IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

PULSED RF OPERATION ANALYSIS*

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Summary

The need for a very low final amplifier output impedance, always associated with class "A" operation, requires a very large power waste in the final tube.

The recently suggested pulsed rf operation, while saving a large amount of power, increases the inherent final amplifier non linearity.

A method is presented for avoiding the "large signal non linear analysis" and it is shown how each component of the beam induced voltage depends upon all the beam harmonics via some "coupling coefficients" which are evaluated.

Introduction

One of the requirements of the rf systems for proton accelerators and storage rings is the low impedance they must present to the accelerated beams.

Beyond some limits the low output impedance can be achieved only by means of feedback amplifiers, of which the common anode amplifier is a good example. This implies that the system has to react strongly to the beam.

Moreover, the final amplifier that drives an accelerating cavity has to work in a very large dynamical range always involving the nonlinear region of the active element characteristics.

The inherent nonlinear characteristics of the electron tubes (the cut off and the saturation are important aspects of these nonlinearities) make the circuit analysis hopelessly complicated and each case must be treated numerically, thus excluding the possiblity of even the simplest analytical prediction.

The problem becomes less serious if we can assume that the beam and the drive are periodic functions of the time with the same period T. In the absence of coherent longitudinal beam oscillations, this hypothesis is verified.

If we assume this periodicity, then the tube parameters which depend upon the applied signals can be treated as periodic functions of time only, with the same period T, and the problem is reduced to the analysis of a linear, time dependent electrical network. This analysis can be performed in the time or frequency domain. We shall discuss both approaches, since they show different aspects of the same phenomenon.

Transient Analysis In Class "C" Operation

The tube nonlinearities are always ignored when the tube is operated in class 'a'. This is very reasonable because class 'a' is selected in order to achieve a nearly linear operation; thus, the nonlinearities can be taken into account only in the second order approximation. Under class 'a' operation the amplifier output impedance can be defined, in the sense of Thevenin's theorem.

On the other hand, class 'a' operation demands a very large quiescent current in the tube and one has to pay in power consumption and in oversizing the tube the advantages, not always clear, of having a linear final amplifier.

It has been suggested (BNL 51246) that the final amplifier could exibit the same good performance even if operated in class 'c', as long as the final amplifier circuit contains a strong feedback. (For instance, a cathode follower which is an amplifier with a feedback ratio approaching one in the frequency band pass).

The advantages come from the fact that class 'c' operation greatly reduces the power wasted in the tube and, consequently, allows the use of smaller tubes than those required by class'a' amplifiers. The class 'a' operation is, by definition, a nonlinear one and any attempt to "linearize" the problem would not be justified.

A first step forward is to recognize that, when operated in class 'c', the tube behaves like a switch in series with a resistor. During the part of the rf period the tube is on, the power supply is connected to the load through the resistor; during the part of the rf period the tube is off, the load is disconnected from the power supply or, in other words, the tube behaves as an infinite resistance.

We can schematize the feedback amplifier with a parallel tuned circuit driven by two ideal current generators: the generator GT that stands for the current due to the tube and the current generator GB that schematizes the beam action. Due to the supposed existing feedback the generator GT depends upon the voltage across the circuit; moreover, due to the tube nonlinearity, the resistor also depends upon this voltage. This scheme can be extended to many feedback amplifiers connected to a resonant load, and one would have to specify the circuit parameters accordingly to the actual amplifier under analysis.

If we suppose that both the beam and the drive are periodic functions of time with the same period, then, applying the model described above, we can make the two generators independent upon the voltage across the circuit, but with parameters chosen so as to account for both the feedback and the tube nonlinearity. In this way the resistor becomes a time dependent resistor. Moreover, because we are concerned only with the beam induced voltage, we can set the current coming from the generator GT equal to zero and we are left with the circuit drawn in Fig. 1.

With reference to Fig. 1,

$$R(t) = R(t + T),\ I(t) = I(t + T) \text{ and } T = 2\pi\sqrt{L.C}$$

is the period.

The function R(t) can be approximated using a set of step functions of suitable amplitudes and delays.

The most important conditions for estimating the system behavior are met if we assume that the resistor has only two values, R and infinite, and if the beam is simulated by a train of delta functions. A very good approximation is obtained if we select the finite resistor value equal to the inverse of the mean tube

* Work performed under the auspices of the U.S. Dept. of Energy.

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transconductance for a cathode follower amplifier.

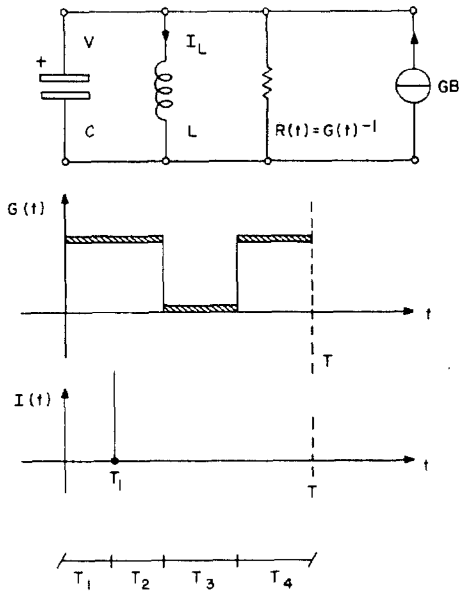

Fig. 1. The parameters L and C stand for the 'cavity-amplifier' fundamental mode. The resistor R is a time function resistor whose value depends upon the cavity losses, the tube, and the feedback ratio.

The circuit analysis of the model described above is straightforward, and was carried out with the state variable method, where the state variables V and I are the voltage in the capacitor and the current in the inductor respectively.

We divide the interval T in four parts, as follows:

1. The resistor is given the value R. The initial conditions are V_o and I_o. The time ranges from zero to T_1. At the end of the first interval the state variable values are V_1 and I_1.

2. The resistor is unchanged. The time ranges from zero to T_2. Because at the beginning of this interval the beam is supposed to excite the circuit, then the initial conditions are $V_1 + q/C$ and I_1. At the end of the second interval the state variable values are V_2 and I_2.

3. The resistor value goes to the infinity (the tube is off). The time ranges from zero to T_3. The initial conditions are V_2 and I_2. At the end of the third interval the state variable values are V_3 and I_3.

4. The tube is again on and the resistor is given the value R. The time ranges from zero to T_4. The initial conditions are V_3 and I_3. The state variable values at the end of this interval are V_4 and I_4.

The various sub-intervals are not independent because we must have $T=T_1+T_2+T_3+T_4$, and the best amplifier performance can be obtained only under appropriate selection of these sub-intervals. It will be shown that the sub-intervals T_3 is by far the most critical in reducing the wake voltage at the beginning of the next rf cycle.

Let $|A|, |R|, |S|, |T|$ be the matrices that account for the tranformations in each of the four sub-interval.

Then we can write:

$$|X_4| = |T|.|S|.|R|.|A|.|X_o| + |T|.|S|.|B|.|q| = |G|\cdot|X_o| + |F|$$

where $|X_4|$ and $|X_o|$ are the final and initial state vectors, respectively, $|B|$ is the matrix that accounts for the effect of the beam, and $|q|$ is the vector (o,q) describing the amplitude of the delta function that represents the beam.

Imposing the periodicity condition $|X_4| = |X_o|$ we obtain the linear system:

$$\{|I| - |G|\}|X_o| = |F|$$

where $|I|$ is the unity matrix; the solution of the system is the periodic vector describing the conditions at the beginning of each cycle.

In Fig. 2 we give an example where the parameters are chosen so as to fit the case of the ISABELLE accelerating amplifiers. (C=1.2 E-8, L=3.82 E-5, R=5 or infinite, q=3.4 E-5). The time T_1 (measured in degrees, with the total interval T=360°) is allowed to vary between 10 and 120 degrees and T_3 is allowed to vary in the same interval. It is important to recognize that a proper choice of the cut-off interval can greatly reduce the wake voltage on the condenser and that, depending on this device, the wake voltage appears positive or negative, thus passing through a true zero.

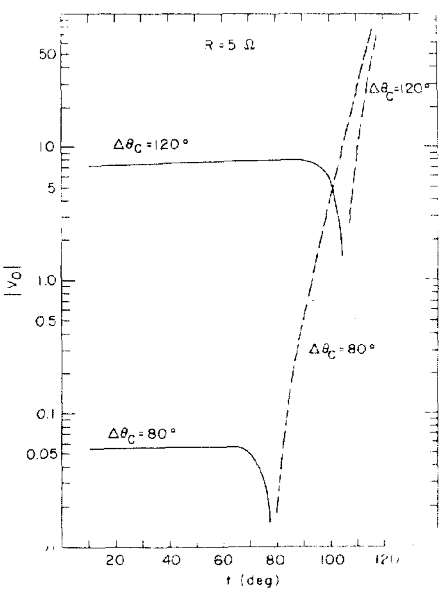

Fig. 2. Voltage at the end of the rf cycle on the capacitor versus T_1. Interval T_3 is the parameter. The solid lines indicates positive voltage, the dashed lines indicates negative voltages.

Changing the value of the resistor R from 5 to 10 and even to 15Ω does not appreciably change the numerical results (for R=28Ω the circuit is critically damped when the tube is on).

Frequency Analysis of a System With Time Varying Parameters

The behavior of the final amplifier discussed in the previous section cannot be described in terms of an impedance in the conventional sense. As we shall see, a modification of this concept, suitable for time varying systems is attempted.

In a system where the parameters do not vary with time, the relationship between the current driver (the beam) and the voltage is defined in terms of the Green's function $h(\tau)$ which depends only on the time interval τ between the occurrance of the δ-function of current and the time at which the voltage is observed:

$$V(t) = \int_{-\infty}^{+\infty} h(\tau)\ i(t-\tau)d\tau \tag{1}$$

The convolution theorem then gives, for the Fourier transforms of the functions in Eq. 1:

$$\tilde{V}(\omega) = \tilde{h}(\omega)\cdot\tilde{i}(\omega) \tag{1a}$$

In a system where the parameters vary with time, the Green's function is also function of this "absolute" time t:

$$h = h(t,\tau)$$

If the circuit parameters vary periodically with time, as in the model described in the previous section, the Green's function can be expanded in Fourier series:

$$h(t,\tau) = \sum_{-\infty}^{+\infty} h_n(\tau)\cdot e^{jn\omega_o t} \tag{3}$$

where $\omega_o = 2\pi/T$, and τ represents the time interval between the occurance of the δ-function of current and the observation of the voltage.

The voltage at time t is given by

$$V(t) = \int_{-\infty}^{+\infty} h(t,\tau)\ i(t-\tau)d\tau = \sum_{-\infty}^{+\infty} F_n(t)\ e^{jn\omega_o t} \tag{4}$$

where

$$F_n(t) = \int_{-\infty}^{+\infty} h_n(\tau)\ i(t-\tau)d\tau$$

The Fourier transform of Eq. 4 gives:

$$V(\omega) = \sum_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F_n(t)\ e^{-j(\omega-n\omega_o)t}\ dt$$

$$= \sum_{-\infty}^{+\infty} \tilde{F}_n(\omega-n\omega_o) = \sum_{-\infty}^{+\infty} \tilde{h}_n(\omega-n\omega_o)\cdot\tilde{i}\ (\omega-n\omega_o) \tag{5}$$

where $\tilde{h}_n$ and $\tilde{i}_n$ denote the $(\omega - n\ \omega_o)$-th Fourier transforms of $h_n(\tau)$, $i(\tau)$. Equation 5 shows that an infinite number of harmonics of the driving current contributes to one harmonic of the induced voltage. In particular, if the driving current is periodic with the same period $T = 2\pi/\omega_o$, substituting its Fourier transform in Eq. 5 and antitransforming we obtain the induced voltage $V(t)$ that must be periodic with the same period T. Equating coefficients with the same frequency we obtain:

$$V_m = \sum_{k=-\infty}^{+\infty} I_k\cdot\tilde{h}_{m-k}(k\omega_o) \tag{6}$$

where I_k is the amplitude of the k-th Fourier coefficient and $\tilde{h}_{m-k}(k\ \omega_o)$ denotes the k-th harmonic of the Fourier transform, with respect to τ (delay event-observation), of the (m-k)th Fourier coefficient, with respect to the "absolute" time, of the Green's function.

Equations 5 and 6 show that the conventional definition of impedance does not apply to systems with time varying parameters. In such systems, expression 1a is replaced by the linear combination 5 or 6 of all the harmonics of the driving current.

In practice, the calculation of the $\tilde{h}$ terms, which we shall call the "coupling coefficients", is cumbersome and involved but for a very simple circuit. In order to give an idea of the relative importance of these coupling terms, we consider the simplest decay model, the R-C circuit and we choose its decay time τ_o to be aproximately the same as in the example of the previous section (Fig. 1).

Moreover, we suppose that the conductance G varies with time according to the function $G = G_o\cdot(1-\varepsilon\cos\omega_o)$, $\varepsilon \geqslant 0$. This smooth variation of the conductance is obviously more physical then the step function assumed in the model of the previous section.

After some manipulation we find, for the $h_n(\tau)$ terms of Eq. 3,

$$h_n(\tau) = \frac{1}{C}\ e^{-\tau/\tau_o}\ e^{-jn\omega_o\tau/2} I_n\left(\frac{2\varepsilon}{\omega_o\tau_o}\ \sin\frac{\omega_o\tau}{2}\right)u(\tau) \tag{7}$$

where I_n is the modified Bessel function of order n, and $u(t)$ is the step function, $u(\tau) = 0$ for $\tau < 0$, $u(t) = 1$ for $\tau \geqslant 0$.

Under the assumption, justified in our example, $\tau_o \ll 2\pi/\omega_o$ ($\tau_o = 0.06\ \mu$sec, $2\pi/\omega_o = 4.27\ \mu$sec), the sine in the argument of the Bessel function can be replaced by the arc, and the Fourier transform of 7 is easily found:

$$\tilde{h}_{m-k}(k\omega_o) =$$

$$= \frac{1}{C}\ \frac{(\varepsilon/\tau_o)^{|m-k|}}{\left\{\left[\frac{1-\varepsilon^2}{\tau_o^2} - \left(\frac{m+k}{2}\right)^2\omega_o^2 + \frac{j\omega_o}{\tau_o}(m+k)\right]^{1/2} + \frac{1}{\tau_o} + \frac{j}{2}(m+k)\omega_o\right\}^{|m-k|}}$$

$$\cdot\ \frac{1}{\left[\frac{1-\varepsilon^2}{\tau_o^2} - \left(\frac{m+k}{2}\right)^2\omega_o^2 + \frac{j\omega_o}{\tau_o}\ (m+k)\right]^{1/2}}$$

In Fig. 3 we have considered the terms $\tilde{h}_{1-k}(k\omega_o)$ which couple all the harmonics of the current to the first harmonic of the voltage. In Fig. 3 we plot the ratio of the moduli

$$|\tilde{h}_{1-k}(k\omega_o)| / |\tilde{h}_o(k\omega_o)|$$

i.e., the modulus of the coupling coefficients normalized to the term $|\tilde{h}_o(k\omega_o)|$ which, alone, would define the voltage in a system with non time varying coefficients. In other words, $|\tilde{h}_o(k\omega_o)|$ couples the kth harmonic of the current to the kth harmonic of the voltage. The points in Fig. 3 refer to three values of the modulation = 0.1, 0.5, 0.9. Figure 3 shows that, as the amplitude of the modulation increases, the coefficients which couple the different harmonics k of the current to the first harmonic of the voltage become important with respect to the value corresponding to k = 1, which couples the same harmonic of the voltage and current. This value $|h_0(k)|$, at the limit $\epsilon \to 0$, reduces to the impedance as defined in a system with non time varying parameters, as one can verify applying this limit to Eq. 8).

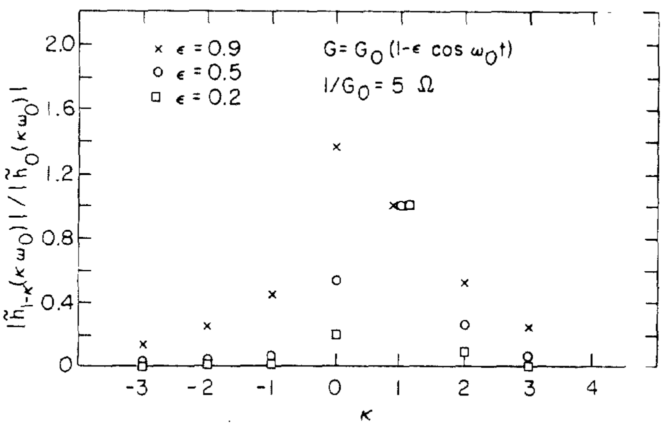

Fig. 3. Coupling coefficients for different values of the modulation of the conductance. These coefficients couple all the harmonics of the current to the first harmonic of the voltage.

Conclusion

The introduction of a network with time varying parameters seems to provide a method of obtaining analytical results as an alternative to the direct nonlinear treatment. It must be noted that the coupling coefficients, that seem somewhat disturbing, depends upon the non linearity which is inherent to any tube characteristics. Therefore a certain amount of coupling is to be expected even in the so called class A_1 operation. In any case, the extent of what is tolerable in terms of tube nonlinearity is an important question, still to be addressed.