## COLLECTIVE LONGITUDINAL MOTION IN INTENSE ION BEAMS

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# Abstract

Inertial Confinement Fusion (ICF) using high energy heavy ion beams requires tight focussing of the igniting ion beams in longitudinal, as well as transverse, space at the pellet target. This sets significant requirements for stability in longitudinal motion. It has been previously noted that this motion can contain significant resistive wall instability. Results of numerical simulations of this instability in perturbed bunched beams are presented and analyzed. It is found that reflection of perturbations off bunch ends is distorted and delayed by space charge forces and that "soliton" waves can appear after reflection.

#### Introduction

In earlier papers<sup>1,2,3</sup> we have explored some characteristics of longitudinal motion in intense ion beams. In references 1 and 2, longitudinal motion in bunched beams with "space charge" forces is studied analytically and in reference 3 longitudinal motion in coasting and bunched beams with "space charge" and "resistive wall" forces is explored analytically and numerically. In reference 3, we find that the "space charge" force causes wave propagation of disturbances in an equilibrium distribution and that a resistive force can cause unstable growth of these disturbances. In this paper we extend our discussion of this wave instability and note characteristics of the "wave reflection" which occurs when these disturbances reach the ends of a beam bunch. "Soliton" formation is observed in reflection, as described below.

### Equations of Motion

We choose the longitudinal position within the bunch z and the distance along the accelerator s as our dependent and independent variables. We assume, as an approximation, that transverse motion is completely decoupled and the bunch is not accelerated. Then for the equation of motion we take the sum of three forces:

1) a "space charge" force

$$-\mathbf{A} \frac{\partial \lambda}{\partial z} = \frac{-\mathbf{q}^2 \mathbf{e}^2}{M\beta^2 \mathbf{c}^2} \frac{\mathbf{g}}{4\pi\epsilon_0} \frac{\partial \lambda}{\partial z}$$
(1)

where qe, M, and  $\beta c$  are the ion charge, mass and velocity, g a geometric factor, and  $\lambda$  the ion charge density.

2) a "resistive wall" force

$$-B\lambda = \frac{-q^2 e^2 R^{\dagger}}{MBc} \lambda$$
 (2)

where R' is the resistive coupling per meter.

3) a bunching force F(z,s) provided by external fields.

\*Operated by Universities Research Association, Inc. under contract with the U.S. Department of Energy. Our equation of motion is nonrelativistic:

$$z'' = \frac{d^2z}{ds^2} = F(z,s) - A \frac{d\lambda}{dz} - B\lambda$$
(3)

To demonstrate the coasting beam instability, we start with an initial distribution

$$f_{o}(z,z') = N' \delta(z')$$

where N' is the initial ion density and the velocity distribution is taken as a  $\delta$ -function to approximate the low velocity spread case of an ICF driver.

We take a perturbation:

$$f_{1} = f_{1}(z^{*}) e^{i(kz-\omega s)}$$
 (4)

and solve the linearized Vlasov equation for  $\omega(k),$  obtaining

$$\omega^2 = N'(A k^2 - ikB)$$
(5)

When Ak >> B, as is true for HIF linacs, we find:

$$\operatorname{Re}\left(\frac{\omega}{k}\right) = \pm \sqrt{N^{\dagger}A} \qquad \operatorname{Im}\left(\frac{\omega}{k}\right) \cong \pm \frac{1}{2} \frac{k}{|k|} \frac{B}{\sqrt{A}} \qquad (6)$$

Since  $\operatorname{Re}(\frac{\omega}{k}) \cong \pm \sqrt{N'A}$  is independent of k, the wave velocity and group velocity are equal, which means disturbances propagate together as coherent wave packets along the beam. Also the motion is unstable, since  $\operatorname{Im}(\omega) \neq 0$ . A forward propagating wave decays  $(\operatorname{Im}(\omega)<0)$  and a backward wave grows.

Typical parameters for ICF can be substituted into equations 5 to find sample values of  $\text{Re}(\omega/k)$  and  $\text{Im}(\omega)$ . For example with R' = 100  $\Omega/m$ , N' =  $3\times10^{13}$  ions/m, q = 4, g = 2, M = 240 m and  $\beta$  = .33 we find  $\text{Re}(\omega/k) = 7.4\times10^{-3}$  and  $\text{Im}(\omega) = \pm.002$  m<sup>-1</sup>, which gives a growth distance (500 m) smaller than typical ICF design lengths.

This behavior has been seen in numerical simulation of longitudinal motion in beam bunches with HIF parameters, using a program first developed by Buchanan, Neil and Cooper." In figures 1 we show an initial disturbance in the center of a beam bunch splitting into forward "fast" and backward "slow" wave packets which decay and grow respectively, in agreement with equation 5.

#### Reflection of Wave Packets at Bunch Ends

In bunched beams the wave packets of a perturbation will reach the bunch ends in a finite time. A naive expectation is that a growing slow



Figure 1-A-D. Wave propagation in a perturbed beam bunch with "resistive-wall" and "space charge" forces. The perturbation splits into a decaying "fast" wave and a growing slow wave. The upper graphs show  $I(\tau) \propto \lambda(-z)$ ; the lower graphs show the phase space distribution  $f(t, \Delta E) \propto f(-z, z')$ .

wave will reach the end of the bunch, be immediately reflected to a decaying "fast" wave by the external bunching field and produce no net instability.

Simulation indicates that reflection is substantially delayed in HIF beams,<sup>3</sup> in fact, more than would be expected by considering the motion of single particles past the bunch ends to reflection by the bunching force. Reflection is a collective process strongly influenced by space charge forces. It can be understood by approximating the bunch end by a parabolic distribution<sup>1</sup> with N particles, characteristic length  $z_0$ , emittance  $\varepsilon_1$ , and a linear bunching force balancing the debunching force

$$z_0" = \frac{d^2 z_0}{ds^2} = 0 \cong \frac{\varepsilon_L}{z_0} + \frac{3}{2} \frac{A N}{z_0 z} - K z_0$$
(7)

In HIF beams  $\varepsilon_L$  is small and will be ignored below. A disturbance reaching the end of the bunch perturbs zo to  $z_0 + \Delta z_0(s)$ . The equation for  $\Delta z_0$  is:

$$\frac{d^2 \Delta z_0}{ds^2} = \simeq \left( \frac{-3AN}{z_0^3} - K \right) \quad \Delta z_0 \approx -3 \ K \ \Delta z_0 \tag{8}$$

Reflection occurs when  $\Delta z_0$  oscillates over a phase  $\pi,$  that is,

$$s = \frac{\pi}{\sqrt{3K}} = \pi \sqrt{\frac{2z_0^3}{9AN}}$$
 (9)

or ightarrow 1km with typical HIF parameters. This approximate analysis agrees with the numerical simulations,  $^3$  as shown in Figures 2.



Figure 2-A-G. Reflection of perturbation off bunch ends with zero "resistive wall." Envelope oscillation is noticeable as well as particle oscillations to large  $\Delta p/p$  at the ends.

### Velocity Wave Formation

In simulations with resistance, an interesting phenomenon can occur when a large perturbation is reflected off the bunch end. The return wave is changed in character from the initial wave and changes the average energy of the bunch. In Figure 3A we show a phase space distribution from a numerical simulation showing this characteristic wave form.

Similar behavior can be seen in a simple coasting beam model in which we collapse the velocity distribution to delta functions as in section 1. The distribution is

$$f(z,z') = \lambda_0 \ \delta \ (z') \qquad z > z_0 + V_w s$$
$$f(z,z') = \lambda_1 \ \delta \ (z'+V_0) \qquad z < z_0 + V_w s$$

This has a "velocity" wave propagating with speed  $\Psi_{_{\rm LI}}$  and with amplitude Vø, as shown in Figure 3B.

This ansatz must be self-consistent, which means:

1) The particle flux entering the wave front must equal the particle flux leaving the wave front:

$$\Psi_{W} \lambda_{0} = \lambda_{1} (\Psi_{W} + \Psi_{0}), \qquad (10)$$

2) Particles in the distribution must receive an

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Figure 3A. Spontaneous appearance of "soliton" mode in f(z,z') after reflection of perturbation off bunch end in bunched beam with "resistive wall".



λ(z)

Figure 3B. Description of "soliton" mode in beam; f(z,z') and  $\lambda(z)$  are displayed.

impulse  $-V_0$  from the space charge force as the wave front passes through them. The space charge force is found from equation (1)

$$F = -A \frac{d\lambda}{dz} \cong -A \frac{(\lambda_0 - \lambda_1)}{\delta}$$
(11)

where we have stretched the discontinuity of figure 3B

over a distance  $\delta.$  The impulse received as the wave passes is:

$$I = -A \quad \frac{(\lambda_0 - \lambda_1)}{\delta} \cdot \frac{\delta}{V_w} = -A \quad \frac{(\lambda_0 - \lambda_1)}{V_w} = -V_0 \quad (12)$$

Equations (10) and (12) may be solved for  ${\tt V}_{\tt W},$  obtaining:

 $V_{\omega} = \sqrt{A\lambda_1}$ 

This is the same wave velocity obtained above for a harmonic wave, except that  $\lambda_0$  is replaced by  $\lambda_1$ . This treatment is for a forward-going "fast" wave; a similar "slow" wave can be formed by changing the sign of  $V_{_{\rm W}}$  and following the self consistent conditions.

The above description is valid for coasting beam, zero resistance motion. Maschke<sup>5</sup> has also noted that similar waves may appear in those conditions and calls these waves "solitons".

Our simulations show that these "solitons" can appear spontaneously in <u>bunched</u> beam with finite resistance when disturbances are reflected off the bunch ends. Their behavior is in agreement with the coasting beam constraints.

Analysis of stability in this characteristic wave propagation mode is important in determining longitudinal stability. Numerical simulation so far indicates that the returning "fast" soliton does not decay as predicted for harmonic waves above. Further numerical and analytic study is necessary.

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### References

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