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BUNCHER SYSTEM PARAMETER OPTIMIZATION*

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Abstract

A least-squares algorithm is presented to calculate the RF amplitudes and cavity spacings for a series of buncher cavities each resonating at a frequency that is a multiple of a fundamental frequency of interest. The longitudinal phase-space distribution, obtained by particle tracing through the bunching system, is compared to a desired distribution function of energy and phase. The buncher cavity parameters are adjusted to minimize the difference between these two distributions. Examples are given for zero space charge. We indicate the manner in which the method can be extended to include space charge using the 3-D space-charge calculation procedure described separately.[1]

Introduction

Until the recent advent of the radio frequency quadrupole (RFQ), traditional longitudinal beam matching into a linear accelerator was accomplished using one or more RF cavities resonating at the fundamental linac frequency or one of the harmonics. We thought that by making a judicious choice of buncher-cavity frequencies, amplitudes, and spacings, we could produce any longitudinal phase-space distribution required to match into a linac. We have written a computer code to do the parameter optimization. The program requires the desired matched distribution in energy vs phase at a given point in time, the number of buncher RF cavities, and their individual frequencies. The computer program then determines the optimum RF field amplitudes in the individual cavities plus the cavity spacings.

The rest of the paper is arranged as follows. We start by showing the results obtained for a series of equally spaced RF cavities each resonating at one particular frequency (a multiple of the fundamental). The RF amplitudes are set to those values determined by a Fourier expansion representing the desired energy vs phase distribution assuming a zero spacing between cavities. (A zero spacing makes the problem linear.) We show results for this method where the cavity spacing is small compared to the longitudinal focal length. We next derive a solution to the general nonlinear problem where we have a series of cavities with arbitrary spacings and RF field amplitudes. Results are obtained for some specific examples.

Linear Problem

We consider a series of pill-box cavities operating in the TM_{010}-like mode and compare this with a single multiple-frequency buncher. The single-cavity buncher resonates at 10 frequencies, with the Fourier coefficients matched to a straight-line waveform of energy vs phase. The maximum applied voltage is 12.6 kV, which will bunch a 250-KeV beam in 30 cm at 440 x 10^6 Hz. The multiple cavities have their amplitudes set to the same value as the corresponding frequency amplitude in the single multiple-frequency buncher. The phases of the cavities are set such that the synchronous particle receives no impulse. The separation between the cavity centers is 2.5 cm, and the cavities are ordered such that the highest frequencies occur first (case 1). We also consider the geometry in which the order of the cavity frequencies is reversed (case 2). Figure 1 shows the results. The solid line shows

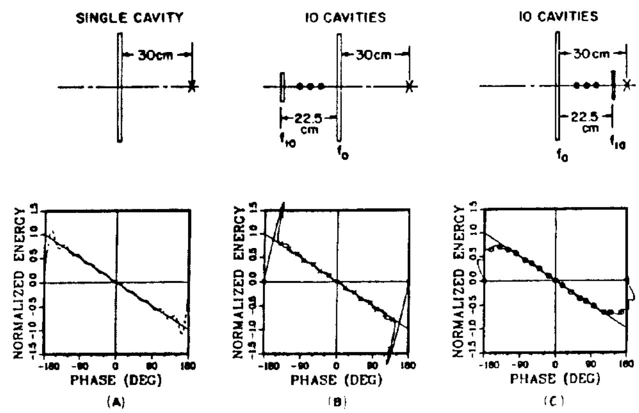

Fig. 1. Fourier Fit to Linear β vs φ distribution, 10 frequencies.

the desired linear ramp and the dashed line the waveform for the single, 10-frequency cavity. The (X's) correspond to the waveform for the multiple-cavity buncher at the fundamental frequency buncher position for case 1. The (O's) correspond to the cavities in reverse order where the beam was drifted back to the fundamental frequency-buncher position (case 2). The poor fit near 180° for case 1 occurs because all the Fourier amplitudes near 180° have the same sign, and decrease to zero at 180°. The effect of each cavity is to decrease the magnitude of the particle's phase such that it sees a larger amplitude than it should in each succeeding cavity.

At first it seems remarkable that this series of cavities, separated by a total distance of 22.5 cm compared to the 30-cm bunching distance, reproduces the effect of the single-cavity buncher so well. But for a linear waveform and case 1, we can show that the particle's total phase change traveling between the nth harmonic cavity and the fundamental frequency cavity that is due to the nth harmonic cavity is at most ($\pi D/L$), where D is the spacing between individual cavities and L is the length for bunching.

General Nonlinear Problem

Given a set of buncher cavities with fixed frequency (some multiple of the fundamental frequency of interest), determine the optimum values for the cavity spacings and RF amplitudes where the particle beam is required to have a given distribution of energy vs phase with respect to the synchronous particle at the buncher section's end (see Fig. 2). Because of symmetry considerations the relative phase of the RF electric fields in the cavities are fixed such that the electric field seen by the synchronous particle passing through a cavity is zero.

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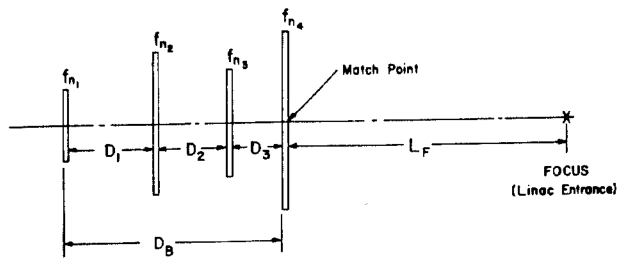


Fig. 2. Multiple cavity buncher system.

The desired particle distribution in energy vs phase is compared using a least squares procedure with that obtained by tracing particles through the buncher system, using some nominal physical parameter set. (Space-charge forces could be included if space-charge impulses were given to the particles as they were transported through the system). The physical parameters of the RF buncher cavities are varied to determine their effect on the least squares sum. A new set of physical buncher parameters is calculated to minimize the least squares sum and the iterative procedure is repeated until a stable solution is obtained. Define: f_i is the frequency of the i^{th} buncher cavity, λ_0 is the wavelength for the fundamental frequency, V is the velocity of a particle, c is the velocity of light, β is V/c, T_0 is the time at which the synchronous particle is at its initial position, T_F is the time when the synchronous particle reaches a final position (where the match is made) downstream of the buncher system. A subscript s denotes a synchronous particle parameter. The time of interest is the total time it takes the synchronous particle to travel from the initial position to the final position (see Fig. 2)

$$T_s = T_F - T_0 = (L_0 + \sum_{i=1}^{N-1} D_i + L_F)/V_s \qquad (1)$$

where L_0 is the distance between the initial position of the synchronous particle and the first buncher cavity, D_i is the distance between the i^{th} and $(i+1)^{th}$ buncher cavity, L_F is the distance between the last buncher cavity and the "match point", and N as the number of buncher cavities. We define X_0 as the initial position of a particle with respect to the synchronous particle at time T_0, and X_F as the final position of a particle with respect to the synchronous particle at time T_F. Then T_s is

$$T_s = \frac{L_0 - X_0}{V_s} + \sum_{i=1}^{N-1} \frac{D_i}{V_i} + \frac{L_F + X_F}{V_N} \qquad (2)$$

where V_i is the particle's velocity after the i^{th} buncher cavity, and V_N is the final particle velocity.

We determine X_F by combining Eqs. (1) and (2) to obtain

$$X_F = V_N \left[\frac{X_0}{V_s} + \sum_{i=1}^{N-1} D_i \left(\frac{1}{V_s} - \frac{1}{V_i}\right) + L_F \left(\frac{1}{V_s} - \frac{1}{V_N}\right)\right] \qquad (3)$$

The energy of a particle after the j^{th} cavity is

$$W_j = W_0 + \sum_{i=1}^{j} A_i \sin \varphi_i \quad ; \; j \leq N \qquad (4)$$

where W_0 is the initial energy, φ_i is the RF phase when the particle passes through the cavity and A_i is the maximum electric field amplitude in the cavity. We obtain the final particle energy for j = N. The fitting procedure will optimize the A_i's and D_i's. We have

$$\varphi_i = \varphi_{0i} + 2\pi f_i T_i \qquad (5)$$

where φ_{0i} is the RF phase in the i^{th} cavity at time T_0, and T_i is the time difference between T_0 and the time the particle passes through the i^{th} buncher cavity.

$$T_i = \frac{L_0 - X_0}{V_s} + \sum_{j=1}^{i-1} D_j/V_j \qquad (6)$$

We determine φ_{0i} from the condition that $\varphi_i = 0$ for the synchronous particle which means setting $V_j = V_s$ for all j and that $X_0 = 0$ in Eqs. [(5) and (6)] giving

$$\varphi_i = 2\pi f_i \left[-\frac{X_0}{V_s} + \sum_{j=1}^{i-1} D_j \left(\frac{1}{V_j} - \frac{1}{V_s}\right)\right] \qquad (7)$$

The energy vs position (phase) distribution is required to have a certain functional form W = G (X) at the time T_F. The particle data consist of a set of X_0's ($X_0{}^k$, k = 1,..., M points). Find the final particle positions $X_F{}^k$ and energy $W_F{}^k$ after transport through the buncher and minimize in the least square sense the functions $(G(X_F{}^k) - W_F{}^k)$ for all k (particles) by varying the buncher amplitudes A_i and distances between the cavities D_i. Minimize the function

$$I = \sum_{k=1}^{M} \left(G(X_F^k) - W_F^k\right)^2 \qquad (8)$$

with respect to the A_i's and D_i's.

$$\frac{\partial I}{\partial(A_i, D_i)} = 2 \sum_{K=1}^{M} \left(G(X_F^K) - W_F^K\right)\left[\frac{\partial G}{\partial X} \frac{\partial X}{\partial(A_i, D_i)}\Bigg|_{x=X_F^K} - \frac{\partial W_F^K}{\partial(A_i, D_i)}\right] = 0 \qquad (9)$$

Equations (9) comprise a set of nonlinear equations in the variables A and D. These equations are solved iteratively using a nonlinear least-squares fitting package. The partial derivatives $\partial(X_F, W_F)/\partial(A_i, D_i)$ can be evaluated numerically using particle transport through the buncher section. Only the partial derivative $\partial G/\partial X$ depends on the desired distribution function.

Initial Conditions

We assume that the buncher system has a longitudinal focal length equal to the distance between the final downstream buncher cavity and the entrance to the linac L_F, and that all cavities occupy the same position as the most downstream cavity. Given that the buncher produces a linear distribution in energy vs phase, we can calculate the maximum voltage (V_{max}) needed to bunch from ±180° in phase at the buncher to 0° in phase at the linac entrance assuming no space charge. The initial voltage amplitudes in the separate cavities are then set to produce a linear saw-tooth wave with maximum voltage (V_{max}) for overlapping cavities.

The synchronous particle gains no energy in the buncher. A particle displaced a distance 1/2 $\beta\lambda$ upstream of the synchronous particle should gain energy eV_{max} from the buncher. We have

$$eV_{max} = M_0c^2\left[\frac{1}{\sqrt{1-(V_F/c)^2}} - \frac{1}{\sqrt{1-(V_s/c)^2}}\right]$$

(M_0 is the particles rest mass.) We obtain V_F from Eq. (3) where $D_i = 0$ (the cavities are together), $X_F = 0$ (for a focus),

$X_0 = V_s\lambda/2c$, and $V_F = V_N$

$$V_F = L_F\Big/\left(\frac{L_F}{V_s} + \frac{\lambda}{2c}\right)$$

Given that J = (cavity frequency/fundamental frequency), the initial starting voltages are set to

$A_i = (-1)^{J_i}\ V_{max}/J_i$

Results

Figure 3 shows the results for a multiple cavity bunching system where the distance between each

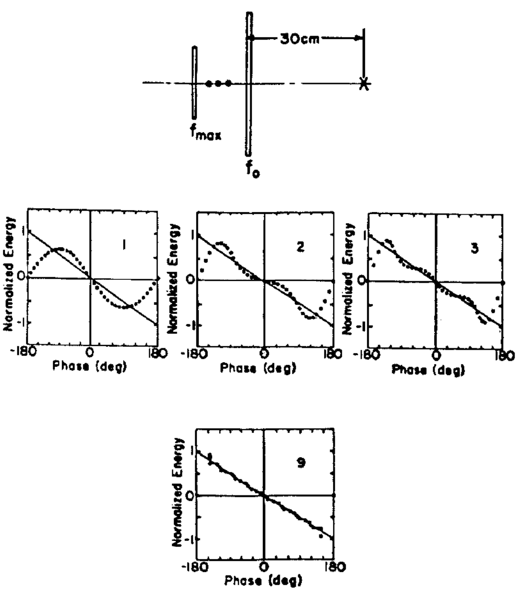
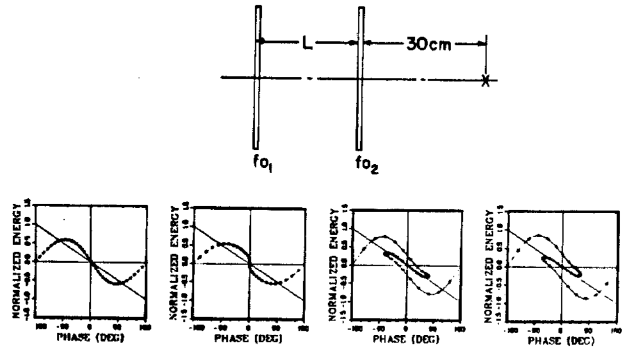


Fig. 3. Least-squares fit for cavity RF amplitudes with 2.5 cm cavity spacing.

cavity was fixed at 2.5 cm. We assume a 30-cm bunching length (distance between the final, downstream buncher cavity and the linac), a 440 x 10^6-Hz linac frequency and a 250-KeV proton energy. The number of cavities were varied from 1 to 9 with the highest frequency cavity occuring first to the beam. Figure 3 shows the phase-space distribution that ideally should be linear. The distance between each point in the figure corresponds to 10° of dc phase. The 9-cavity buncher result shown in Fig. 3 should be compared to Fig. 1 (b). Notice the large deviation near ±180° in Fig. 1 (b) has been eliminated in Fig. 3 by the fitting procedure.

Figure 4 shows results obtained with two cavities both resonating at the fundamental frequency of 440 x 10^6 Hz. The longitudinal focal length was again set at 30 cm. The RF amplitudes were obtained



Ao_1	.34	.26	.52	.42
Ao_2	.42	.49	.84	.81
L	10 CM	50 CM	60 CM	80 CM

Fig. 4. Two fundametal frequency cavities, vary the separation, find the amplitudes.

by fitting to a linear phase-space distribution in energy vs phase. Figure 4 shows results for a fixed-cavity separation (L) of 10, 50, 60, and 80 cm. There is a radically different solution obtained between 50 cm and 60 cm. The relative cavity amplitudes are given below the figure.

We can operate the program in a mode where the program tries to place particles inside an upright ellipse at the buncher focus (see Fig. 5). The program minimizes the sum of squares (raised to a

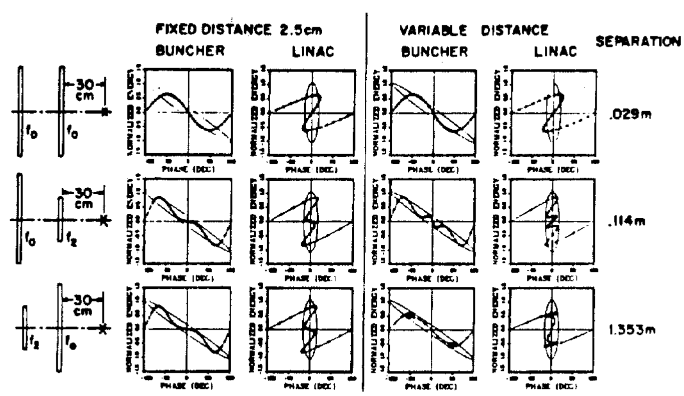


Fig. 5. Two cavity Buncher systems.

given power) of the distances in phase space from the origin for the set of particles. The distances in phase and energy are weighted by the semimajor and semiminor axis of the desired phase ellipse.

Results for a 2-cavity buncher system are shown in Fig. 5. A 250-KeV proton beam was required to bunch in 30 cm. We assumed an initial beam of 361 particles spaced uniformly over 360° in phase with no energy spread. We show cases where both cavities resonate at the fundamental frequency, where the first cavity resonates at the fundamental and the second at the second harmonic, and where the frequency order is reversed. The phase space distribution is given at the end of the buncher section and at the longitudinal focus (linac entrance). The buncher-cavity separation was fixed both at 2.5 cm and also was allowed to vary.

Reference

1. W. P. Lysenko and E. A. Wadlinger, "Three-Dimensional Space-Charge Calculation Method," Proc. of this Conf.