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COMPUTATION OF INTRABEAM CHARGE EXCHANGE RATE*

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Summary

ANL's planned Accelerator Development Facility (ADF) for heavy-ion fusion will test and demonstrate virtually all of the beam manipulations thought to be necessary for a heavy-ion inertial confinement fusion driver. The relatively simple upgrade of adding synchrotron acceleration capability to the storage ring would also allow important energy deposition and materials experiments. The feasibility of this upgrade depends critically on the beam loss rate from charge exchange scattering. A computer program has been written for the purpose of obtaining a better estimate for this lifetime. The code assumes a K-V transverse distribution folded into a Neuffer longitudinal distribution. The emittance ellipse parameters, along with various estimates for the velocity-dependent ion-ion cross sections, are read in and a numerical integration is performed over the distribution, yielding a value for the loss rate $-1/N^2\, dN/dt$. Preliminary estimates indicate that this mode of beam loss presents no obstacle to upgrading the ADF.

Introduction

The ADF,[1] as presently planned, will produce 40 mA of Xe^{+8} at 220 MeV. Synchrotron acceleration capability to reach a kinetic energy of 10 GeV, if feasible, would increase the targetable ion-beam power by more than an order of magnitude with a relatively small cost increment. This upgrade would add to the ability to study important physics related to transport, focusing, energy deposition and equation-of-state of dense target materials. Specifically, 2 x 2 transverse splitting, final transport, and focusing could all be studied under conditions of realistically high beam intensity. Further, with an anticipated 3 kJ of beam energy delivered to a spot size of order 0.5 mm radius on a foil target, target temperatures of $k_B T \approx 50$ eV could be achieved. With this facility, the first experimental study of heavy ion energy deposition physics in high density plasma would become possible.

A critical issue in determining the practical feasibility of the ADF upgrade is the beam lifetime against intrabeam charge exchange. Synchrotron acceleration time would be between 0.1 and 1 second. The existing published, very rough estimate[2] gives a lifetime of about this same time; thus a better estimate is required. One of the uncertainties arises from a mathematical simplification used in all previous work - the use of orbit-averaged values for the collision velocities and cross section. We have carried out a more careful numerical study involving integration of the loss rate over the 6-dimensional beam, assuming a K-V transverse distribution in phase space. We also examined the available information on cross sections in more detail. Our conclusion, that the beam lifetime would, in fact, be long compared with the synchrotron ramp time, adds to the credibility of the ADF upgrade.

Loss Rate Integral

Quasiclassically, the rate at which ions change their charge state and, thus, are lost from the beam is

$$\begin{aligned}\alpha &= -\,dN/dt \\ &= \int d\underset{\sim}{x}\, d\underset{\sim}{v}\, d\underset{\sim}{v}'\, f(\underset{\sim}{x},\underset{\sim}{v})\, f(\underset{\sim}{x},\underset{\sim}{v}') \\ &\quad \times\ |\underset{\sim}{v}-\underset{\sim}{v}'|\ \sigma\,(|\underset{\sim}{v}-\underset{\sim}{v}'|)\end{aligned} \tag{1}$$

where $f(\underset{\sim}{x},\underset{\sim}{v})$ is the 6-dimensional distribution function describing the beam's phase space density, and σ is the cross section for loss of each ion

$$\sigma = \sigma\,(\text{charge exchange}) + \tfrac{1}{2}\,\sigma\,(\text{ionization})\ . \tag{2}$$

For f, we choose a K-V transverse distribution, multiplied by Neuffer's[3] suggested longitudinal distribution function. Properly normalized, this gives

$$\begin{aligned} f = {} & \frac{3N}{2\pi^3\, \varepsilon_x\, \varepsilon_y\, \varepsilon_z\, \beta^3\, \gamma^3} \\ & \times \left[1 - \left(\frac{z}{z_0}\right)^2 - \left(\frac{z_0}{\varepsilon_z}\right)^2 \left(\frac{v_z - \beta c}{\beta c} - \frac{z_0'}{z_0}\, z\right)^2\right]^{1/2} \\ & \times\ \Theta\left[1 - \left(\frac{z}{z_0}\right)^2 - \left(\frac{z_0}{\varepsilon_z}\right)^2 \left(\frac{v_z - \beta c}{\beta c} - \frac{z_0'}{z_0}\, z\right)^2\right] \\ & \times\ \delta\left[\left(\frac{x}{x_0}\right)^2 + \left(\frac{x_0}{\varepsilon_x}\right)^2 \left(\frac{v_x}{\beta c} - \frac{x_0'}{x_0}\, x\right)^2 \right. \\ & \left. +\ \left(\frac{y}{y_0}\right)^2 + \left(\frac{y_0}{\varepsilon_y}\right)^2 \left(\frac{v_y}{\beta c} - \frac{y_0'}{y_0}\, y\right)^2 - 1\right]\ , \end{aligned} \tag{3}$$

where the meanings of the symbols are as follows:

N = the number of particles in the bunch

$\pi\varepsilon_x$, $\pi\varepsilon_y$, $\pi\varepsilon_z$ = the beam geometrical emittance in the three planes

x_0, y_0, z_0 = the bunch half-lengths in the three directions

x_0', y_0', z_0' = the derivatives of y_0, y_0, z_0 with respect to circumferential distance around the ring

βc = the beam central velocity

Θ = the Heaviside step function.

Subsituting Eq. (3) into Eq. (1) and changing integration variables yields

$$\alpha = \frac{9N^2}{4\pi^6\, x_0\, y_0\, z_0} \int d\underset{\sim}{x}\, d\underset{\sim}{u}\, d\underset{\sim}{u}'\, \sqrt{1 - z^2 - u_z^2}$$

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$$\times \sqrt{1 - z^2 - u_z'^2}\ \tau\ \sigma(\tau)\ \delta(x^2 + y^2 + u_x^2 + u_y^2 - 1)$$

$$\times\ \delta(u_x^2 + u_y^2 - u_x'^2 - u_y'^2)\ \theta(1 - z^2 - u_z^2)\ \theta(1 - z^2 - u_z'^2)\ , \tag{4}$$

where

$$\tau = \beta c\{\left[(\varepsilon_x/x_o)(u_x - u_x')\right]^2 + \left[(\varepsilon_y/y_o)(u_y - u_y')\right]^2 + \left[(\varepsilon_z/z_o)(u_z - u_z')\right]^2\}^{\frac{1}{2}}\ . \tag{5}$$

We note that the angles, x_o', y_o', and z_o', have disappeared from Eq. (4). Straightforward manipulations further reduce Eq. (4) to

$$\alpha = \frac{3N^2}{\pi^5 x_o y_o z_o} \int_0^1 d\rho \int_0^{2\pi} d\theta_1 \int_0^{2\pi} d\theta_2 \int_{-1}^1 du \int_0^{|u|} dv$$

$$\times\ \rho\ (1 - v^2)^{3/2} \left[(1 + m)\ E(m)\ -\ (1 - m)\ K(m)\right]$$

$$\times\ \tau\ \sigma(\tau)\ ,$$

where

$$m = (1 - u^2)/(1 - v^2)$$

$$\tau = \beta c\ \{\left[(\varepsilon_x\ \rho/x_o)\ (\cos\theta_1 - \cos\theta_2)\right]^2 + \left[(\varepsilon_y\ \rho/y_o)\ (\sin\theta_1 - \sin\theta_2)\right]^2 + \left[(\varepsilon_z/z_o)\ (u - v)\right]^2\}^{\frac{1}{2}}\ . \tag{6}$$

In Eq. (6), E and K are the standard elliptic integrals

$$E(m) = \int_0^{\pi/2} (1 - m\sin^2\theta)^{\frac{1}{2}}\ d\theta$$

$$K(m) = \int_0^{\pi/2} (1 - m\sin^2\theta)^{-\frac{1}{2}}\ d\theta$$

Numerical Results

A computer program has been written to predict beam lifetime based on Eq. (6). Beam emittance, velocity, and bunch length (z_o) are input, as are tables of values for as a function of v and for x_o and y_o as functions of s (the measure of position around the ring). For each entered value of s, α/N^2 is evaluated by a Monte Carlo integration and is printed. Finally, an average loss rate is obtained by a trapezoidal rule average over the range of entered values.

The cross sections used were derived from the data of Angel, Dunn, Neill, and Gilbody,[4] who performed a Xe^{+1} - Xe^{+1} crossed beam experiment and reported the cross section for production of Xe^{+2} in one of the beams. Assuming negligible cross section for production of higher charge states, this means that their reported numbers correspond to

$$\tfrac{1}{2}\left[\sigma(\text{charge exchange}) + \sigma(\text{ionization})\right]\ . \tag{8}$$

Assuming a branching ratio of 1, we have, therefore, multiplied their results by 1.5 to obtain the combination (2). The tighter binding of Xe^{+8} is crudely accounted for by multiplying the reported Xe^{+1} cross sections by the square of the ratio of the radius of the outer electron shell in Xe^{+8} to the Xe^{+1} radius; this ratio is 0.15.[5] These approximations are supported to some extent by the observations that the geometric size of the Xe^{+1} ion, $r^2{}_{val} = 4.25 \times 10^{-16}\ cm^2$, is in rough agreement with the experimental data. Table 1 lists the resulting values of cross section versus collision velocity.

Beam parameters were chosen to correspond to the expected values in the ADF storage ring:

$$\varepsilon_x = \varepsilon_y = 10^{-2}\ \text{cm rad}$$

$$\varepsilon_z = 2 \times 10^{-3}\ \text{cm rad}$$

$$\beta = 0.06$$

$$z_o = 4278.54\ \text{cm}\ , \tag{9}$$

where the value of z_o corresponds to a bunching factor of 1/2. Figure 1 shows the anticipated transverse beam dimensions through one of the 4 ring superperiods, along with the value of α/N^2. The standard deviation from the Monte Carlo integration is less than 1%. From the average value $\alpha/N^2 = 6.3 \times 10^{-16}\ sec^{-1}$, and assuming, as expected, that $N = 2.5 \times 10^{12}$, we find that the beam lifetime at injection is about 10.6 minutes.

For comparison, the analytically estimated lifetime is

$$T = 4\pi\ \bar{x}_o\ \bar{y}_o\ z_o/3N\bar{v}\ \sigma(\bar{v}) \tag{10}$$

where the orbit-averaged beam parameters are

$$\bar{x}_o = \bar{y}_o = (\varepsilon_x R/\nu)^{\frac{1}{2}}$$

$$\bar{v} = \beta c\ (\varepsilon_x \nu/R)^{\frac{1}{2}}\ . \tag{11}$$

In Eq. (11), R is the ring radius and ν is the transverse tune. Substituting into Eq. (10) the values of 27.24 m and 5.25 for R and ν yields a predicted lifetime of 6.9 minutes.

The maximum loss rate will occur at the high energy end of the cycle. Assuming that ε_x, ε_y, ε_z, and z_o all scale like $(\beta\gamma)^{-1}$ (i.e., preserving normalized emittance and fractional momentum spread), and that the CSL transverse beam parameters remain unchanged, the loss rates are those shown in Fig. 2; the average rate is $\alpha/N^2 = 1.3 \times 10^{-13}\ sec^{-1}$. For the anticipated intensity, the average lifetime, even at this maximum loss rate, is about 3 seconds.

Conclusion

Losses from a Xe^{+8} beam due to intrabeam charge exchange have been computed from a detailed integral over the phase space distribution function. The assumed cross section was derived from published experimental data on Xe^{+1} - Xe^{+1} scattering, scaled down for Xe^{+8} by the ratio of geometrical ion sizes. Beam emittance parameters were those relevant to possible upgrading of ANL's ADF. The lifetime was found to be around 10 minutes at injection and still acceptably long (3 seconds) at full energy (10 GeV). If the loss rate changes roughly linearly through a one-second acceleration time, apporoximately 14% of the beam would be lost due to this mechanism. This appears to be an accceptably small loss for such a research facility.

Refereneces

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TABLE 1

v (cm/sec)	σ (cm^2)
0.0×10^7	0.9×10^{-17}
1.0	1.2
2.0	1.8
3.0	2.7
4.0	4.5
5.0	5.8
6.0	6.7
7.0	7.3
8.0	6.7
9.0	8.5
10.0	13.0

Effective charge changing cross section for Xe^{+8} - Xe^{+8} as a function of relative velocity.

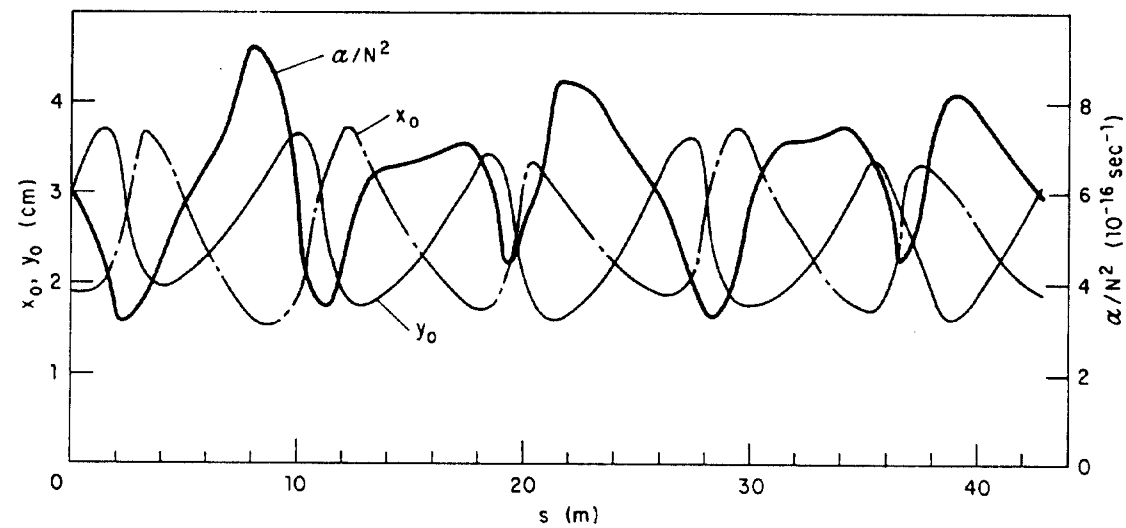

Fig. 1 Beam transverse dimensions and rate parameter α/N^2 as a function of position in one superperiod of ANL's ADF storage ring. T = 220 MeV; $\varepsilon_x = \varepsilon_y = 0.01$ cm rad; $\varepsilon_z = 0.0017$ cm rad; $B_f = \frac{1}{2}$.

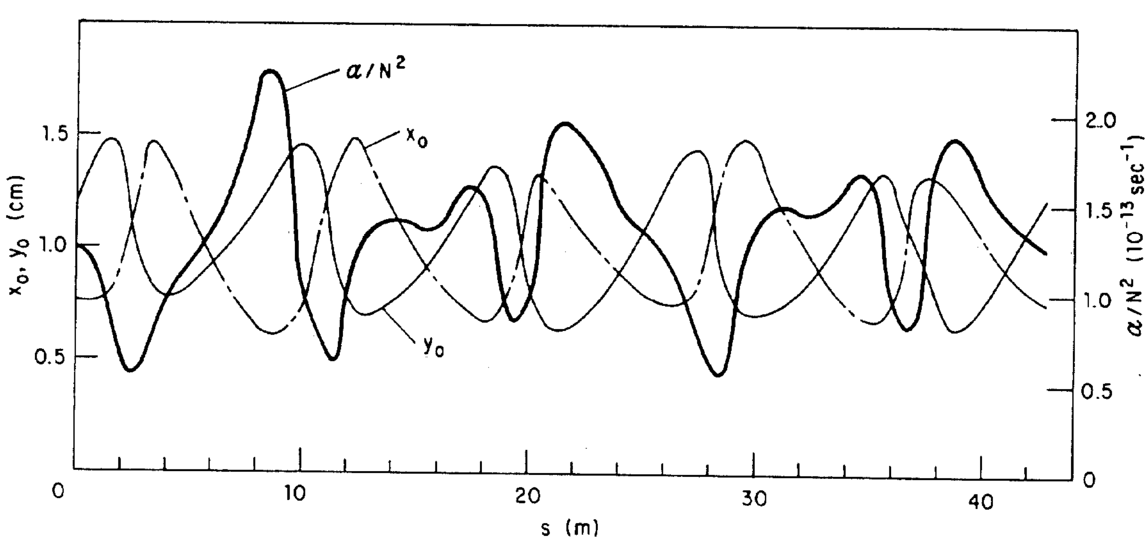

Fig. 2 Same as Fig. 1, T= 10 GeV; $\varepsilon_x = \varepsilon_y = 0.00157$ cm rad; $\varepsilon_z = 2.6 \times 10^{-4}$ cm rad; $B_f = 0.0742$.