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FREE ELECTRON LASERS*

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Introduction

It is the purpose of this paper to explain the various operating modes of the "free electron laser" in a manner that is easy for accelerator designers to understand.[1] The successful operation of the "free electron laser" by the group at Stanford, directed by John Madey,[2,3] along with the availability of high power electron beams has stimulated a great deal of interest in the use of the "free electron laser" (FEL) to produce high power tunable laser beams. At the last National Accelerator Conference held in San Francisco, Pellegrini[4] presented a paper on the FEL. Since his paper contains an excellent discussion of the work preceding and following Madey's experiments, we will not repeat it here. Often the analysis of the FEL, as well as the methods envisioned for the FEL operation, started with the assumptions that the period and amplitude of the wiggler field were constant and the magnetic field uniform in the transverse direction. A great deal of the earlier work, using the techniques of the laser and plasma physics community, was presented at the 1977 Telluride Conference.[5] Here Colson developed the equations of motion for a single electron moving through a wiggler in the presence of an electromagnetic wave propagating along the wiggler axis.

The similarity of these equations of motion to those used by accelerator physicists, designing radio frequency accelerating systems, allowed us[6] to use the ideas developed for the acceleration of charged particles to guide the design of the FEL. The FEL is viewed as a decelerator in which the usual longitudinal accelerating field of a microwave cavity is replaced by the transverse decelerating field of a laser. The FEL uses a periodic transverse magnetic field (wiggler field) to provide the coupling between the longitudinally directed electrons and the transverse laser field. At the 1979 Telluride Conference[7] many of the papers used terms like: coupling between the betatron and synchrotron oscillations, stationary and decelerating buckets, phase displacement, adiabatic capture, and detrapping. Accelerator physics has both influenced FEL design and introduced a new jargon into the field.

Since the equations of motion have been derived before,[6] the main emphasis of this paper is to demonstrate the similarity of these equations to those studied by the accelerator physicist for many years. In the following survey of the field, we will use accelerator methods to discuss the present FEL designs. Because this survey is mainly tutorial the derivation of the equations is presented in a physically intuitive fashion rather than in a strictly rigorous manner.

Equations of Motion

When an electron travels through a periodic transverse magnetic field, the electron is given a transverse velocity which allows it to either receive or give energy to the transverse electric field of a plane electromagnetic wave. If the longitudinal velocity of the electron is such that the electron slips behind the radiation wave by one radiation wave length while traveling a distance of one magnetic field period, the transverse velocity of the electron remains in resonance with the electric field of the radiation. Such electrons will continue to have their energy increased or decreased, depending on the phase of their transverse velocity relative to the radiation field. We define this phase by

$$\psi = \left\{ \int_0^z \left[k_w(z_1) + k_s(z_1)\right] dz_1 - \omega_s t \right\} \tag{1.1}$$

where z is the longitudinal position of the electron at time t; k_w and k_s are the wave numbers of the wiggler magnetic field and the signal (or optical field), which we allow to be functions of z, and ω_s is the frequency of the signal field. The equation of motion for ψ is given by

$$\dot{\psi} = \left(k_w + k_s\right)\dot{z} - \omega_s \tag{1.2}$$

If we substitute $\omega_s = k_s c$ we see that $\dot{\psi} = 0$ corresponds to the resonant condition described above. The longitudinal velocity $\dot{z}$ depends upon the electron energy by the relativistic approximation $(c - v) = c/2\gamma^2$ and upon the transverse velocity by the relation $v^2 = v_x^2 + v_y^2 + v_z^2$. The transverse velocity in the wiggler is sinusoidal with wave number k_w and rms amplitude equal to ca_w/γ, where we have introduced the dimensionless vector potential for the wiggler field

$$a_w = \left(\frac{e}{mc^2}\right)\frac{B_w}{k_w} \tag{1.3}$$

with B_w the rms magnetic field of the wiggler, and γ the energy parameter of the electron. We use the assumptions that $v_x/v << 1$ and $v_y/v << 1$ to obtain

$$\psi' = k_w - \frac{k_s}{2\gamma^2}\left(1 + a_w^2\right) \tag{1.4}$$

where we have changed the independent variable from t to z by noting that $dz = v_z dt \approx cdt$, and the prime denotes the z derivative. The rate of change in the energy of the electron is proportional to the product of its transverse velocity (produced by the wiggler field) and the transverse electric field of the radiation and is given by

$$\gamma' = -\frac{k_s a_s a_s}{\gamma}\sin\psi \tag{1.5}$$

where we introduced the dimensionless vector potential for the signal field

$$a_s = \left(\frac{e}{mc^2}\right)\frac{E_s}{k_s} \tag{1.6}$$

with E_s the rms electric field strength of the signal field. For completeness we need the equation that describes the signal field a_s; this we obtain by observing that the energy lost by the electrons increases the energy in the optical field. This gives the following expression

$$a_s^2(z) = a_s^2(0) - \frac{Z_0 J}{k_s mc^2}\left[\bar{\gamma}(z) - \bar{\gamma}(0)\right] \tag{1.7}$$

where J is the current density of the beam, Z_0 is the impedance of free space, and the bar over γ indicates the average over all of the electrons. There is, of course, another equation for the phase of the signal field, or equivalently the wave number k_s, which we will discuss later. It should be noted that certain assumptions, discussed by Kroll et al.,[6] have been made in deriving the previous equations. Given the initial conditions $\psi(0)$ and $\gamma(0)$ of every electron along with the expressions for $k_w(z)$, $a_w(z)$ and $a_s(z)$ the equations of motion [Eqs. (1.4) and (1.5)] may be integrated to yield the values of ψ and γ as functions

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of the longitudinal position z. In principle this procedure could be used to choose the optimum functional form for $k_w(z)$ and $a_w(z)$.

The equations of motion given above are similar to those used to describe the rf acceleration of charged particles; therefore, it is possible to understand much about the motion of the electrons without the need to integrate these equations for every electron.

Motion About the Synchronous Energy

In an analysis similar to that used in rf accelerator theory, we define the synchronous or resonant particle by its energy, γ_r, and its phase ψ_r, by

$$\gamma_r^2(z) = \frac{k_s}{2k_w}\left(1 + a_w^2\right) \tag{2.1}$$

and

$$\gamma_r'(z) = -\frac{k_s a_s a_w}{\gamma_r}\sin\psi_r \tag{2.2}$$

From Eq. (2.1) it follows that $\psi_r' = 0$ for an electron at the resonant energy. It is possible to look at Eqs. (2.1) and (2.2) as definitions of γ_r and ψ_r assuming that k_w, a_w and a_s are known functions of z. However, it is also possible to consider these equations as design equations to yield the desired functions ψ_r, γ_r and a_s. An analogous approach is used in proton linacs to determine the drift tube lengths once the energy gain and stable phase angle are chosen. Note that one is restricted in the choice for γ_r' and a_s since ψ_r is undefined if

$$\left|k_s a_s a_w\right| < \left|\gamma_r \gamma_r'\right| \tag{2.3}$$

This is also analogous to the case of a protron linac where the voltage would be too low to achieve the desired energy gain.

In the latter part of this paper we will discuss how the different operating modes of the FEL relate to the choice for the functions a_w, k_w, a_s, k_s, ψ_r and γ_r. First, we must study the motion of the electron with phase and energy different from the resonant value. Again we follow rf accelerator theory, and define the difference between the electron energy and the resonant energy by

$$\delta\gamma = \gamma - \gamma_r \tag{2.4}$$

and use the assumption that $|\delta\gamma| << \gamma_r$ to obtain the following equations of motion

$$\delta\gamma' = -\frac{k_s a_s a_w}{\gamma_r}\left(\sin\psi - \sin\psi_r\right) \tag{2.5}$$

and

$$\psi' = k_s\left(\frac{1}{\gamma_r^2} + \frac{a_w^2}{\gamma_r^2}\right)\frac{\delta\gamma}{\gamma_r} = 2k_w\left(\frac{\delta\gamma}{\gamma_r}\right) \tag{2.6}$$

Equations (2.5) and (2.6) are extremely familiar to us as accelerator physicists, and for the case where the parameter changes in the above equations are adiabatic we can immediately draw the trajectories of an electron in the phase plane.

These trajectories are shown in Fig. 2.1 for the

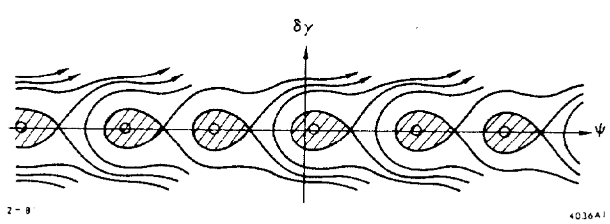

Fig. 2.1. Phase curves $\delta\gamma$ versus ψ.

case where $\sin\psi_r < 0$. For constant parameter wigglers $\psi_r = 0$. The sign of ψ_r has been chosen opposite to that used by accelerator designers; when $0 < \psi_r < \pi/2$, the resonant electron is decelerated. The closed trajectories correspond to electrons trapped in buckets and which perform synchrotron oscillations about the resonant phase and energy.

The maximum stable phase curve or bucket for $-\pi < \psi < \pi$ is shown in Fig. 2.2. It follows from Eqs.

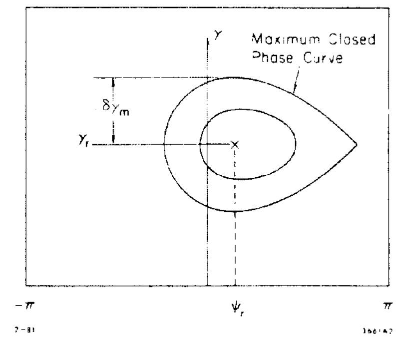

Fig. 2.2. Phase Curves $\delta\gamma$ versus ψ.

(2.5) and (2.6) that the maximum value of $\delta\gamma$ for which a particle may be trapped in a bucket is

$$\delta\gamma_m = 2\gamma_r\sqrt{\frac{a_s a_w}{1 + a_w^2}}\;\Gamma\left(\psi_r\right) \tag{2.7}$$

with

$$\Gamma\left(\psi_r\right) = \sqrt{\cos\psi_r - \left(\frac{\pi}{2}\sin\psi_r - \psi_r\right)\sin\psi_r} \tag{2.8}$$

We note that Γ varies between its maximum value of one (at $\psi_r = 0$) to a value of zero (at $\psi_r = \pi/2$). The area of the bucket shown in

$$A = 16\,\gamma_r\sqrt{\frac{a_s a_w}{1 + a_w^2}}\;\alpha\left(\psi_r\right) \tag{2.9}$$

where $\alpha(\psi_r)$ is the usual moving bucket parameter which varies between a value of one (at $\psi_r = 0$) to a value of zero (at $\psi_r = \pi/2$). We see that we have the usual condition—similar to that for rf accelerator systems—that both the height and area of the bucket decrease as the deceleration (or acceleration) rate increases until at a resonant phase of $\pi/2$ the bucket disappears.

The small amplitude or linear oscillation frequency also follows from Eqs. (2.5) and (2.6) and is

$$\Omega_\ell = 2k_w\sqrt{\frac{a_s a_s \cos\psi_r}{1 + a_w^2}} = k_w\sqrt{\frac{\cos\psi_r}{\Gamma\left(\psi_r\right)}}\left(\frac{\delta\gamma_m}{\gamma}\right) \tag{2.10}$$

Constant Parameter Wiggler

The "standard operational mode" of the FEL, which was the mode used in the Madey experiment, is one in which the wiggler wave number k_w and field amplitude a_w are constant. The resonant phase $\psi_r = 0$ and the resonant energy γ_r is constant given by

$$\gamma_r^2 = k_s\left(\frac{1 + a_w^2}{2k_w}\right) \tag{3.1}$$

In this mode the buckets are nonaccelerating or stationary. At first glance it is a little difficult to understand how such a device can work since electrons injected near the resonant energy with a uniform phase distribution will have their energy oscillate about the resonant energy (which remains constant). The key to successful operation of the FEL in this mode is to inject the electrons above the resonant energy and to allow them to complete only a fraction of an oscillation, as shown in Fig. 3.1.

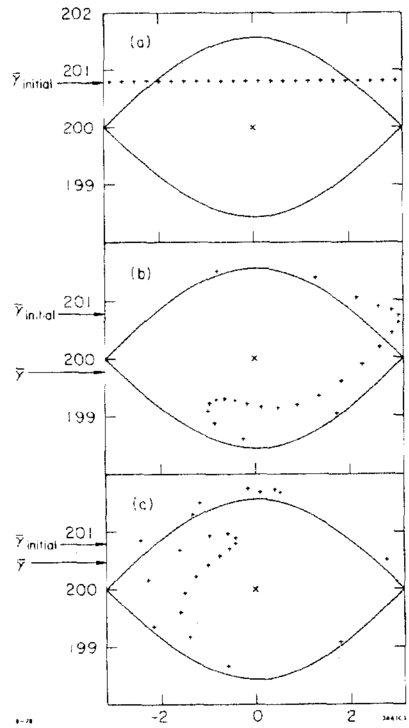

Fig. 3.1. Evolution of the electron energy distribution. (a) Initial distribution; (b) after one-half oscillation; (c) after nearly one oscillation.

The electrons with either large or small energy deviations (compared to $\delta\gamma_m$) will have only a small average energy change, while electrons with an initial energy deviation $\delta\gamma_i \sim \delta\gamma_m$ and which perform approximately one-half of an oscillation will have their average energy reduced the most. For maximum gain it is important to choose both the initial energy and the wiggler length correctly. Fig. 3.2 shows how for a fixed wiggler length, the energy extracted from the beam depends upon the initial energy.

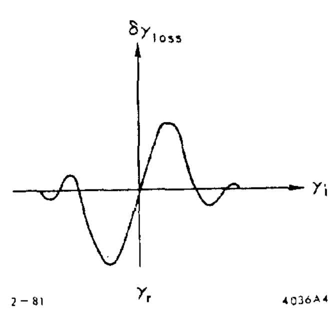

Fig. 3.2. Gain curve. Energy loss of electrons versus initial electron energy or signal frequency.

It is clear that either we can regard the electron energy as the quantity which differs from its resonant value or we can regard the signal frequency ($\omega_s = k_s c$) as the quantity which differs from its resonant value. The relation relating these two viewpoints is

$$\frac{\delta\omega_s}{\omega_s} = 2\left(\frac{\delta\gamma}{\gamma}\right) \qquad (3.2)$$

Therefore, Fig. 3.2 may also be regarded as a plot of the signal intensity increase as a function of the signal frequency. In laser physics we refer to such a curve as the gain curve. The width of this gain curve is of the order of the bucket height ($\delta\gamma_m/\gamma$), so that for a beam of electrons with an initial energy spread much larger than the bucket height, only a small fraction will transfer energy to the optical wave. We see from Fig. 3.1 that the maximum energy loss by the electrons is given by

$$\left(\frac{\delta\gamma}{\gamma}\right)_{\text{max loss}} \sim \left(\frac{\delta\gamma_m}{\gamma}\right) = 2\sqrt{\frac{a_s a_w}{1+a_w^2}} \qquad (3.3)$$

while electrons emerge from the wiggler with an energy spread

$$\left(\frac{\delta\gamma}{\gamma}\right)_{\text{spread}} \sim \left(\frac{\delta\gamma_m}{\gamma}\right) \quad . \qquad (3.4)$$

The optimum wiggler length for maximum energy transfer from the electrons to the signal wave follows from Eq. (2.10)

$$L = \frac{\pi}{\Omega_\ell} = \frac{\lambda_w}{2\left(\frac{\delta\gamma_m}{\gamma}\right)} \qquad (3.5)$$

with λ_w the wiggler period = $2\pi/k_w$. When this equation is combined with Eq. (3.3) we find that the maximum energy that can be extracted from the electrons in this mode of operation is given by the simple relationship

$$\left(\frac{\delta\gamma}{\gamma}\right)_{\text{max loss}} \sim \frac{1}{2N} \qquad (3.6)$$

where N is the number of wiggler magnet periods. The fact that, for a constant parameter wiggler, the average energy spread produced by the FEL is always equal or greater than the average energy loss follows from a more general theorem proved by Madey.[8] This places a severe restriction on the efficiency of this type of FEL. Renieri[9] has shown that if such an FEL is operated in a storage ring, with the synchrotron radiation used to damp the energy spread due to the FEL, the maximum obtainable laser power is related to the power radiated into synchrotron radiation by

$$P_{\text{laser}} \leq 2P_{\text{synch}}\left(\frac{\delta\gamma_m}{\gamma}\right) \quad . \qquad (3.7)$$

There are FEL-storage ring projects, which use the radiation damping to limit the energy spread of the electrons, at Orsay, Frascati, Brookhaven and Novosibirsk.

It should be noted that the equations of motion derived above are Hamiltonian, guaranteeing that the phase area is conserved. The net increase in the energy spread is due to a combination of phase area filamentation and a smearing of the optical phase angle as the electrons travel around the storage ring from the end of the FEL back to the entrance.

Deacon[10] has analyzed the operation of a constant parameter FEL in an isochronous storage ring. As an electron travels around the ring, such a device maintains the phase relationship between the electron and the optical wave. The electron is trapped in the optical bucket and can transfer energy from the low frequency rf cavity to the high frequency optical cavity. He discusses the design of a storage ring capable of restricting the spread in the longitudinal position of the electrons to be less than a fraction of the optical wave length. This requires a very low momentum compaction factor, and the longitudinal focusing is very weak in the absence of the optical wave. At the present time, this idea has not been explored experimentally.

Other Wiggler Schemes

While it is gratifying to note that the results of the constant parameter wiggler may be obtained by the graphic use of optical buckets, the real use of this technique is in the FEL design of a variable parameter wiggler. By using this technique it is easy to devise schemes where a significant fraction of the electrons are captured in a decelerating bucket as shown in Fig. 4.1.

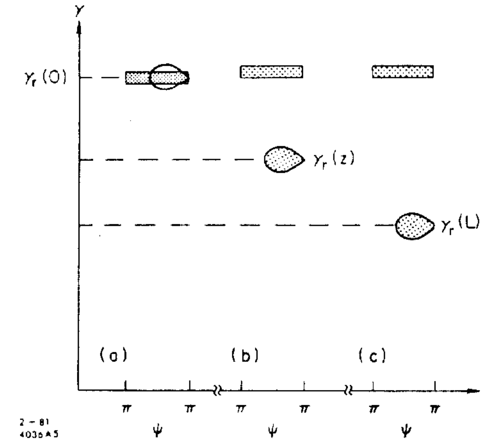

Fig. 4.1. Electron phase space distribution: (a) at the wiggler entrance, (b) at the wiggler center, and (c) at the wiggler exit.

Electrons which are trapped in the bucket will have their average energy reduced by an amount equal to the decrease in the resonant energy given by $\Delta\gamma_r = [\gamma_r(0) - \gamma_r(L)]$. In order to simplify the design of such an operational mode it is useful to consider the case where ψ_r is chosen constant between 0 and $\pi/2$. It follows from Eqs. (2.1) and (2.2) that the change in the resonant energy is related to the change in the wiggler parameters by

$$\Delta\left(\gamma_r^2\right) = \frac{k_s}{2}\,\Delta\left(\frac{1+a_w^2}{k_w}\right) \tag{4.1}$$

with the constraint that the wiggler length is

$$L = \frac{\left(\dfrac{\Delta\left(\gamma_r^2\right)}{2k_s a_s \bar{a}_w}\right)}{\sin\psi_r} \tag{4.2}$$

where $\bar{a}_w$ is the value of a_w averaged over z. From these equations we see that, for a long enough wiggler, it is possible to produce a desired change in the resonant energy by an appropriate change in the wiggler parameters, a_w and k_w, as a function of z. The choice for the value of the resonant phase is determined by a compromise between the desire to minimize the wiggler length (large ψ_r) and the desire to capture a large fraction of the electrons (small ψ_r). The optimum is near $\pi/4$. From Fig. 4.1 we see that in order to trap a significant fraction of the electrons it is important that the electrons enter with an energy spread less than the bucket height [Eq. (2.7)]. In addition, the transverse emittance ε produces an equivalent energy spread due to the fact that a spread in the transverse dimension produces a spread in a_w while a spread in the transverse velocity produces a spread in the longitudinal velocity. This additional equivalent energy spread is given by[11]

$$\left(\frac{\delta\gamma}{\delta}\right)_{\text{equiv}} = \frac{k_w a_w}{2\left(1+a_w^2\right)}\;\frac{\gamma\varepsilon}{\pi}\quad . \tag{4.3}$$

As we see from Eq. (2.7) the bucket height is proportional to the square root of the laser field, i.e., the fourth root of the laser intensity! The energy spread and the emittance from most linacs require a rather large laser intensity to produce the necessary bucket area. Diffraction effects relate the radius of the optical beam and thus the optical intensity to the FEL length, while both the FEL length and the optical field are related to the energy extracted from the electrons by Eq. (4.2). All of these relationships must be combined to obtain a consistent set of parameters and then some type of optimization performed.[7] This type of procedure nearly always requires a good quality electron beam with a high peak current to obtain the desired optical gain and energy extraction from the electrons in a single pass FEL.

At the present time in this country there are three single pass experiments under way to study this mode of FEL operation as illustrated in Fig. 4.2.

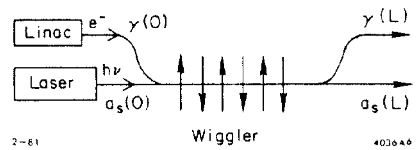

Fig. 4.2. Proof of principle experiment.

These experiments are located at TRW, LASL and Math. Sci. N.W. All experiments use a 10.6 μm laser beam (CO_2 laser) and a SmCo permanent magnetic wiggler (similar to that used in the SPEAR storage ring). The electron beam energies vary from 20 to 25 MeV. MSNW is using the Boeing linac and TRW is using the EGG linac, while LASL is updating one of their linacs for these experiments. These experiments should be completed soon, and we eagerly await the results.

Another scheme,[12] designated as a gain expanded FEL, has been proposed by the Stanford group to reduce the sensitivity of the optical gain to the electron energy. This method utilizes a transverse gradient in the wiggler magnetic field which produces a dispersion (i.e., different energy electrons have different equilibrium orbits) in the FEL. The wiggler field is chosen so that, regardless of its energy, an electron entering on its equilibrium orbit will travel through the wiggler with an average longitudinal velocity which is independent of its energy. The resonant condition is given by [Eq. (2.1)]

$$k_w = \frac{k_s}{2\gamma^2}\left(1 + a_w^2\right) \tag{4.4}$$

we see that the dispersion $\eta = \gamma(dx/d\gamma)$, and the transverse variation of the magnetic field must satisfy the relation

$$\eta\,\frac{da_w}{dx} = \frac{\left(1+a_w^2\right)}{a_w}\quad . \tag{4.5}$$

There are two operating regimes for this device. The first occurs when the synchrotron frequency is small compared to the betatron frequency. In this regime the coupling between the synchrotron and the betatron motion results in a decrease of the electron energy accompanied by an increase in the transverse betatron amplitude. There is hope, by using a separated function wiggler, that cancellation of the driven transverse oscillations may be achieved. In the second or high field regime the gain expanded FEL behaves similarly to the variable parameter FEL. The electrons are trapped in optical buckets and as their energy decreases they move transversely to a lower magnetic field. This regime has many of the same advantages and problems as the variable parameter FEL discussed above.

A phase area displacement scheme would be useful to decelerate electrons with only a modest increase in the electron energy spread. This method allows all of the electrons to be decelerated even when the initial energy spread (including the effective energy spread from the transverse emittance) is much larger than the bucket height. This scheme is shown in Fig. 4.3.

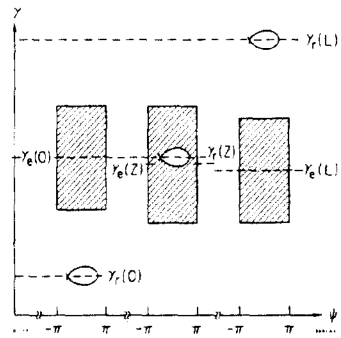

Fig. 4.3. Position of empty bucket and phase area of electrons at various positions in the FEL.

Phase area displacement could be used in a single pass device where the energy is too large to use the capture-deceleration scheme or in a storage ring where is is desirable to be able to decelerate all of the electrons with a large energy spread while minimizing the increase in the energy spread. This method requires a rather long wiggler to insure that the change in the resonant energy is adiabatic.

There are many other types of schemes that have been proposed to overcome some of the difficulties mentioned above. The use of the optical bucket concept has been instrumental in both the conception and the interpretation of many of these new schemes.

References

1. The formulation used in this paper to describe the operation of the "free electron laser" was developed at a summer workshop in La Jolla in 1978, by a collaboration with N. M. Kroll, R. Novick and M. N. Rosenbluth. Much of what is in this paper presented is due to this work.
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