

COLLECTIVE EFFECTS ON DCI[†]

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Summary

New measurements of the dependance of energy spread and bunch length on beam current are reported. They are checked against the combined potential well and turbulent lengthening models. Broad band impedance is deduced from the data. Betatron frequency shifts are compared to the first head-tail modes derived from the impedance.

Bunch Length and Energy Dispersion in the Beam

Measurements of bunch lengthening and increased energy spread made on DCI have been already published¹. In this paper the turbulent lengthening model shows a good agreement for the dependance with current, RF voltage and energy but the impedance deduced from bunch length values was too large to explain the variation of the energy dispersion with current. In the present paper, we take into account the effect of the potential well distortion.

New measurements have been made at a lower energy where the increased synchrotron contribution to the beam vertical dimension is dominant. At .7 GeV, $V = 250$ kV, the increase of σ_z is about a factor 2 for a beam intensity of 200 mA.

Above the turbulent threshold, the Keil-Schnell criterion² applied to bunched beams³ gives :

$$\sigma_z^2 = U_z \beta_z \left(\frac{\sigma_E}{E} \right)_0^2 + \frac{1}{\sqrt{2\pi}} \frac{\eta_z^2 R}{E/e\alpha} \left| \frac{Z}{r} \right| \frac{I}{\sigma_\lambda} \quad (1)$$

where U_z is the betatron invariant, β_z and η_z are the local betatron and dispersion functions at the source point in the vertical bending magnet, $(\sigma_E/E)_0$ is the energy dispersion at zero current, R the average ring radius, α the momentum compaction factor, E the beam energy, I the intensity per bunch and σ_λ the r.m.s. bunch length.

$|Z/r|$ is the absolute value of the impedance of the chamber divided by the mode number ω/ω_0 . The simultaneous measurement of bunch length and vertical dimension allows one to determine this quantity.

The bunch length was measured using the signal delivered by a short electrode. The signal was integrated and corrected for small errors introduced by the network⁴. When the intensity increases, the shape is no longer gaussian. The value σ_λ used in all the fits is the second moment of the signal, corrected for a resolution estimated at 170 ps. A photodiode of 250 ps resolution gave a FWHM 11 % higher than $2.36 \sigma_\lambda$ of the electrode. In this experiment the bunch length varies between 200 and 800 ps.

The variation of the energy dispersion with the longitudinal density is shown on figure 1. The plot is linear, in agreement with an impedance approximately frequency independent.

In order to describe the broad band longitudinal impedance we suppose⁵ a resonator with $Q = 1$ quality factor and a resonance frequency $\omega_r/2\pi = 1.3$ GHz (Fig. 2).

In our case, the bunch is long ($\sigma_\lambda \gg c/\omega_r$), and in the frequency range involved, the ratio $|Z/r|$ remains close to its zero frequency value $|Z/r|_0$, in agreement with our results.

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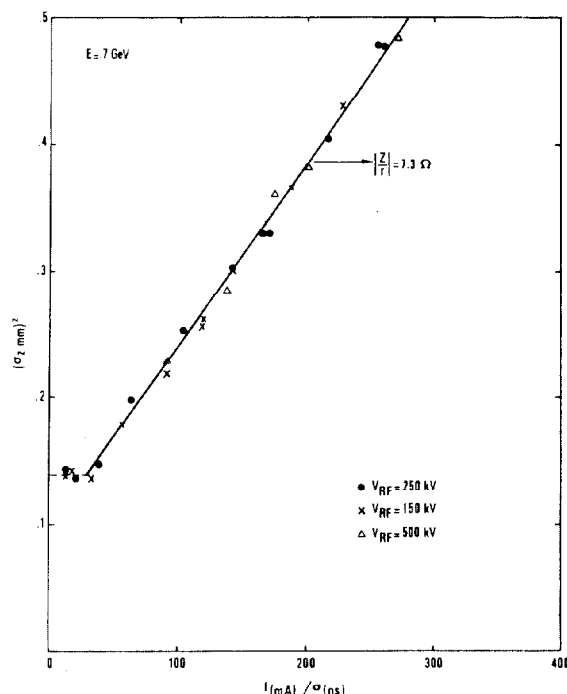


Fig. 1 : Turbulent effect. Vertical beam dimension versus longitudinal density

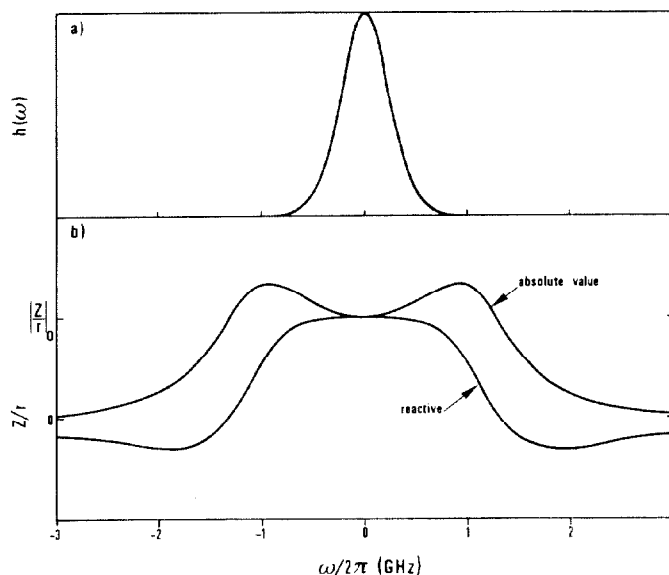


Fig. 2 : a) $h(\omega)$ power spectrum of a Gaussian bunch $\sigma_\lambda = 500$ ps
b) impedance of a resonator $Q = 1$, $\omega_r/2\pi = 1.3$ GHz

Below the turbulent threshold, the vertical size is consistent with a coupling factor $\kappa^2 = 2 \times 10^{-2}$ and the extrapolation of the straight line at zero current is in agreement with the theoretical value $\eta_z = .69$ m. The slope yields a value of $|Z/r| = 7.3 \Omega$.

Assuming that the bunch length results from a combination of potential well and turbulent effects we get :

$$\left(\frac{\sigma}{R}\right)^3 = \frac{\sqrt{2\pi} I}{h V \cos \phi_s} \left\{ \left(\frac{Z_i}{r}\right)_{\text{eff}} + \left|\frac{Z}{r}\right| \right\} I > I_{\text{th}} \quad (2)$$

where $(Z_i/r)_{\text{eff}}$ is the effective impedance deduced from the reactive part of the impedance integrated over the beam frequency spectrum :

$$\left(\frac{Z_i}{r}\right)_{\text{eff}} = \int_0^\infty \frac{Z_i(\omega)}{r} h(\omega) d\omega ,$$

$$h(\omega) = \frac{\sigma}{c\sqrt{\pi}} e^{-\omega^2 \sigma^2 / c^2} \text{ with } \int_0^\infty h(\omega) d\omega = 1 .$$

Again it appears (Fig. 2) that for long bunches $(Z_i/r)_{\text{eff}}$ is roughly equal to $|Z/r|_0$.

This is clear on figure 3 where $A\sigma_z^2 (V \cos \phi_s)^{1/3}$ is plotted versus $I^{1/3}$.

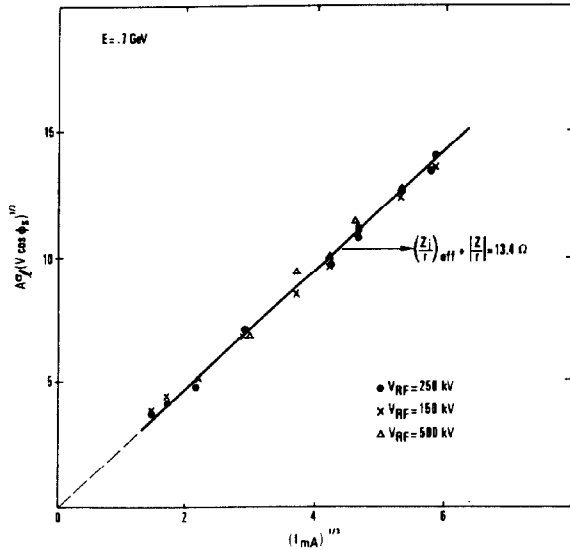


Fig. 3 : Combined potential well and turbulent bunch lengthening effects. Bunch length versus current

We deduce $(Z_i/r)_{\text{eff}} + |Z/r| = 13.4 \Omega$. With the value of $|Z/r| = 7.3 \Omega$ one gets $(Z_i/r)_{\text{eff}} = 6.1 \Omega$. Both values are nearly equal and can be related to

$$\left|\frac{Z}{r}\right|_0 \approx 7 \Omega .$$

The experimental values of σ^2 versus $(V \cos \phi_s)^{1/3} I^{2/3}$, on figure 4, fit accurately with the computed straight line from the combined potential well and turbulent effects :

$$\sigma_z^2 = U_z \beta_z \left(\frac{\sigma_E}{E}\right)_0^2 + \frac{\eta_z^2 h^{1/3} \left|\frac{Z}{r}\right| (V \cos \phi_s)^{1/3} I^{2/3}}{(2\pi)^{2/3} \alpha \frac{E}{e} \left[\left(\frac{Z_i}{r}\right)_{\text{eff}} + \left|\frac{Z}{r}\right| \right]^{1/3}} \quad (3)$$

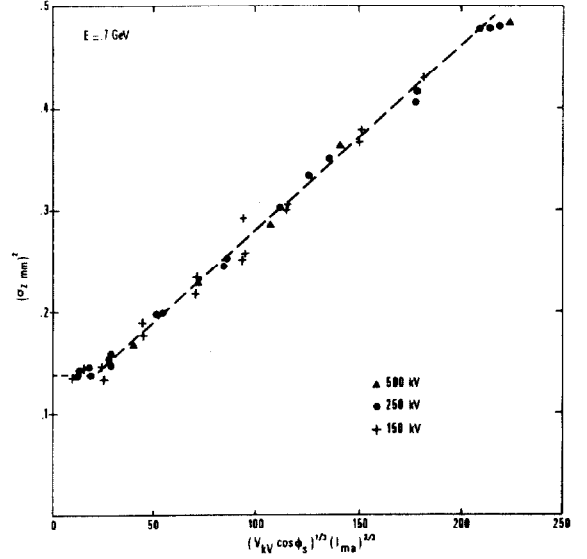


Fig. 4 : Combined potential well and turbulent effects. Vertical beam dimension versus current.

Vertical Betatron Frequency Shifts

Additional measurements of vertical betatron frequency shifts were also taken at .7 GeV and compared with the first head-tail modes⁶ for long bunches :

$$\Delta Q_m \approx \frac{I R^2 e}{(m+1) 2 Q_z E L} Z_T \quad (4)$$

L being the full bunch length and Z_T the transverse coupling impedance.

Taking $L = 2\sqrt{2} \sigma$ we get from equation (2) :

$$\Delta Q_0 \approx \frac{R h^{1/3} (V \cos \phi_s)^{1/3} I^{2/3} Z_T}{4\sqrt{2} Q_z E/e (\sqrt{2\pi})^{1/3} \left[\left(\frac{Z_i}{r}\right)_{\text{eff}} + \left|\frac{Z}{r}\right| \right]^{1/3}}$$

The two frequencies observed on DCI are plotted on figure 5 versus

$$\frac{I^{2/3} (V \cos \phi_s)^{1/3}}{E}$$

A linear variation is observed which permits the definition of $Z_T : Z_T = 2.8 \times 10^5 \Omega/\text{m}$.

Notice that the measurements previously made at 1 GeV lie on a slightly different curve.

From the relation linking the transversal and longitudinal impedances $Z_T = (2R/b^2) |Z/r|_0$ we get $b = 2.7$ cm. This value is somewhat small for the DCI vacuum chamber.

However there are several approximations in this computation, namely the length taken into account is questionable and the fact that DCI's vacuum chamber is not smooth.

It is worth-while mentioning that if the working point crosses the 2/3 resonance, one sees that the beam is excited on the TV screen and eventually can be lost, both for mode 0 and mode 1.

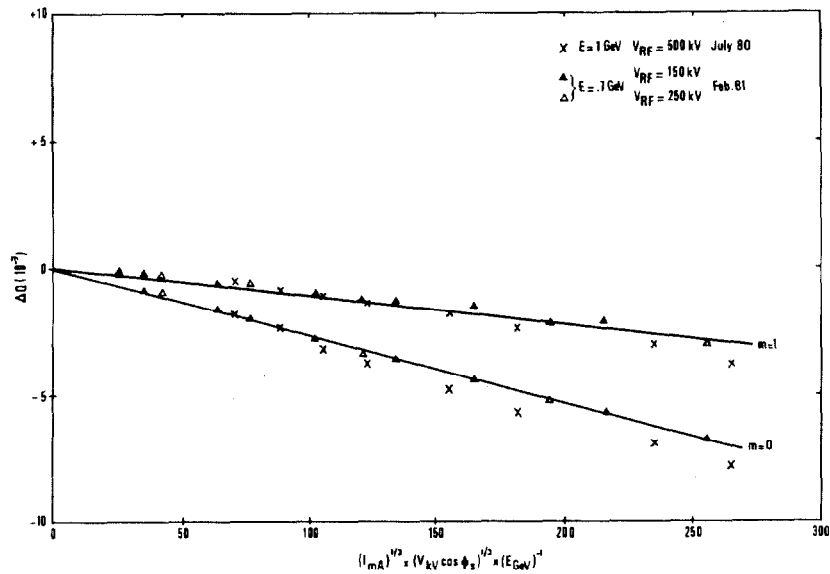


Fig. 5 : Tune shift versus current

Acknowledgments

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References

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