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DETERMINATION OF TRANSVERSE PHASE-SPACE AND MOMENTUM ERROR FROM SIZE MEASUREMENTS ALONG THE 50-MeV H^- RCS INJECTION LINE*

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Summary

The 50-MeV H^- injection line for the RCS at Argonne National Laboratory has 16 quadrupole and eight bending magnets. Horizontal and vertical profiles can be obtained at 12 wire scanner positions. Size information from these profiles can be used to determine the three ellipses parameters in each plane required to describe the transverse phase space. Those locations that have dispersion permit the momentum error to be used as a fourth fitting parameter. The assumed accuracy of the size measurements provides an error matrix that predicts the rms errors of the fitted parameters.

Description

Let

$$\begin{pmatrix} A_i & B_i & C_i \\ D_i & E_i & F_i \\ 0 & 0 & 1 \end{pmatrix}$$

be the transfer matrix from the beginning of the beam line to wire scanner i for the vector

$$\begin{pmatrix} X \\ X' \\ \frac{\Delta P}{P} \end{pmatrix}$$

where X, X' stands for either the initial horizontal or vertical positions and slope, and $\Delta P/P$ the momentum error of an ion. All X, X' are assumed to lie on ellipse contours described by

$$X^2 + (\alpha X + \beta X')^2 = \beta\varepsilon$$

where

$\sqrt{\beta\varepsilon}$ = the half size,

α = the orientation, and

$\pi\varepsilon$ = the area of the contour.

At wire scanner i, all ions from the same contour fall on an ellipse having the same area described by the parameters

$$\beta_i = A_i^2\beta - 2A_iB_i\alpha + C_i^2\gamma$$

$$\alpha_i = \alpha(A_iE_i + B_iD_i) - B_iE_i\gamma - A_iD_i\beta$$

$$\gamma = (1+\alpha^2)/\beta$$

Let $\pi\varepsilon$ be the area of a particular limiting contour that includes a specified percentage of the total number of ions. After adding the dispersion, the calculated percentage width at wire scanner i is given by

$$W(C)_i = 2\left(\sqrt{A_i^2BE - 2A_iB_i\alpha E + B_i^2\gamma E} + \left|C_i\frac{\Delta P}{P}\right|\right)$$

The initial parameters of the beam are those values of B, α, E, $\Delta P/P$ which most nearly predict the measured percentage widths $W(M)_i$ at all wire scanners. We assume these best values to be those that minimize the expression[1]

$$\chi^2 = \sum_i \frac{\Delta W_i^2}{E_i^2}$$

where $\Delta W_i = W(C)_i - W(M)_i$. Thus, we wish to obtain solutions of

$$\frac{\partial\chi^2}{\partial\mu_j} = 0$$

$$\mu_1 = B,\ \mu_2 = E,\ \mu_3 = \alpha,\ \mu_4 = \frac{\Delta P}{P}$$

To solve these equations, one substitutes trial values for μ_j and calculates corrections $\partial\mu_j$ by solving

$$\sum_j M_{kj}\,\delta\mu_j = -D_k \qquad k = 1, 2, 3, 4$$

where

$$M_{kj} = \sum_i \frac{1}{E_i^2}\left(\frac{\partial\Delta W_i}{\partial\mu_k}\frac{\partial\Delta W_i}{\partial\mu_j} + \Delta W_i\frac{\partial^2\Delta W_i}{\partial\mu_k\partial\mu_j}\right)$$

$$D_k = \sum_i \frac{\Delta W_i}{E_i^2}\frac{\partial\Delta W_i}{\partial\mu_k}$$

This procedure is iterated with $\mu_j + \delta\mu_j$, replacing μ_j until $\delta\mu_j$ is sufficiently small. The diagonal elements of the reciprocal of the converged M_{kj} matrix are the predicted rms errors of the parameters. The off-diagonal elements are a measure of the correlations of the parameters. (For more details of the program and measurements, see Refs. 2 and 3.)

Results

The RCS injection beam line is shown schematically in Fig. 1 and in more detail in Figs. 2 and 3. Dipole, focusing quadrupole, and defocusing quadrupole magnets are represented, respectively, by on-axis, above-axis, and below-axis rectangles in Figs. 2 and 3. The measurements consist of moving both vertical and horizontal thin wires through the beam and measuring the intensity of the scattered electrons from the H^- ions at 0.02 in. intervals. This raw data is spline-fitted and the widths of a specified percentage of the beam is calculated from the smoothed results.

Figures 2 and 3 show the measured beam sizes and the predicted beam characteristics for a typical running condition of the line. In Fig. 2, the dispersion width is shown by the inner two curves. The outer curves show the total beam size. The measurements are for 90% of the beam and the estimated errors are 10%. The normalized χ^2 is χ^2/σ where σ is the number of degrees of freedom ($\sigma = N-4$ for the horizontal and $N-3$ for the vertical. N is the number of measurements).

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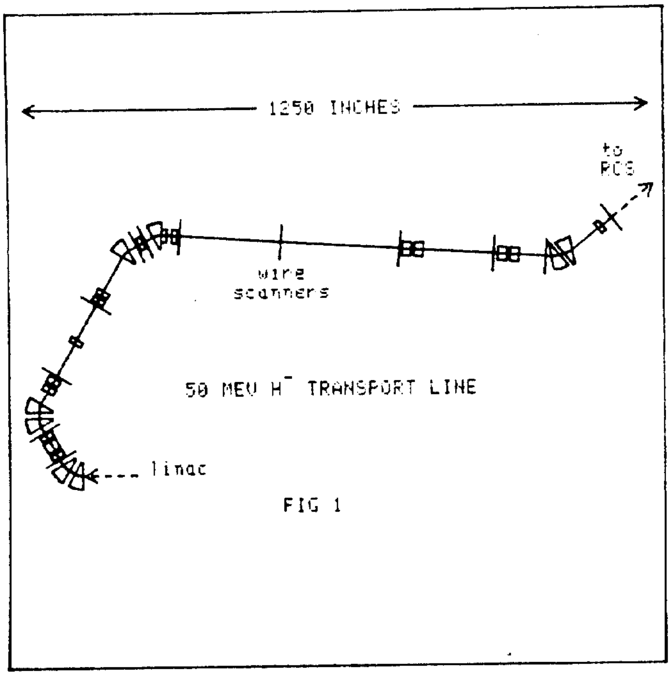

FIG 1

The larger rms errors assigned to the horizontal parameters are caused by a relatively strong correlation between the transverse beam quality and the dispersion in the fitting equation. If the horizontal beam quality, ε, is fixed at 0.25 mrad in. (1 standard deviation larger and smaller than the horizontal and vertical predictions), the other three parameters can be varied for a new fit to the data. The resulting fit is almost as good with the predicted momentum spread reduced to 0.12% and the estimated errors reduced by a factor of 2.

It should be noted that the triplet system between wire scanners 1 and 2, and the defocusing quadrupole between 5 and 6 can be adjusted for zero dispersion at wire scanners 4 - 10. Measurements taken under these conditions would permit independent predictions of the horizontal phase space parameters. The normal running condition for the beam, however, is as shown.

Figures 4 and 5 show the predicted results from measurements taken on a different day from those of Figs. 2 and 3 and using a different technique. Instead of the entire beam line, only the first three wire scanners were used. Four separate sets of profiles were measured at these scanner positions with different field gradients on the intervening quadrupoles. After

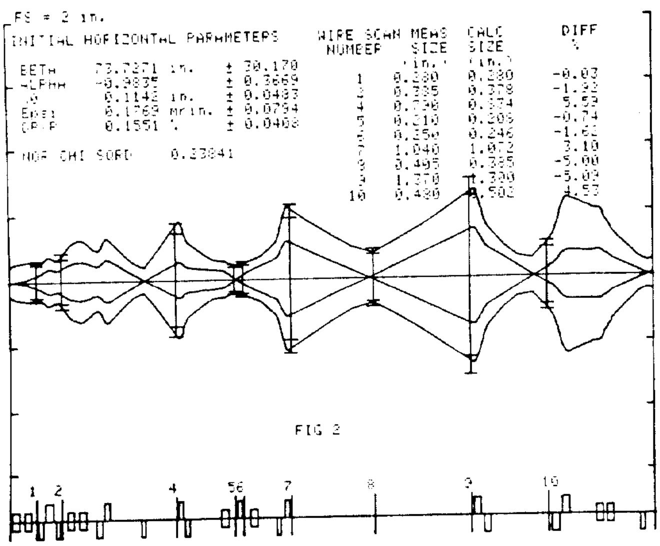

FIG 2

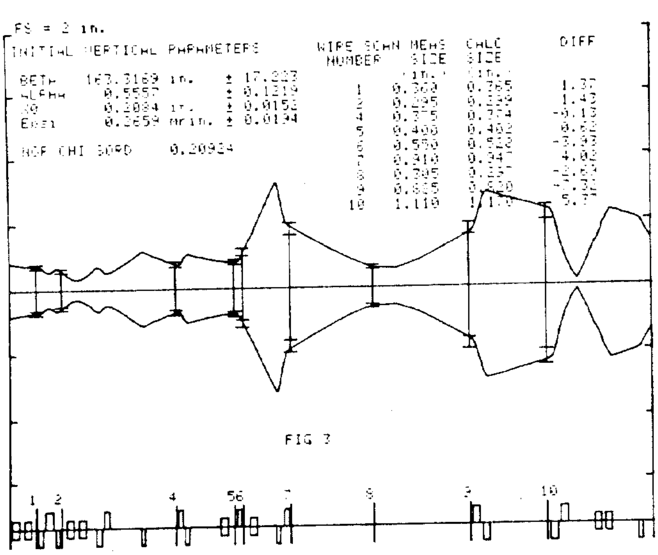

FIG 3

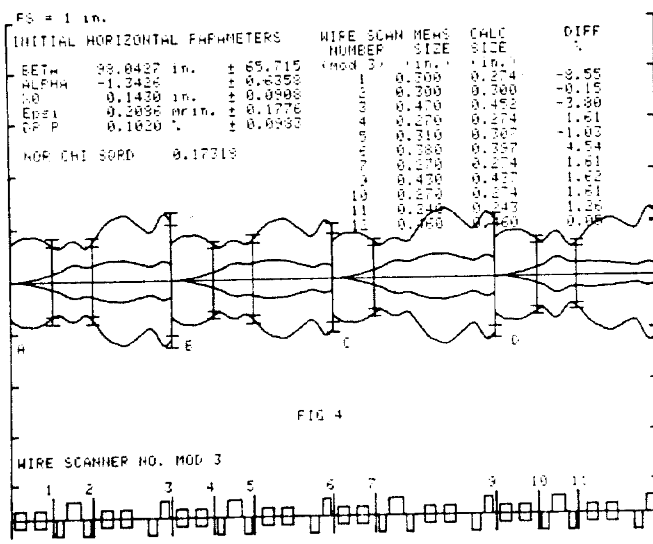

FIG 4

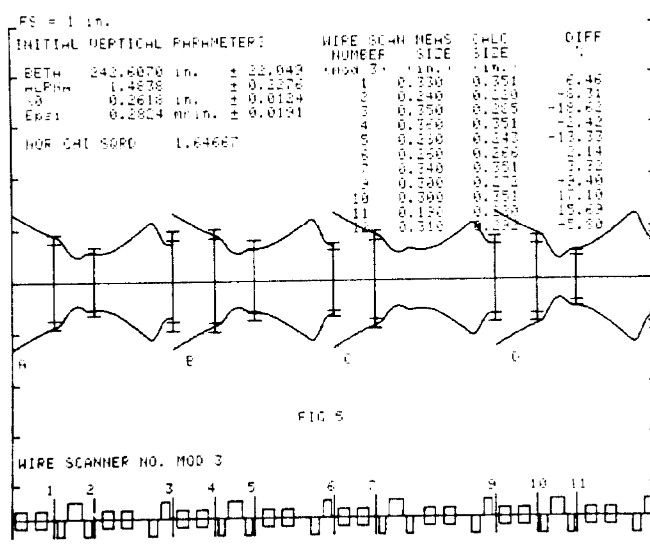

FIG 5

discarding one of the measurements (wire scanner 2 from the third set), a total of 11 measured widths were used to predict the beam parameters. In the figures, the beam from the linac starts over again at position A, B, C, D.

For these results also, the correlation between the transverse and dispersive contributions to the widths lead to large rms errors for the horizontal predictions. Fixing the horizontal beam quality at 0.275 mrad in. leads to almost as good a fit with reduced rms errors. The predicted momentum spread is reduced to 0.07%.

References

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3. Y. Cho, E.A. Crosbie, and H. Takeda, *Determination of 50-MeV Line Beam Characteristics*, Argonne National Laboratory IAS Note #80-6 (July 28, 1980).