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STUDY OF THE BEAM BREAKUP MODE IN LINEAR INDUCTION ACCELERATORS FOR HEAVY IONS*

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Abstract

A simple theoretical study and numerical estimate is presented for the transverse amplitude growth of a nonrelativistic heavy ion beam in an induction linac, as envisaged for use in commercial power plants, due to the nonregenerative coherent beam breakup mode. An equivalent electrical circuit has been used to represent the accelerating induction modules. Our calculation shows that for the parameters of interest, the beam breakup amplitude for a heavy ion beam grows extremely slowly in the time scales of interest, to magnitudes insignificant for transport purposes. It is concluded that the coherent beam breakup mode does not pose any serious threat to the stability of a high current (kA) heavy ion beam in an induction linac.

I. Introduction

High current heavy ion beams are being actively studied as potential drivers for inertial confinement fusion. Such high current nonneutral beams are subject to coherent and incoherent, transverse and longitudinal, collective instabilities arising from the beam space charge (self-force) and its interaction with the environment (external impedances, cavities etc.). In this paper, we study the growing coherent transverse motion of a high current (~kA) heavy ion beam due to an oscillatory transverse mode (analogous to TM_{110} mode of a pill-box cavity) excited by the beam in the accelerating modules. The subject has been studied extensively in connection with electron linacs by several authors[1-5], who computed the upper limit of transportable total charge set by the growth of beam breakup amplitude. However no such study has been reported for heavy ion beams transported by induction linacs.

II. Model For Transport

Our theoretical model of transport is a semi-infinite series of identical accelerating induction modules with identical focussing elements between them (see Fig. 1). If the beam centroid is off center (or if the beam is centered in an azimuthally asymmetric structure), it will excite a transversely deflecting mode in the modules. The induced electromagnetic fields act on later parts of the beam, causing a transverse motion of the beam centroid. The amplitude of the coherent beam oscillation increases from head to tail in a bunch, the cavity excitation increases in time at any location and increases in distance along the accelerator.

Our analysis is based on the following assumptions:

(a) Only one effective resonant mode of frequency Ω and quality factor Q is of significance.

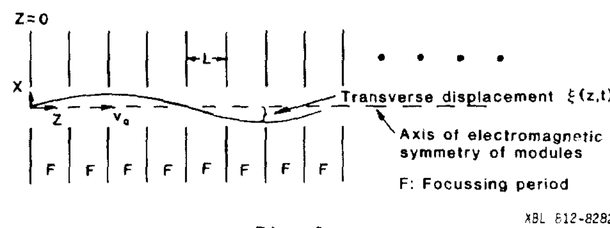

Fig. 1

(b) Focussing can be treated in the smooth approximation i.e. focussing fields of quadrupoles or interrupted solenoids can be replaced by their average values.

(c) There is no acceleration.

(d) The process is 'non-regenerative', i.e. there is no propagation of electromagnetic fields from one induction module to the next and information is carried only by perturbations on the beam.

(e) The rate of amplitude growth is small compared to Ω.

III. Induction Module Response

An induction linac module differs drastically from an r.f. cavity in its response to excitation by a particle beam. There is no accelerating mode as such[6]; the longitudinal interaction of beam and module is best represented by an equivalent circuit involving the external drive, corresponding typically to a frequency of a few megacycles and strongly overdamped by the low drive-impedance. For the asymmetric modes of interest to the beam breakup phenomenon, the module looks like a pill-box with conducting end walls and a lossy outer wall traversed longitudinally by one or more conducting straps. Accordingly, we take as a model the excitation of the TM_{110} mode of a pill-box cavity with a radius of about half a meter and a Q of about 10.

The vector potential can be written as:

$$A_z = A(t)\ J_1\left(\frac{\Omega}{c}\, r\right) \cos\theta$$

and A satisfies the differential equation:

$$\ddot{A} + \alpha\dot{A} + \Omega^2 A = \frac{\mu_0 \Omega^3 I}{\pi c j_1^{\,2} J_0(j_1) J_2(j_1)}\ \xi(\tau) \tag{1}$$

where I is the beam current, $\xi(\tau)$ is the transverse beam displacement at time τ following beam arrival at the module and the other symbols have their conventional meanings. In traversing the cavity, the beam experiences a change in slope (see Fig. 2) given by:

$$\Delta\xi' = -\frac{(Ze)\ell}{mv_0} B_y = \frac{(Ze)}{2mv_0} \cdot \frac{\ell\Omega}{c} A$$

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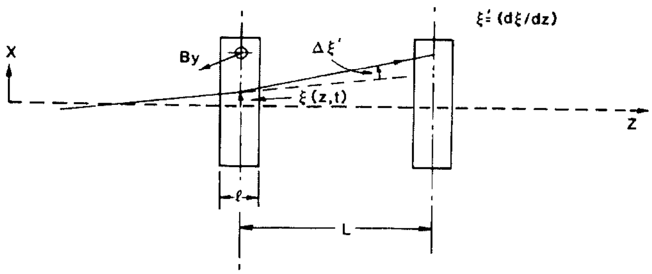

Fig. 2

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where Z is the charge state of the ions, ℓ the effective cavity length (including a transit time factor) and mv_0 the particle momentum. Using the solution of eqn. (1), we then have:

$$\frac{\Delta\xi'}{L} = -iG\int_0^\tau dt\, e^{-\alpha(\tau-t)}\,\xi(t)\left[e^{i\Omega(\tau-t)} - e^{-i\Omega(\tau-t)}\right]$$

where $$G = \frac{Z^2 r_p\,\ell}{ALv_0}\left[\frac{\dot{N}\Omega^3}{c^2 j_1{}^2 J_0(j_1)J_2(j_1)}\right]$$

with $r_p = (e^2/4\pi\epsilon_0 m_p c^2)$(classical proton radius), L the distance between modules, $\dot{N}$ the current in particles per second, and A the atomic number.

IV. Equation of Motion and Solution in Closed Form

In the approximation that both focussing and the impulses from the modules can be replaced by their average values (smooth approximation), the transverse displacement is then determined by an integro-differential equation:

$$\left[\frac{\partial^2}{\partial z^2} + \omega_\beta{}^2\right]\xi(z,\tau) = -iG\int_0^\tau dt\, e^{-\alpha(\tau-t)} \times \xi(z,t)\left[e^{i\Omega(\tau-t)} - e^{-i\Omega(\tau-t)}\right]$$

where ω_β is the coherent spatial betatron frequency. Then, with a change of variable,

$$\xi(z,\tau) = e^{-\alpha\tau}\left[X(z,\tau)e^{i\Omega\tau} + X^*(z,\tau)e^{-i\Omega\tau}\right] \qquad (2)$$

where $X(z,\tau)$ is a slowly varying function of τ, we arrive at the equation[7]:

$$\left[\frac{\partial^2}{\partial z^2} + \omega_\beta{}^2\right]X(z,\tau) = -iG\int_0^\tau dt\, X(z,t) \qquad (3)$$

We have neglected a rapidly varying term in $e^{2i\Omega\tau}$ in arriving at eqn. (3). We now take a Laplace transform of eqn. (3) in τ obtaining:

$$\frac{\partial^2\tilde{x}(z,s)}{\partial z^2} + \left[\omega_\beta{}^2 + \frac{iG}{s}\right]\tilde{x}(z,s) = 0$$

with the immediate solution:

$$\tilde{x}(z,s) = \tilde{x}(0,s)\,\mathrm{Cos}\left[\left(\omega_\beta{}^2 + \frac{iG}{s}\right)^{1/2} z\right]$$

For an initial displacement

$$\xi(0,\tau) = d\,e^{-\alpha\tau}\,\mathrm{Cos}\,\Omega\tau$$

we have $x(0,\tau) = \frac{d}{2}$ and $\tilde{x}(0,s) = \frac{d}{2s}$

Thus: $$\tilde{x}(z,s) = \frac{d}{2s}\,\mathrm{Cos}\left[\left(\omega_\beta{}^2 + \frac{iG}{s}\right)^{1/2} z\right]$$

Using infinite and binomial series expansions for the cosine and $(\omega_\beta{}^2 + iG/s)^n$ respectively and making use of the Laplace inversion formula

$$L^{-1}\left(\frac{1}{s^{n+1}}\right) = \frac{s^n}{n!}$$

we get an expression for $X(z,\tau)$ involving a double sum over integers, one of which can be summed in closed form to give spherical Bessel functions. We finally get:

$$X(z,\tau) = \frac{d}{2}\sum_{\ell=0}^{\infty}(-i)^\ell\,\frac{1}{(\ell!)^2}\left(\frac{Gz\tau}{2\omega_\beta}\right)^\ell(\omega_\beta z)j_\ell(\omega_\beta z) \qquad (4)$$

After a few betatron wavelengths down the accelerator, $\omega_\beta z \gg 1$ and we use:

$$j_{\ell-1}(\omega_\beta z)\xrightarrow[(\omega_\beta z)\to\infty]{}\frac{1}{(\omega_\beta z)}\,\mathrm{Cos}\left[\omega_\beta z - \tfrac{1}{2}\ell\pi\right] \qquad (5)$$

Using (4), (5) and (2), we finally arrive at the expression for the transverse beam displacement $\xi(z,\tau)$ at location z and time τ following the arrival of the front of the beam, in closed form, as follows:

$$\xi(z,\tau) = \frac{d}{2}e^{-\alpha\tau}\left[\mathrm{Cos}(\omega_\beta z - \Omega\tau)I_0\left(\sqrt{\frac{2Gz\tau}{\omega_\beta}}\right) + \mathrm{Cos}(\omega_\beta z + \Omega\tau)\,J_0\left(\sqrt{\frac{2Gz\tau}{\omega_\beta}}\right)\right] \qquad (6)$$

$$(\omega_\beta z \gg 1)$$

where J_0 and I_0 are zero-order Bessel and modified Bessel functions respectively.

We note that in the limit of no focussing at all ($\omega_\beta = 0$), we have:

$$x(z,\tau) = \frac{d}{2}\sum_{\ell=0}^{\infty}(-i)^\ell\,\frac{(Gz^2\tau)^\ell}{\ell!(2\ell)!}$$

so that the absolute square of the slowly varying amplitude grows as:

$$|X(z,\tau)|^2 = \frac{d^2}{4}\sum_{n=0}^{\infty}(Gz^2\tau)^{2n}\sum_{m=0}^{2n}\frac{(-1)^{n-m}}{m!(2m)!(2n-m)!(4n-2m)!}$$

in agreement with Panofsky and Bander[2] and hence is expected to scale similarly as

$$|X(z,\tau)|^2 \sim e^s \quad \text{with} \quad s = (Gz^2\tau)^{1/3}$$

V. Numercial Estimates:

We observe from expression (6) that the beam displacement is damped on the whole if $\alpha > (Gz/2\omega_\beta)$; if $\alpha \ll (Gz/2\omega_\beta)$, the maximum in τ of the amplitude of displacement comes at $\tau = (Gz/2\omega_\beta\alpha^2)$ and has a magnitude:

$$x = \frac{d}{2}\sqrt{\frac{\alpha\omega_\beta}{2\pi Gz}}\; e^{Gz/2\omega_\beta\alpha}$$

As a numerical example, we consider an induction linac that accelerates singly charged Uranium ions, with a 30° phase advance between modules. Example beam parameters[8] for two significant cases and parameters of equivalent induction module cavities are listed in Table I below.

TABLE I

	A	B
Ion	Uranium	Uranium
Charge State, Z	+1	+1
Atomic Number, A	238	238
Beam Energy	3 MJ	10 MJ
Kinetic Energy	10 GeV	10 GeV
$\mathring{N}\tau$	$\sim 2 \times 10^{15}$	$\sim 3 \times 10^{15}$
Resonant frequency, Ω	300 MHz	300 MHz
Q of cavity ($\alpha = \Omega/2Q$)	10	10
Effective length of cavity, ℓ	5 cm.	5 cm.
Distance between modules, L	1 meter	1 meter
Phase advance per module, $\omega_\beta L$	30°	30°

With these parameters, we find that the transverse beam breakup amplitude is damped for $z < 540$ km for Case a with 3 kA current and $z < 180$km for Case B with 10 kA current. These distances are much larger than the length of about 10 kms. visualized for Inertial Confinement Fusion drivers. Beyond these distances, the magnitude of maximum amplitude grows with distance down the machine as

$$x = \frac{d}{2}\, \delta\, z^{-1/2}\, e^{\gamma z}\ , \quad \delta = (4\pi\gamma)^{-1/2}$$

where $\gamma = 1.85 \times 10^{-5}\ m^{-1}$ for Case A and $\gamma = 5.55 \times 10^{-6}\ m^{-1}$ for Case B.

Conversely, for a 10 km. long machine, the beam breakup amplitude starts becoming significant when the product (QI) is about 1850 kA. For a Q of about 10 as in Table I, this implies no growth of tranvserse amplitude up to a current of 185 kA. For a beam carrying about 10 kA current, as envisaged in typical ICF drivers, we would need a Q of at least 200 for transverse oscillations to start to grow.

VI. Conclusion

As is evident from the estimates above, the damping due to the low-Q, heavily loaded induction modules is dominant over the cumulative buildup of the beam breakup mode and prevents growth of transverse oscillation amplitude for large distances of the order of hundreds of kilometers or equivalently up to high currents of hundreds of kiloamperes! For an accelerated beam, the total pulse duration is usually much shorter than the time at which maximum growth occurs for very large distances. We conclude that high current heavy ion beams in induction linacs are safe against the beam breakup mode in general. However, induction modules driven asymmetrically in azimuth, could be dangerous for beam transport against the beam breakup mode, since the beam would have no equilibrium orbit at all in such a case.

References

1. R.L. Gluckstern, D.E. Nagle and W.M. Visscher, A Note on Transverse Beam Instabilities in Multisection Linacs, Proceedings of the 1966 Linear Accelerator Conference, 1966, Los Alamos Document LA-3609, p. 281.

2. W.K.H. Panofsky and M. Bander, Asymptotic Theory of Beam Breakup in Linear Accelerators, The Review of Scientific Instruments, 1968, Vol. 39, No.2.

3. V. Kelvin Neil and Richard K. Cooper, Coherent Instabilities in High Current Linear Induction Accelerators, Particle Accelerators, 1970, Vol. 1, pp. 111-120.

4. R. Helm and G. Loew, Beam Breakup, Chapter B.1.4. in Linear Accelerators (Eds. P.M. Lapostolle and A.L. Septier), North Holland Book Co., Amsterdam, 1970.

5. V.K. Neil, L.S. Hall and R.K. Cooper, Further Theoretical Studies of the Beam Breakup Instability, Particle Accelerators, 1979, Vol. 9, pp. 213-222.

6. A. Faltens, Proceedings of the Heavy Ion Fusion Workshop, LBL-10301 and SLAC-Pub-2575, 1979, p. 182.

7. This equation is derived also in Ref. 4.

8. W.B. Herrmannsfeldt, Heavy Ion Accelerator Study Session, Proceedings of the Heavy Ion Fusion Workshop, LBL-10301 and SLAC-Pub-2575, 1979, pp. 1-5.