

CHARACTERISTICS OF MAGNETIC FOCUSING AND CHROMATICITY CORRECTION SYSTEM FOR TARN

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Summary

TARN is designed by a separated function FODO lattice. Its main magnet system consists of eight dipole and sixteen quadrupole magnets. The relatively lower injection energy of TARN ($\beta \approx 0.134$) leads to large transverse coupling impedance and the relatively smaller intensity limit from transverse coherent instability (6×10^8 ions/pulse). In order to surmount the instability by Landau damping, a chromaticity control system with twelve sextupole magnets are designed and fabricated. The field properties of all the magnets are measured precisely before alignment and the result is taken into account by the calculation with the computer code SYNCH. The closed orbit is obtained by iteration and the nonlinear elements such as sextupole magnets are linearized in the neighbourhood of the closed orbit. The work line is also experimentally studied by an RF knock-out method. The chromaticities of TARN without correction sextupole magnets were measured at -1.59 and -0.47 in horizontal and vertical directions, respectively.

1. Introduction

TARN (Test Accumulation Ring for the NUMATRON project) has been constructed in order to test the feasibility of a beam accumulation method which uses the combination of a multi-turn injection into the transverse phase space and an RF stacking into the longitudinal phase space.¹⁾ The beam dynamics are also to be studied by the ring. It is designed to be able to accumulate heavy ions up to N^{+5} with the kinetic energy of 8.5 MeV/u.²⁾ The mean radius of the ring is 5.06 m, which is determined considering the synchronization between the RF system of TARN and that of the injector SF cyclotron.

It is important to choose an optimum operation point to accumulate ions in the storage ring. For the case of TARN, an RF stacking is applied and the momentum spread of the accumulated beam is large (2.5 %) and from the point of view of avoiding the lower order single particle resonances, it is desirable to make the chromaticity as small as possible, while some amount of chromaticity is needed to surmount the transverse coherent instability by Landau damping.^{3),4)}

For the purpose of controlling the work line, the chromaticities in both horizontal and vertical directions are to be tunable and two families of correction sextupole magnets are needed.

The number of betatron oscillations per revolution is calculated by the computer code SYNCH,⁵⁾ and it is also experimentally studied by an RF knock-out method.⁶⁾

2. Transverse Coherent Instability

The transverse resistive instabilities are found by various high energy accelerators in the world.^{7),8)} The stability condition against the transverse coherent instability (called as TCI hereafter) is given by the relation:⁹⁾

$$\left| \frac{Z_{\perp}}{Z_0} \right| < 2F \frac{A}{q^2} \frac{Y}{Nr_c} v | (n - v) \tilde{\eta} + v' \left| \frac{\Delta p}{p} \right| \quad (1)$$

where the notation is as follows

$Z_0 (= 120 \pi \Omega)$ is the impedance of space,

F is a form factor depending upon the shape of momentum distributions, a value of 0.45 is appropriate for the present case,
 N is the total number of accumulated ions,
 r_c is the classical radius of such a particle as has a unit atomic mass ($m_0 c^2 = 931.5$ MeV) and is equal to $\frac{1}{4\pi\epsilon_0} \frac{e^2}{m_0 c^2} = 1.57 \times 10^{-18}$ m,
 n is the mode number and the nearest integer greater than $\frac{v}{\beta}$ should be used as this value,
 $\tilde{\eta}$ is equal to $\frac{1}{Y} - \frac{1}{Y_T}$ (Y_T is the transition gamma),
 $\frac{\Delta p}{p}$ is full width of fractional momentum spread at half height
 v is the number of betatron oscillation per revolution
 v' is the chromaticity ($= \frac{dv}{d(\frac{\Delta p}{p})}$)

and A and q are the mass number and charge state of the accumulated ion, respectively.

For the ideal case where the beam and the vacuum chamber have constant circular cross sections, transverse coupling impedance $Z_{\perp}(\Omega/m)$ is given by the formula:⁴⁾

$$Z_{\perp} = iRZ_0 \left[\frac{1}{\beta^2 \gamma^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) - (1+i) \frac{\delta}{b^3} \right] \quad (2)$$

where R is the mean radius of the machine and a and b are the radii of the beam and vacuum chamber respectively and δ is the skin depth of the chamber wall. As known from Eq. (2), the coupling impedance Z_{\perp} becomes larger for lower value of β as is the case of TARN ($\beta \approx 0.134$). Then the intensity limit of the TCI is estimated to be 6×10^8 ions/pulse without the correction sextupole magnets. The e-folding growth rate, τ , is given by

$$\frac{1}{\tau} = \frac{N\beta c r_c q^2 / A}{2\pi v \gamma \beta^3} \sqrt{\frac{2R}{Z_0 \sigma \beta (n-v)}} \quad (3)$$

where σ is the conductivity of the chamber wall ($1.37 \times 10^6 \Omega^{-1} m^{-1}$ for stainless steel) and τ is estimated to be 0.2 sec if 2×10^{10} ions of N^{5+} are stored. It is

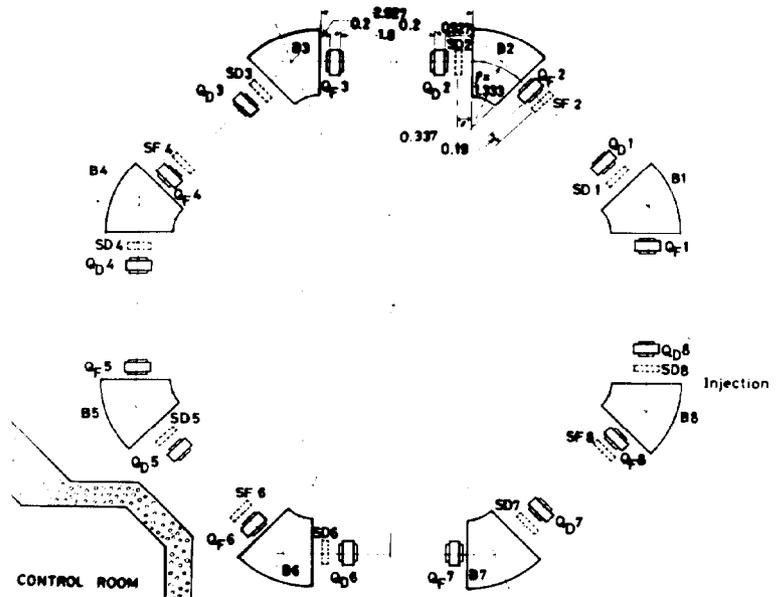


Fig. 1 Arrangement of Magnet for TARN.

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known from Eq. (1) that the intensity limit can be raised by increasing the size of chromaticity ν' .

3. Chromaticity Correction System

The arrangement of the magnets of main lattice for TARN is shown in Fig. 1 by solid lines. It consists of 8 dipole and 16 quadrupole magnets and has a separated function FODO structure. Its superperiodicity is 8. The rather higher superperiodicity is preferred so as to avoid the lower order sector resonances. The ν -value is chosen around 2.25 both in horizontal and vertical directions. The beta and dispersion functions along the central orbit calculated by the computer code SYNCH is shown in Fig. 2.

In order to control the work line in the tune diagram, it is necessary that chromaticities in horizontal and vertical directions ($\xi_x \equiv \nu'_x = \frac{d\nu_x}{d(\frac{\Delta p}{p})}$ and $\xi_z \equiv \nu'_z = \frac{d\nu_z}{d(\frac{\Delta p}{p})}$) can be varied independently.

The contributions of sextupole magnets to the chromaticities are given by

$$\begin{aligned} \xi_x &= \frac{1}{4\pi} \int \frac{B''}{B\rho} \eta \beta_x ds \\ \xi_z &= \frac{1}{4\pi} \int \frac{B''}{B\rho} \eta \beta_z ds \end{aligned} \quad (4)$$

In the present case, the wavelength of betatron oscillation is about 14 m, which is long enough compared with the core length of the sextupole magnet (0.1 m).

Therefore beta and dispersion functions can be assumed to be constant in the sextupole magnets with good approximation. Then Eq. (4) can be written as

$$\begin{aligned} \xi_x &= \frac{1}{4\pi} \frac{\langle \eta \rangle \langle \beta_x \rangle}{\beta\rho} \int B'' ds \\ \xi_z &= \frac{1}{4\pi} \frac{\langle \eta \rangle \langle \beta_z \rangle}{\beta\rho} \int B'' ds \end{aligned} \quad (5)$$

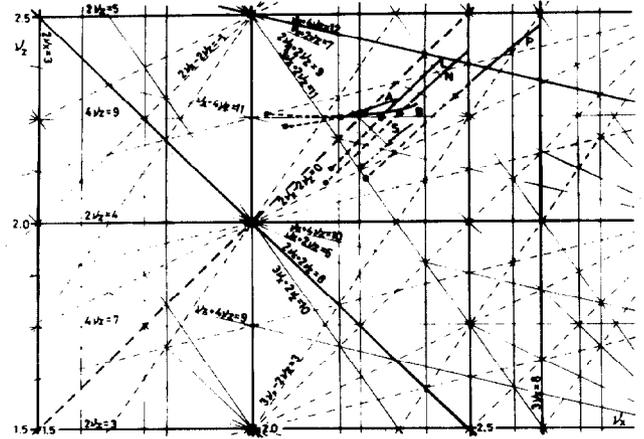
where $\langle \beta_x \rangle$, $\langle \beta_z \rangle$ and $\langle \eta \rangle$ are beta and dispersion functions in the sextupole magnet. The sextupole magnets should be placed at the two places where $\langle \beta_x \rangle$, $\langle \beta_z \rangle$ and $\langle \eta \rangle$ take different values from each other so as to introduce two degrees of freedom and are aligned at the places shown by dashed lines in Fig. 1.

Numerical calculation is made for various work lines with the use of computer code SYNCH. It calculates the closed orbits for various fractional momenta ($\frac{\Delta p}{p}$) and then the non linear elements as sextupole magnets are linearized in a neighbourhood of the closed orbit. The sextupole magnet is treated as a thin lens by the relation

$$\begin{aligned} \frac{dx}{ds} \Big|_{\text{out}} - \frac{dx}{ds} \Big|_{\text{in}} &= - \frac{B''l}{(B\rho)_0 \left(1 + \frac{\Delta p}{p}\right)^2} (x^2 - z^2) \\ \frac{dz}{ds} \Big|_{\text{out}} - \frac{dz}{ds} \Big|_{\text{in}} &= \frac{B''l}{(B\rho)_0 \left(1 + \frac{\Delta p}{p}\right)} xz \end{aligned} \quad (6)$$

Table Intensity Limits of TCI

Line	Configuration	Chromaticity (ν'_x/ν'_z)	$B''l$ (G/cm)	N_{max}
A	No Correction Ideal Magnet	-4.35/-1.07	—	1.2×10^9
B	No. Correction Realistic Magnet	-5.74/-0.25	—	5.9×10^8
L	8 SD's	-4.19/-4.28	144.6	3.7×10^9
N	8 SD's & 4 SF's	-5.83/-5.53	228.5 -152.7	3.8×10^9
P	"	-7.13/-6.56	291.7 -250.1	4.4×10^9
S	"	-1.49/-1.45	-35.1 300.0	1.5×10^9



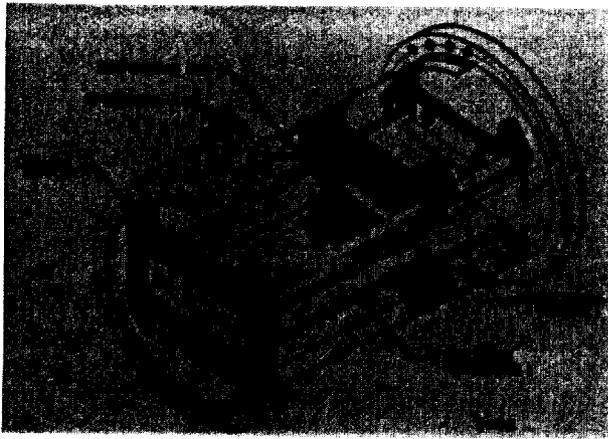


Fig. 4 Illustration of Electrodes for RF Knock-out.

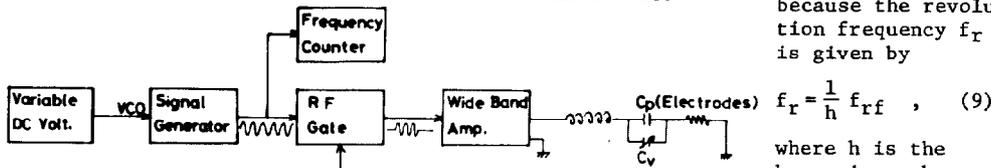


Fig. 5 Block Diagram of the RF Knock-out System.

measured to be $140 \text{ kg/m}^3 \sim 200 \text{ kg/m}^3$.¹¹⁾

Sextupole Magnet

The pole shape of the magnet is chosen to satisfy a cubic equation $3x^2z - z^3 = \pm r_0^3$. The core length is determined to be 100 mm and the maximum value of the integrated sextupole component, $\int B'' ds$, is designed to be 650 G/cm.

5. ν -measurement by an RF Knock-out Method

The ν -value can be measured by applying a transverse RF electric field which resonates with the betatron oscillation (RF Knock-out).⁶⁾ In Fig. 4 the illustration of electrodes installed in the vacuum chamber is given. The resonance condition is given by the relation

$$f_{K0} = m \cdot f_r + c \cdot f_r \quad (m = 0, \pm 1, \pm 2 \dots) \quad (7)$$

where f_{K0} is the frequency of the transverse RF field, f_r is the revolution frequency of the beam and c is the fractional part of ν -value. In the present case, the beam momentum is decreased a little (a few percent) during an RF stacking and the revolution frequency, f_r , is also decreased following the relation

$$\frac{\Delta f_r}{f_r} = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right) \frac{\Delta p}{p} \quad (8)$$

In order to observe the work line, it is necessary to know the resonant frequency, f_{K0} , for each revolution frequency, f_r . For the purpose, a pulsed transverse RF field with duration of 0.5 msec is applied and the timing of the pulsed RF is changed by a delay circuit (Fig. 5). The measurement of ν -value has been done with the use

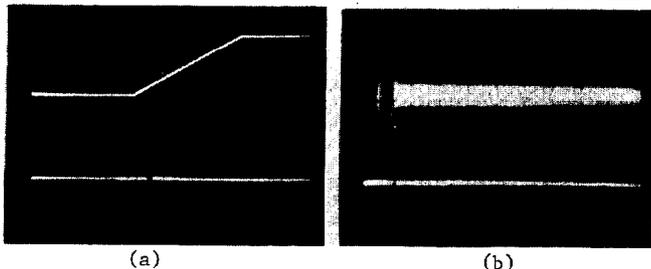


Fig. 6 Signals in the Process of RF Knock-out.

of molecular hydrogen (H_2^+) and α -particle with kinetic energy 7 MeV/u. The upper signal of Fig. 6 (a) is the sweep signal of RF frequency of the RF field, f_{rf} , applied to the accelerating cavity for stacking. The frequency is 7.987 MHz at the base line and 7.841 MHz at the flat top. The lower signal represent the pulsed transverse RF applied to the electrodes (500 V/div.). The upper signal in Fig. 6 (b) is the beam signal picked up by an electrostatic monitor¹³⁾ (5 mV/div.) and the lower signal is the same as Fig. 6 (a). It is clearly observed that when the pulsed transverse RF field with an adequate frequency, f_{K0} , is applied, the beam is lost at the corresponding timing. From the value of f_{K0} and Eq. (7), the fractional part of ν -value, c , is known, because the revolution frequency f_r is given by

$$f_r = \frac{1}{h} f_{rf} \quad (9)$$

where h is the harmonic number and is chosen at 7 for the case of TARN. The measured

ν -values for various fractional momenta are shown in Fig. 7 together with the calculated line. Without correction sextupole magnets, the experimentally obtained chromaticities are -1.59 and -0.47 for horizontal and vertical directions, respectively.

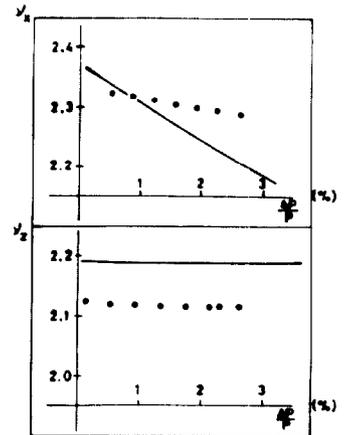


Fig. 7 ν -values for Various Fractional Momenta.

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