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INHOMOGENEITIES IN MAGNETIC FIELD DUE TO VARIATIONS OF LAMINATION GAP HEIGHTS*

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Abstract

It is difficult and expensive to maintain stringent tolerances on the variation in gap heights of dipole magnet laminations. Using the perturbation technique of K. Halbach,[1] whereby one approximates a deformation of a magnet pole tip by a change of the scalar potential on the undeformed pole, we have calculated the tolerance necessary to achieve a given homogeneity of the magnetic field. Tolerances may be significantly relaxed if a systematic shuffling scheme is used to minimize the lamination variations averaged over a distance small compared to the gap height. Results of our calculations are compared with data from field measurements on the bending magnets for the NSLS 700 MeV Ring, for which laminations were carefully shuffled, and for the corresponding prototype magnet which was not shuffled. We conclude that for magnets used in storage rings, very tight gap height tolerances are not necessary, because field variations are reduced by shuffling and significantly cancel out when integrated over the magnet.

Introduction

Fluctuations in the gap height from lamination to lamination result in a variation of the magnetic field in the midplane. We have estimated the magnitude of this variation using the perturbation technique of Halbach,[1] whereby one approximates a deformation of a magnet pole tip by a change of the scalar potential on the undeformed pole. We find that the rms field variation in the midplane is given by

$$\left(\frac{\Delta B}{B_o}\right)_{rms} = \left(\frac{\pi a}{3G}\right)^{1/2}\left(\frac{\sigma}{G}\right) \tag{1}$$

where

$G=2g$ is the full unperturbed gap height,
σ=rms fluctuation in gap height,
a=lamination thickness.

For the NSLS dipole,

$G=55$ mm,
$a=1.5$ mm,

and measurements on the prototype indicate

$\sigma=0.04$ mm,

so the data in Figure 1b should have $(\Delta B/B_o)_{rms} = 1.2\times10^{-4}$.

In the Fourier analysis of the midplane magnetic field, short wavelengths λ are filtered out by a weighting factor

$(\pi G/\lambda)/\sinh(\pi G/\lambda)$.

We expect that the field variations should be characterized by wavelengths on the order of πG. The variation cannot be much faster than this because shorter wavelengths are attenuated, and it cannot be much slower because there are many laminations (115) in a distance of πG (17 cm) so the field variations are expected to average out over such a length. Magnetic measurement shows characteristic wavelengths on the order of 10 cm, and for B_o = 12 KG, a worst case peak to peak variation of about 10 G. This can be compared to $2\sqrt{2}(\Delta B)_{rms}$ = 4.3 G (B_o = 12 KG).

Out of the midplane, the characteristic wavelengths are shorter and the peak to peak variations larger. Also, one sees the effect of errors in aligning the laminations in additon to the effects of errors in the gap height. In fact, the former produce field deviations odd under reflection about the midplane, while the latter produce deviations which are even. For distances y out of the midplane, y not too large, say $y/g<1/2$,

$$\left(\frac{\Delta B}{B_o}\right)_{rms} = \left\{\frac{\pi}{3}\frac{a}{G}\left[\frac{\sigma^2}{G^2}C\left(\frac{y}{g}\right) + \frac{\tilde{\sigma}^2}{G^2}\tilde{C}\left(\frac{y}{g}\right)\right]\right\}^{1/2}, \tag{2}$$

where $g=G/2$, $\tilde{\sigma}$ is the rms error in the stacking of the laminations, and σ is the rms gap height error,

$$C(\alpha) = \frac{6}{\pi^2}\int_0^\infty \frac{dp\; p^2\cosh^2\alpha p}{\sinh^2 p}$$

and

$$\tilde{C}(\alpha) = \frac{6}{\pi^2}\int_0^\infty \frac{dp\; p^2\sinh^2\alpha p}{\cosh^2 p}$$

In particular,

$C(0)=1$, $\tilde{C}(0)=0$,

$C(1/2)=2.0$, $\tilde{C}(1/2)=0.93$.

Taking $\tilde{\sigma}=\sigma$, we find for $y/g=1/2$.

$$\left(\frac{\Delta B}{B_o}\right)_{rms} = 1.7 \times \text{value at } y=0.$$

The data shown in Figure 1 indicate an enhancement by slightly more than a factor of 2, in reasonable agreement with the factor of 1.7 found above.

The fluctuation in the integrated midplane field due to the gap height errors is

$$\left\langle\frac{1}{L}\int dz\,\frac{\Delta B}{B}\right\rangle_{rms} = \frac{1}{\sqrt{N}}\,\frac{\sigma}{G}$$

where $N=L/a$ is the number of laminations in the magnet of length L. For the VUV dipole $\sqrt{N}$ = 30, so even if σ = 0.15 mm (6 mil),

$$\left\langle\frac{1}{L}\int dz\,\frac{\Delta B}{B}\right\rangle_{rms} = 10^{-4}.$$

Of course, this assumes a good shuffling procedure to assure that each magnet contains a random sample from the entire range of gap heights to be found in the inventory of laminations.

Integrated field errors may be further reduced by a shuffling scheme which systematically arranges the laminations to produce a regular periodic variation in gap height. If this periodic variation has a wavelength comparable to the average gap dimension, the resultant modulation of the midplane field is negligible. Such a shuffling procedure requires that laminations be sorted into stacks according to their gap height. Laminations are taken from each stack in

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a fixed sequence to make a magnet. Integrated field errors are caused by the random gap variation within each stack, which is smaller than for the entire ensemble of laminations. Assuming the systematic field variation caused by this shuffling scheme is small enough, the residual random errors are described by the formulas given previously, but with σ now being the rms gap variation within each stack. If the shuffling procedure results in a sinusoidal modulation of the full gap,

$$\Delta g(z)=\Delta g\ \cos(kz+\phi)$$

symmetric about the midplane, then the induced ripple in the magnetic field is given by

$$\frac{\Delta B_y(y,z)}{B_o} = \frac{-k\ \Delta g\ \cos(kz+\phi)\ \cosh(ky)}{2\ \sinh(kg)} \tag{3}$$

This equation is used in the next section to estimate Δg based on measurements of $\Delta B_y/B_o$.

Measurements

We have tested the results of our calculations against measurements of the dimensions and field quality of the NSLS 700 MeV ring dipole magnets. Figure 1 shows magnetic field measurements in the prototype magnet, which was constructed without shuffling from the first small production run of laminations. Gap measurements done on a sample of these laminations showed σ = 0.04 mm. The dark vertical lines in the figure mark the edges of the separate blocks of 117 laminations from which the magnet was assembled. Figure 1a shows the average, $1/2(B_y(y=13.5\text{ mm}, z) + B_y(y=-13.5\text{ mm}, z))$ for the prototype. This average eliminates the effect of lamination misalignment errors, which is odd in y. Ripples in this average are caused only by variation in the gap dimension. The field variations for $0<z<150$ mm are approximately sinuosidal with wavelength 37 mm and amplitude $\frac{\Delta B}{B_o}$ = 4×10^{-4} (8×10^{-4} peak-to-peak).
Substitution in equation (3) shows that a gap modulation of amplitude Δg=0.05 mm would cause the observed ripples in B_y. Equation (3) implies that these ripples should be reduced by a factor $\cosh(2\pi\cdot13.5\text{ mm}/37\text{ mm}) = 5$ for the midplane field, which is shown in Fig. 1b. The observed reduction is about a factor of 4. The region $=-150\text{ mm} <z<0$ in Fig. 1a shows a ripples in B_y with amplitude $\Delta B/B_o = 0.7\times10^{-3}$ and wavelength 75 mm. These are also consistent with a gap variation amplitude of Δg=0.05 mm. These ripples are visible in Fig. 1b with amplitude reduced by the expected factor $\cosh(2\pi\ 13.5\text{ mm}/75\text{ mm}) = 1.7$.

Figure 2 shows field measurements for the first production 700 MeV dipole. The laminations for this magnet had an rms gap variation σ=0.05 mm. The laminations were separated into seventeen stacks, and assembled into blocks in a sequence which was repeated every 24 mm. We might therefore expect to see a ripple with wavelength 24 mm in the field, as well as jumps at the edges of the blocks. Figure 1a is a plot of $1/2(B_y(y=15\text{ mm}, z)+B_y(y=-15\text{ mm}, z))$ for a production magnet. Ripples of wavelength λ = 24 mm are evident in the region $0<z<150$ mm. The amplitude is $\Delta B/B = 2\times10^{-4}$, consistent with a gap error of amplitude 0.04 mm.

These ripples should be reduced in amplitude by a factor of 22 in the midplane. Figure 2b shows the field in the midplane of the magnet. As expected, the short wavelength ripples are not visible. The remaining variations are mostly associated with block boundaries and are due to spaces between the blocks.

Figure 3 shows $1/2(B_y(y=13.5\text{ mm}, z)-B_y(y = -13.5\text{ mm}, z))$ for the prototype dipole. Field errors here are due to misalignment of laminations only. The ripples in the region $-480\text{ mm} <z< -375$ mm are consistent with an alignment error of amplitude 0.02 mm.

The Calculation

Let the unperturbed magnetic field B_o in the dipole magnet be in the y-direction. The lamination lies parallel to the xy-plane, and the z-axis is perpendicular to the laminations. We let $\Delta g_\pm(z)$ represent the variation of the top (+) and bottom (-) of the lamination gap,

$$\Delta g_\pm(z) = \int_{-\infty}^{\infty} dk\ G_\pm(k)\ e^{ikz} \tag{4}$$

The magnetic field can be written in terms of a scalar potential,

$$B = -\nabla\varphi \tag{5}$$

and Halbach's[1] approximation consists of determining the scalar potential variation on the top (+) and bottom (-) of the gap by

$$\Delta\phi_\pm(z) = \frac{\partial\phi}{\partial y}\ \Delta g_\pm(z) = -B_o\ \Delta g_\pm(z). \tag{6}$$

Using this as a boundary condition, we solve Laplace's equation to determine the scalar potential within the gap as

$$\Delta\phi(y,z)= \int_{-\infty}^{\infty}dk\ e^{ikz}[a(k)\cosh ky + b(k)\ \sinh ky], \tag{7}$$

where

$$a(k) = \frac{-B_o}{2\ \cosh kg}\ [G_+(k) + G_-(k)],$$

$$b(k) = \frac{-B_o}{2\ \sinh kg}\ [G_+(k) - G_-(k)],$$

and 2g is the full gap height.

Introducing

$$\Delta g(z) = \Delta g_+(z) - \Delta g_-(z), \tag{8}$$

we can write the following expression for the vertical field in the midplane (y=0):

$$\Delta B_y(0,z) = B_o \int_{-\infty}^{\infty} \frac{dz'}{2\pi}\ F(z-z')\Delta g(z'), \tag{9}$$

where

$$F(z) = \int_{-\infty}^{\infty} \frac{kdk}{2\ \sinh kg}\ e^{ikz}. \tag{10}$$

We are interested in the correlation

$$\langle \Delta B(0,z_1)\Delta B(0,z_2)\rangle = \left(\frac{B_o}{2\pi}\right)^2 \int\int dz_1'\ dz_2'\ F(z_1-z_1')F(z_2-z_2').\langle\Delta g(z_1')\Delta g(z_2')\rangle \tag{11}$$

the average being taken over the distribution of gap fluctuations. We assume no correlation between the gap variations of different laminations, i.e.

$$\langle\Delta g(z_1')\Delta g(z_2')\rangle = \begin{cases} \sigma^2 & ma \le z_1',\ z_2' \le (m+1)a \\ 0 & \text{otherwise,} \end{cases}$$

where a is the lamination thickness. Assuming $a \ll g$ and the $F(z_l - z_l')$ is roughly constant over one lamination, we obtain

$$\langle \Delta B_y(0,z_1)\Delta B_y(0,z_2)\rangle = \sigma^2 \left(\frac{B_o}{2\pi}\right)^2 \cdot \sum_{m=-\infty}^{\infty} a^2 F(z_1 - ma)F(z_2 - ma). \qquad (12)$$

Using (10) together with

$$\sum_m e^{-i(k_1+k_2)ma} = \frac{2\pi}{a}\sum_n \delta\left(k_1+k_2 - \frac{2\pi n}{a}\right),$$

and for $a \ll g$ keeping only the dominant n=0 term, we obtain

$$\left\langle \frac{\Delta B(0,z_1)}{B_o}\,\frac{\Delta B(0,z_2)}{B_o}\right\rangle = \frac{\sigma^2 a}{2\pi g^3}\int_{-\infty}^{\infty}\frac{p^2\,dp}{4\sinh^2 p}\, e^{ip\left(\frac{z_1-z_2}{g}\right)} \qquad (13)$$

For $z_1=z_2$, we find $\langle(\Delta B(0,z)/B_o)^2\rangle_{rms}$ as given in eq. eq. (1).

The preceding discussion is easily generalized to consider the field out of the midplane, and one finds

$$\frac{\Delta B_y(y,z)}{B_o} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dz\,[F(y,z-z')\Delta g(z')+F(y,z-z')\Delta g(z')],$$

where

$$\Delta g(z) = \Delta g_+(z) - \Delta g_-(z), \quad \text{(gap error)}$$

$$\Delta\tilde{g}(z) = \Delta g_+(z) + \Delta g_-(z), \quad \text{(misalignment error)}$$

$$F(y,z) = \int_{-\infty}^{\infty} \frac{k\,dk \cosh ky}{2\sinh kg}\, e^{ikz},$$

$$\tilde{F}(y,z) = \int_{-\infty}^{\infty} \frac{k\,dk \sinh ky}{2\cosh kg}\, e^{ikz},$$

and $\langle(\Delta B(y,z)/B_o)^2\rangle$ is given in Eq. (2).

References

1) K. Halbach, Nucl. Inst. Methods 74, 147 (1969).

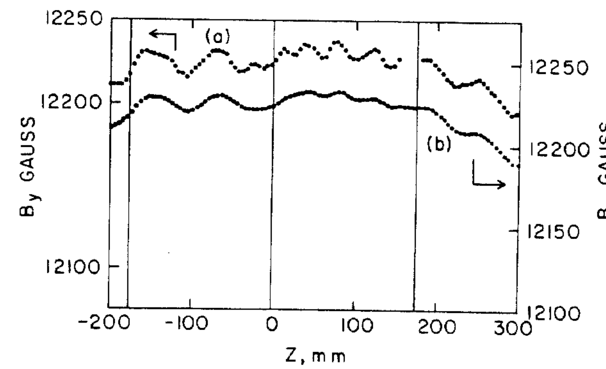

Figure 1. Unshuffled prototype magnet: (a) Average of magnetic field above and below midplane (b) Magnetic field in midplane.

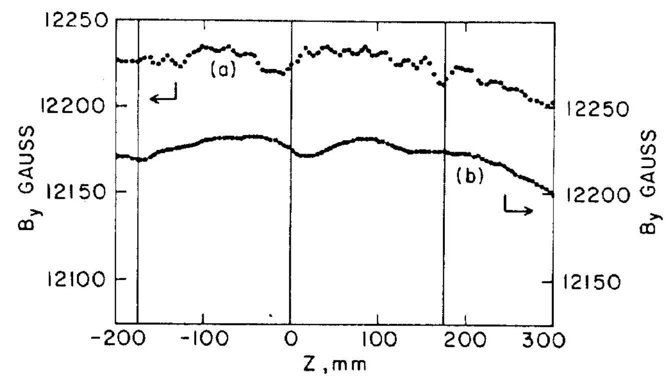

Figure 2. Shuffled production magnet: (a) Average of magnetic field above and below midplane (b) Magnetic field in midplane.

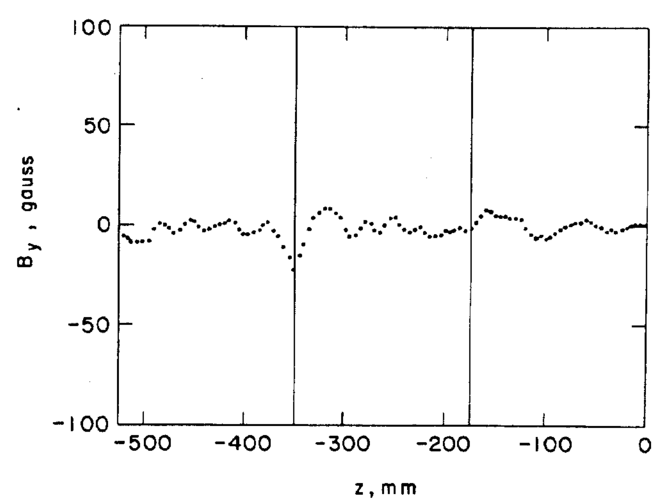

Figure 3. Unshuffled prototype magnets: Difference between field above midplane and field below midplane.