More precisely, we looked for that particular configuration of spatial modes which reproduces itself unmodified after one wiggler passage. From the above definition it follows that each spatial mode, belonging to one SM, must obey the following equations for the energy density (W) and phase (φ) variation

$$\Delta W_i = \alpha W_i, \quad \Delta\varphi_i = \psi \tag{1}$$

where α and ψ are the gain and the advance in phase per pass respectively and are identical for each longitudinal mode belonging to the SM.

Both the energy density and phase variations have been derived in the framework of the small signal analysis of[6].

To better clarify the SMs physical role we can introduce a quantity, linked to both W_i and φ_i, but with a well defined physical meaning, namely[4,5]

$$\zeta(s) = \sum_i \left\{\sqrt{W_i}\,\exp(j\varphi_i)\right\}\exp\left[j\nu_i\,\frac{s_i}{\Delta}\right], \quad j = \sqrt{-1} \tag{2}$$

where

$$\begin{cases} s & \equiv \text{longitudinal electron coordinate} \\ \nu_i & = [1-\omega_i/\omega_o](\Delta\omega/\omega)_o^{-1}, \ \omega_i \equiv \text{i-th laser frequency} \\ \omega_o & = 2\pi c/\lambda_o \equiv \text{resonant frequency} \\ \lambda_o & = [\lambda_q/2\gamma^2](1+K^2) \equiv \text{resonant wavelength} \\ \lambda_q & \equiv \text{wiggler pass, } K = eB_o\lambda_q/\sqrt{2}\,2\pi m_o c^2 \text{ (linear polarized wiggler), } B_o \equiv \text{peak magnetic field} \\ (\Delta\omega/\omega)_o & = \lambda_q/2L_w, \ L_w \equiv \text{wiggler length} \\ \Delta & = \mu_c\sigma_z \equiv \text{e.b. - l.b. vacuum slippage due to the different beam velocities}^{4,5} \\ \mu_c & = \lambda_o/2\sigma_z\cdot(\Delta\omega/\omega)_o^{-1} \equiv \text{coupling parameter}^{4,5,7} \\ \sigma_z & \equiv \text{r.m.s. bunch length} \end{cases} \tag{3}$$

It is readily understood that $\zeta(s)$ is the "slowly varying" part of the laser electric field. We have, elsewhere, shown[4] that the equation defining SMs is

$$q_\gamma\zeta(s) = \int K(s,s_o;\ \mu_c;\ \mu_\varepsilon,\mu_a)\zeta(s_o)ds_o \tag{4}$$

where

$$q_\gamma = 1/g_o\left\{\alpha+\gamma_T/(1-\gamma_T)+2j\left[\psi-\pi/2(\Delta\omega/\omega)_o^{-1}\,g_o\,\Theta\right]\right\} \tag{5}$$

γ_T are the cavity losses and

$$g_o = 2\pi(2\lambda_o/\lambda_q)^{1/2}(I_P/I_o)(L_w\,\lambda_o/\Sigma_L)K^2/(1+K^2)^{3/2}\cdot(\Delta\omega/\omega)_o^{-2}, \quad I_o = ec/r_o$$

Σ_L = l.b. cross section = $\lambda_o L_w/\sqrt{3}$ to minimize the diffraction losses

I_P ≡ peak current

$\Theta = -(2/\pi)(\Delta\omega/\omega)_o\,\omega_o\delta t/g_o$ ≡ delay parameter

$$\delta t = T_c - T_e \tag{6}$$

(T_c, T_e) cavity round trip period and electron bunch-bunch time distance (further insight in the role played by the delay parameter Θ can be found in[4,5,7]. Finally the Kernel of the integral Eq. (4) writes (linear polarized wiggler)

$$\begin{aligned} K(s,s_o;\ \mu_c;\ \mu_\varepsilon,\mu_a) &= \Delta\,\Theta\,\delta'(s-s_o) - \\ &- j(2\pi)^{3/2}/\mu_c\Delta^2\int_{s_o}^{s+\Delta} f(z)dz\ \cdot \\ &\cdot \vartheta(s_o-s)\vartheta(s-s_o+\Delta)(s_o-s)\ \cdot \\ &\cdot \exp\{-1/2[\mu_\varepsilon\pi(s-s_o)]^2\}/(1+j\mu_a\pi\,(s-s_o)/\Delta) \end{aligned} \tag{7}$$

with δ' and ϑ derivative of the Dirac function and step function, respectively, and

$$\begin{cases} \mu_\varepsilon = (\Delta\omega/\omega)_\varepsilon\cdot(\Delta\omega/\omega)_o^{-1} \\ \mu_a = (\Delta\omega/\omega)_a\cdot(\Delta\omega/\omega)_o^{-1} \end{cases} \tag{8}$$

$(\Delta\omega/\omega)_\varepsilon$, $(\Delta\omega/\omega)_a$ being the inhomogeneous bandwidth due to the energy spread and emittance respectively[8], and reads

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$$\begin{cases} (\Delta\omega/\omega)_\varepsilon = 2\sigma_\varepsilon, \ \sigma_\varepsilon \equiv \text{r.m.s. relative energy spread} \\ (\Delta\omega/\omega)_a = (\sqrt{2}\ K/\sqrt{1+K^2})\sigma_a/\sqrt{\lambda_o\ \lambda_q}, \ \sigma_a = \text{r.m.s. emittance} \end{cases} \tag{9}$$

We stress that the SMs can be recognized as the eigenstates of the Eq. (4).

The next step will be the discussion of the numerical analysis of our main Eq. (4) and thus the dependence of the FEL relevant quantities on μ_c, μ_ε, μ_a.

3. Numerical Results

Before discussing the numerical results let us recall that the gain relevant to the first SM can be derived from (5) and reads

$$G = g_o \ \mathrm{Re}\, q_\gamma^1 \ (\Theta;\ \mu_c;\ \mu_\varepsilon,\mu_a) \tag{10}$$

The main goal of the forthcoming analysis will be the study of the numerical dependence of the gain function (10) on the parameters μ_c, μ_ε, μ_a; to this aim it appears expedient to introduce the following quantity

$$\beta(\mu_c;\mu_\varepsilon,\mu_a) = \frac{\mathrm{Re}\, q_\gamma^1(\Theta^\star(\mu_c;\ \mu_\varepsilon,\mu_a);\ \mu_c;\ \mu_\varepsilon,\mu_a)}{\mathrm{Re}\, q_\gamma^1(\Theta^\star(0;\ 0,0);\ 0,\ 0,\ 0)} \tag{11}$$

where $\Theta^\star$ indicates value of the delay parameter for which $\mathrm{Re}\, q_\gamma^1$ assumes the maximum value (and is in turn a function of our main parameters). The quantity at the denominator of (11) (i.e. the maximum gain for $\mu_c=\mu_\varepsilon=\mu_a=0$) was already derived in[4] and is given by

$$\mathrm{Re}\, q_\gamma^1(\Theta^\star(0;\ 0,\ 0);\ 0;\ 0,\ 0) \cong .85. \tag{12}$$

It follows from (9), (10) and (11) that it could be useful to express the gain as follows

$$G/I_p \sim \{7.68\times10^{-4}(\lambda_o/\lambda_q)^{1/2} \cdot$$
$$\cdot K^2/(1+K^2)^{3/2}\ (\Delta\omega/\omega)_o^{-2}\ \beta(\mu_c;\ \mu_\varepsilon,\ \mu_a)\}\mathrm{Amp}^{-1}$$

where the peak current I_p is expressed in Ampére.

In Figure 1 both $\beta(\mu_c;\ 0,\ 0)$ (Fig. 1a) and $\Theta^\star(\mu_c;\ 0,\ 0)$ (Fig. 1b) have been plotted. It appears

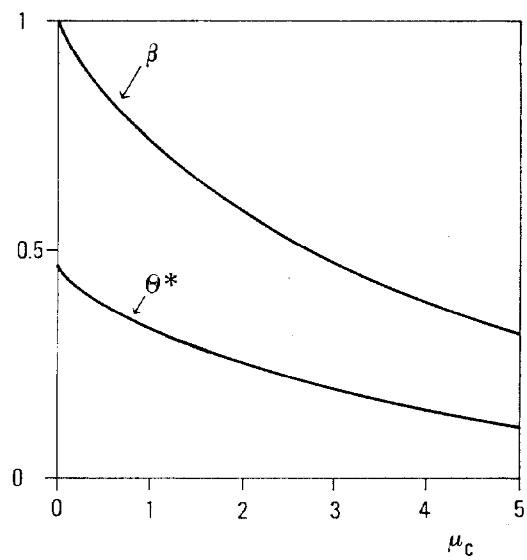

Fig. 1 - β and $\Theta^\star$ vs μ_c for $\mu_\varepsilon = \mu_a = 0$

that to increasing values of μ_c correspond decreasing values of β and $\Theta^\star$.

This result is not a new one[4], and can be understood by noticing that to higher values of μ_c correspond smaller values of σ_z compared to the slippage Δ, and this amounts to a "narrowing" of the gain region for the l.b.. In the mean time the working region for Θ becomes smaller, i.e. the FEL operation becomes more critical (see[4] for further comments).

In Figures 2, 3 have been plotted $\beta(0;\ \mu_\varepsilon,\ \mu_a = 0.0,\ 0.5,\ 1.0,\ 1.5,\ 2.0)$ and $\beta(1;\ \mu_\varepsilon,\ \mu_a = 0.0,\ 1.0,\ 2.0)$ respectively. The behaviour of Fig. 1 is confirmed,

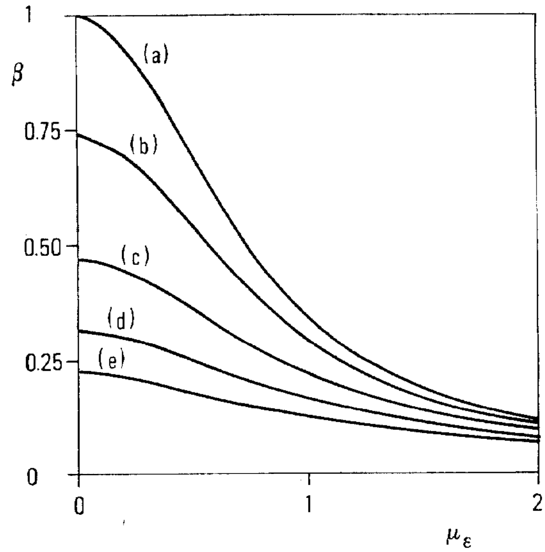

Fig. 2 - β vs μ_ε for $\mu_c = 0$, and $\mu_a = 0.0$ (a), 0.5 (b), 1.0 (c), 1.5 (d), 2.0 (e)

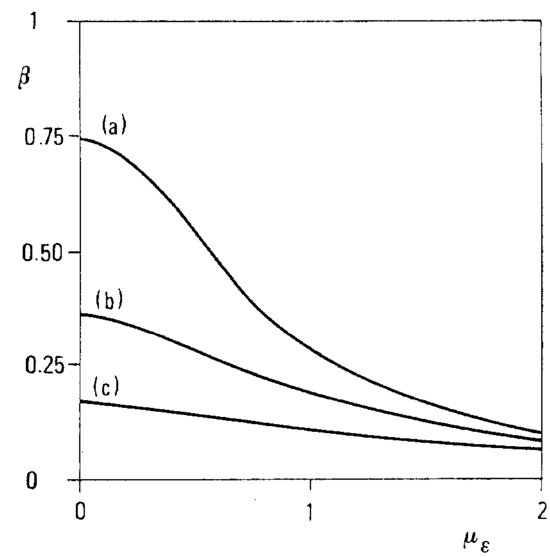

Fig. 3 - β vs μ_ε for $\mu_c = 1$, and $\mu_a = 0.0$ (a), 1.0 (b), 2.0 (c)

indeed $\beta(0;\ \mu_\varepsilon,\mu_a) > \beta(1;\ \mu_\varepsilon,\mu_a)$; furthermore we find the obvious fact that β (and in turn the gain) decreases for increasing values of the energy spread and emittance.

Finally in Fig. 4 it has been plotted $\Theta^\star(1;\ \mu_\varepsilon,\mu_a = 0.0,\ 1.0,\ 2.0)$ which shows that with increasing of both μ_ε and μ_a, $\Theta^\star$ decreases thus indicating a more critical operation with higher values of the energy spread and emittance.

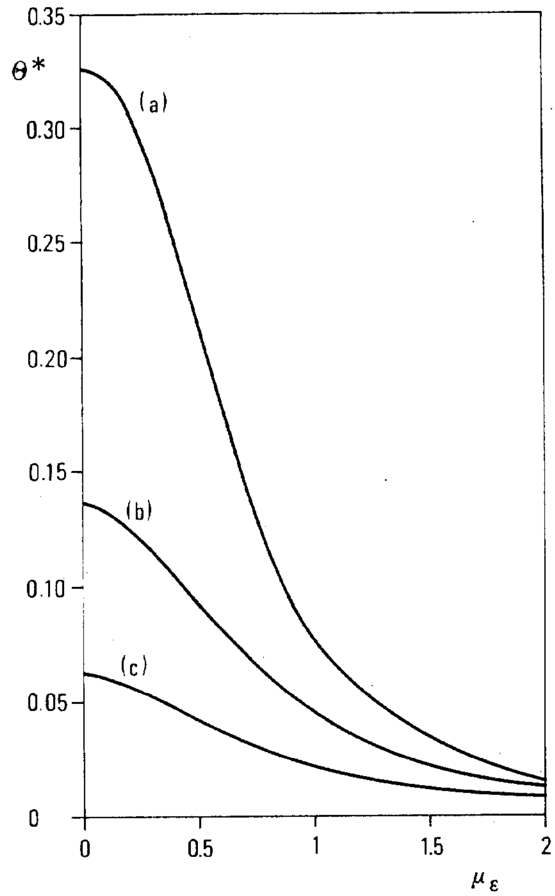

Fig. 4 - $\Theta^{\star}$ vs μ_{ε} for $\mu_c = 1$, and $\mu_a = 0.0$ (a), 1.0 (b), 2.0 (c)

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