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## A UNIQUE SPACE SAVING ACCELERATOR CAVITY DESIGN\*

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### Summary

A cavity with 3 series gaps was designed and modeled to operate at 70 MHz as a SuperHILAC post acceleration buncher (8.5 MeV/A). Because of a cross-coupling scheme, the 3 cells operate in the 1/2  $\beta\lambda$  mode instead of the  $\beta\lambda$  mode of an Alvarez cavity. This coupling results in a cavity with diameter reduced from 3 to less than one meter and a length half that of an Alvarez cavity for the same energy gain. The 3 gaps are electrically in parallel but mechanically in series. The cavity has high Q and shunt impedance. This type of cavity appears to be useful for low velocity beams with  $\beta \leq 0.2$ .

## Introduction

In connection with post-buncher design studies for time-of-flight experiments at the SuperHILAC, we investigated a possibility of feeding R-F voltage into a pair of 1/2 g $\lambda$  drift tube structures from an Alvarez cavity.

1/2 BA accelerating structures orginated by Wideroe in 1928 is being used at a low frequency of a few MHz with lump circuit oscillators. At a higher frequency (10  $\sim$  30 MHz), one may feed R-F voltage into several accelerating gaps from a common resonating line as in a pre-accelerator for the SuperHILAC.<sup>1</sup> At still higher frequencies, utilizing TE modes of a cavity, interdigital H type structures<sup>2</sup> and RFQ<sup>3</sup> have been developed. All of these devices are primarily for low  $\beta$  accelerations. As the energy of the particle increases drift tubes are longer and the resultant capacitive currents give poor shunt impedences compared to Alvarez structures.

In the Alvarez linear accelerators which are using TM<sub>010</sub> modes and have acceleration gaps  $\beta\lambda$ apart, the shunt impedance is higher than 1/2  $\beta\lambda$ modes at the intermediate energies. The total energy loss in an Alvarez accelerator will be further decreased with a lower accelerating voltage but this makes a longer accelerator. Therefore, at a higher energy, it is a general practice to push the R-F voltage to near the electric breakdown limits to make a shorter accelerator. Since the 1/2  $\beta\lambda$  structures have two times more accelerating gaps compared to  $\beta\lambda$ structures at the same wavelength, we have a relatively high shunt impedance if we introduce  $1/2 \beta\lambda$  structure inside of the Alvarez cavity.

A coupling of Wideroe drift tube structures and a TM cavity has been proposed by R. W. Müller<sup>4</sup> at a low frequency application. The basic idea behind the structure can be applied at a higher frequency as a cell of an accelerating structure with a reduced number of drift tubes in a cell.

The SuperHILAC is an Alvarez linear accelerator operating at a frequency f = 70.26 MHz, output energy of 8.5 MeV/A ( $\beta$  = 0.13). The diameter of the resonating tank with drift tubes is about 3m and at the output energy one cert rength is about 0.5m. For

a compact post buncher with multiple accelerating gaps it is desired to reduce spacial dimensions and this has been achieved by introducing a pair of coaxial 1/2  $\beta\lambda$  structures in the axias of the Alvarez cavity as shown in Fig. 1. The diameter of the cavity will be considerably decreased by higher capacitive loading and 3 gap accelerations in a distance of 1  $\beta\lambda$  give shorter accelerating distance. The cavity seems also to have relatively high shunt impedance at the SuperHILAC energy.

In the following sections, we present a view of initial design and test results of a 3 gap,  $3/2 \beta \lambda$  full size model cavity. In the end a possibility of extending SuperHILAC output energy by introducing this new cavity is also given.

# A Foreshortened Radial Transmissions Line

A cell of the Alvarez cavity with drift tubes could be viewed as a foreshortened radial transmission line with a load impedance due to a drift tube structure. Introduction of 1/2  $\beta\lambda$ structures shown in Fig. 1 makes a different load impedance than a simple drift tube with an accelerating gap. A resonant cavity radius could be approximately calculated from the total load impedance Z<sub>T</sub> at the input of the radial line.

We give the following notations used in a radial transmission line:

$$\mathbf{x} = \frac{2\pi}{\lambda} \mathbf{r} \tag{1}$$

$$\theta(\mathbf{x}) = \tan^{-1} \left[ \frac{N_{\Omega}(\mathbf{x})}{J(\mathbf{x})} \right]$$
(2a)

$$\phi(\mathbf{x}) = \tan^{-1} \left[ \frac{J_1(\mathbf{x})}{-N_1(\mathbf{x})} \right]$$
(2b)

Where  $J_o$ ,  $J_1$ ,  $N_o$  and  $N_1$  are usual Bessel and Neuman functions. The characteristic impedance  $Z_o(x)$  is defines as:

$$Z_{o}(x) = y \frac{G_{o}(x)}{G_{1}(x)}$$
 (3)

where

$$G_{o}(x) = [J_{o}^{2}(x) + N_{o}^{2}(x)]^{1/2}$$
 (4a)

$$G_{1}(x) = [J_{1}^{2}(x) + N_{1}^{2}(x)]^{1/2}$$
(4b)  
y = 120 \pi ohms

Then the resonant condition of  $\ensuremath{\text{TM}_{010}}\xspace$  made of the cavity is given by:

$$\theta_{\rm c} = \tan^{-1} \left[ \frac{Z_{\rm ol} \ L \ \sin\theta_1 + j \ Z_{\rm T} \ 2\pi \ r_1 \ \cos\theta_1}{Z_{\rm ol} \ L \ \cos\theta_1 - j \ Z_{\rm T} \ 2\pi \ r_1 \ \sin\theta_1} \right]$$
(5)

where  $\theta_1$ ,  $\phi_1$ , and  $\Xi_{01}$  are  $\vartheta$ ,  $\phi$ ,  $\Xi_0$  values at  $r = r_1$  and  $\theta_c = \theta(x_c)$ . For a given  $r_1$ , L, and  $Z_T x_c$  will be obtained using Eq. 5 and Eq. 2. For a given set of  $r_1$ , L and  $\lambda$ , when the effective capacitance at the center of cavity is

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increased the cavity radius rc will be decreased.

## Introduction of $1/2 \beta \lambda$ Structures in the Cavity

We took a cylinder of a height L and the radius  ${\bf r}_{\rm c}$  . Next we put a pair of co-axial intermediate cylinder of radius r1 extending from the end walls and having a middle gap as shown in Fig. 2. Inside of these intermediate cylinders, we insert a pair of 1/2  $\beta\lambda$  drift tubes along the axis of the cylinder and cross-connect to the intermediate cylinder. Thus we obtain 3 accelerating gaps which are coupled to the outer  $TM_{010}$  mode cavity. As a single cavity, the length of the cylinder L could be any value larger than the central gap. However, as a cell of an accelerating structure, we may take L =  $\beta\lambda$  or L =  $3/2~\beta\lambda$  . For a low  $\beta$  , the total central capacitance  $C_{\mathrm{T}}$  is small and the main ohmic loss is at the outer walls (r =  $r_c$ ), and L = 3/2  $\beta\lambda$  gives a better shunt impedance. As  $\beta$  increases, the ohmic loss at the central structure increases and  $L = \beta \lambda$  will be the better choice for a high shunt impedance. For a first model cavity we took  $L = 3/2 \beta \lambda$  to see the characteristics of the cavity though the final buncher cavity could be  $L = \beta \lambda$  at the SuperHILAC output energy.

The total load impedance  $Z_T$  of the radial line used in Eq. 5 could be found by the measurement from a model. For an approximate calculation we assumed the impedance  $Z_T$  is made out of three parallel impedance of which two are from the two coaxial lines and the one is from the central gap and connections. When the end gap impedances  $Z_g$  are much higher than the characteristic coaxial line impedance  $Z_{co}$ , then:

$$Z_{\rm T} = \frac{Z_{\rm C} Z_{\rm CO} \cot \pi\beta}{2 j Z_{\rm C} + Z_{\rm CO} \cot \pi\beta}$$
(6)

where  $Z_{CO} = 60 \ln(r_1/r_o)$  and  $Z_C$  is the impedance from the central gap and connections. And the central potential  $V_C$  is related to the end gap potential V as:

$$V_{\mu} = V \cos(\pi\beta)$$
(7)

The total ohmic loss except the end walls can be divided into three parts. The power loss  $P_c$  at the outer walls (r =  $r_c$ ) is:

$$P_{c} = \frac{\pi r_{c}}{L} \sigma_{s} V_{c}^{2} \left[ \frac{G_{oc}}{G_{o1} \cos(\theta_{1} - \phi_{1})} \right]^{2}$$
$$\times \left[ \frac{z_{o1}}{jz_{T}^{2} \pi r_{1}} \cos(\theta_{1} - \phi_{c}) + \sin(\phi_{c} - \phi_{1}) \right]^{2}$$

where  $G_{oc}$ ,  $Z_{oc}$  and  $\phi_c$  are taken at  $x = x_c$ ,  $G_{o1}$  and  $Z_{o1}$  are taken at  $x = x_1$ ,  $\sigma_s$  is a surface resistance.

Next the power loss  $P_1$  at the outer surface of the intermediate cylinder (r =  $r_1$ ) is approximated as:

$$P_1 = \frac{L}{2\pi r_1} \left(\frac{V_c}{z_T}\right)^2 \sigma s$$

finally, the power loss in the coaxial parts is:

$$P_{co} = \sigma_{s} \left(\frac{V}{Z_{oc}}\right)^{2} \frac{x_{1} + x_{o}}{2 x_{o} x_{1}} \beta \left[1 - \frac{\sin 2\pi\beta}{2\pi\beta}\right]$$

Then the total energy loss  $P_{\rm T}$  is:

$$P_{T} = P_{C} + P_{1} + P_{co}$$

Examples

We take f = 7 x 10<sup>7</sup> and we assume  $Z_g >> Z_{CO}$  and also we assume  $Z_c >> Z_{CO}$ 

a)  $\beta = 0.1 r_0 = 5 cm r_1 = 20 cm L = \beta \lambda$ 

Then  $r_{\rm c}$  = 1.32m and which is somewhat smaller than the drift tube loaded Alvarez cavity radius 1.5m and from Eq. 8,  $P_{\rm T}$  is:

$$P_{\rm T} = 6 \times 10^{-5} \sigma_{\rm s} V^2$$

The energy gain of the new structure is about 3 times of V, the shunt impedance of the new structure is about 3.5 times better than Alvarez structure.

b) 
$$\beta = 0.13 r_0 = 5 cm, r_1 = 20 cm, L = 3/2\beta$$

This is a set of parameters for a model cavity. When we assume the impedance of the central gap  $Z_c$  as  $Z_{\rm C}$  >>  $Z_{\rm CO}$ , Eq. 6 gives  $Z_{\rm T}$   $\approx$  96 ohms. However, in the model used for present studies, the cross connecting rods are extending inside of the  $r = r_1$  cylinder to the point 1/4  $\beta\lambda$  and, it virtually, has an effect of increasing ro. Initially we build a cavity of  $r_c = 0.5m$ , which is corresponding to  $Z_T = 70$  ohms, and it resonated at a frequency of 56.2 MHz, instead of 70 MHz. Using this frequency we calculated back the impedance  ${\rm Z}_{\rm T}$ from eq. 5. Its value is about 45 ohms, and this impedance gives 35.5 cm at 70 MHz. Then we build the second model with  $r_c = 35.5$  cm then the resonance frequency was measured to be 69 MHz which was close to the designed value even if outer cavity was near to the coaxial line than a radial line. The calculated shunt impedance is about one half of a normal Alvarez cavity. However the shunt impedance could be above the Alvarez cavity if we change r1 from 20 cm to 30 cm with an increased  $r_c$ .

#### Model Studies

A full scale, 3 gap, prototype buncher shown by Fig. 3 was constructed to determine resonant frequencies, Q values and shunt impedances, relative gap voltages and phases, and coupling and tuning loop size. The results of the model studies are summarized in Table 1. The cavity was made of 0.032"

CAVITY DIMENSIONS(cm) & RESULTS

L	h	1	1	10	1	'e	to(MHZ)	٥	Ra
83.8	35.6	22.9	5.	5	20	50	56.2	10,000	27x108
83.8	35.6	22.9	5	5	20	47.8	58.2		
83.6	35.6	22.9	5	6	20	35	69.9	8,500	18x 106
83.8	35.6	22.9	5	5	20	20	188	5,000	2.7= 106
	L 83.8 83.8 83.8 83.8	L h 63.8 35.8 63.8 35.8 63.8 35.8 63.8 35.6 63.8 35.6	L h 1 63.6 35.6 22.9 63.6 35.6 22.9 63.6 35.6 22.9 63.8 35.6 22.9	L h 1 9 63.6 35.6 22.9 5 63.6 35.6 22.9 5 63.6 35.6 22.9 5 63.8 35.6 22.9 5	L h 1 8 r0   03.8 35.4 22.4 3 4   03.4 35.4 22.4 5 5   63.4 35.4 22.9 5 5   63.4 35.6 22.9 5 5   63.8 35.6 22.9 5 5	L h 1 3 r0 r1   83.8 35.4 23.4 5 5 20   83.8 35.4 23.9 5 5 20   83.8 35.4 23.9 5 5 20   83.8 35.4 23.8 5 5 20   83.8 35.6 23.9 5 5 20	L h 1 9 r0 r1 re   03.8 35.4 23.4 5 5 20 50   03.8 35.4 23.9 5 5 20 47.0   03.4 35.4 23.9 5 5 20 47.0   03.4 35.4 23.0 5 5 20 35   03.4 35.6 23.0 5 5 20 35   03.4 35.6 23.0 5 5 20 20	L h 1 9 rg r1 rc fo(MH2)   83.8 35.6 22.0 3 5 20 50 58.2   83.8 35.6 22.0 3 5 20 50 58.2   83.8 35.6 22.0 5 5 20 47.6 68.2   83.8 35.6 22.0 5 5 20 35 60.0   83.8 35.6 22.0 5 5 20 20 100	L h 1 9 rg r1 rc fs(MHZ) O   83.8 35.8 22.4 3 5 20 50 58.2 10,000   93.8 35.8 22.4 3 5 20 56.2 10,000   93.8 35.8 22.9 5 5 20 47.8 68.2   93.8 35.6 22.9 5 5 20 35 69.9 8,300   93.8 35.6 22.9 5 5 20 20 148 5,000

GL 813-8405

### Table 1 - Cavity Dimensions (cm) & Results

copper sheet metal. The resonant frequencies and Q values were measured with a HP 8640B signal generator and a HP 8405A vector voltmeter as a detector of amplitude and phase. A 0.25" diameter by 0.25" long brass bead was introduced into the gaps along the cavity center line. This purturbation method<sup>5</sup> causes a resonant frequency change in proportion to the square of the gap electric fields. This data together with the cavity Q also allowed the shunt impedance to be estimated. By integrating the measured electric fields across each gap we found the

(8)

relative gap voltages to be equal within 5%. The phase of the gap fields were determined by adding a capactive voltage divider to each gap and observing the relative phase with the vector voltmeter. They were found to agree with predictions. A 10 cm x 10 cm square tuning loop of 1/4" diameter cooper tubing was tested in the H field at one end of the cavity. The range of resonant frequency change was about 76 KHz. A 50 ohm coupling loop was found to be approximately 4 cm in diameter. It appears there are no other resonant modes near the operating frequency, e.g. In our first model of f = 56.2 MHz, the next higher mode was at f = 179 MHz.

The shunt impedance,  $R_s$ , shown in Table 1 at f = 69.9 MHz is better than the estimated values from Eq. 8 and comparable to the Alvarez cavity. This difference could orginate in an overestimate of the power loss P<sub>1</sub> of the intermediate cylinder.

## Discussions

A pair of coaxial 1/2  $\beta\lambda$  accelerating structure with 3 gaps excited by outer TM<sub>010</sub> cavity, modeled as a capacitively loaded radial transmission line, resonated at the calculated frequency once the load impedance was known. The present model, L = 3/2  $\beta\lambda$  $r_0$  = 5 cm,  $r_1$  = 20 cm,  $r_c$  = 35 cm at  $\beta$  = 0.13 resonating at 70 MHz is a compact cavity with a comparable shunt impedance of Alvarez cavity. However, the shunt impedance will be considerably increased if:

a) We change the cross connection method of which the rod is now extending to the inside r = r<sub>1</sub> cylinder.

b) We change  $r_1 = 20$  cm to the bigger value to increase  $Z_{co}$  and at the same time reduce the resistance of the  $r = r_1$  cylinder.

c) We change L = 3/2  $\beta\lambda$  to L =  $\beta\lambda$  in order to reduce ohmic heating at r = r1 cylinder.

Finally if we introduce this new structure for SuperHILAC acceleration scheme, the same energy could be reached in one-half of the present length with about the same electric power. When we extend applying this structure to a higher energy, for an example,  $\beta = 0.2$  (  $\sim 20$  MeV/A), we may take  $r_0 =$ 5 cm,  $r_1 = 50$  cm, then  $Z_T$  could be made about 60 ohms. When we take  $L = \beta\lambda$ , then  $x_c =$ 1.5 ( $r_c > 1m$  if  $f = 7 \times 10^6$ ), the calculated shunt impedance is comparable to the corresponding Alvarez cavity. Therefore, the length of the machine could be reduced one-half of the Alvarez accelerator with a modest increase of R-F power.

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Fig. 1 L =  $\beta\lambda$  Cell



Fig. 2 L =  $3/2 \beta \lambda$  Cell



Fig. 3 Buncher Design Concept