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BEAM-BEAM INTERACTIONS: A SUMMARY OF EXPERIMENTAL EVIDENCE

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Summary

The performance of existing colliding beam accelerators (mainly electron-positron) and experimental results on the beam-beam interaction (two beams) are discussed.

Introduction

All colliding beam accelerators in operation, except the ISR at CERN, are electron machines and a discussion of experimental results necessarily deals mostly with the latter. The following formulae refer to bunched gaussian beams, in head-on collision.

The basic parameters generally used to describe incoherent beam-beam interaction are derived from the linear lens model[1)2)]. The strength of the lens equivalent to a gaussian beam with standard deviations σ_x and σ_y, is given by:

$$\frac{4\pi}{\beta_x^*}\xi_x = \frac{2 r_o i_b}{e f_o \gamma \sigma_x^{*2}(1+\sigma_y^*/\sigma_x^*)} = \frac{\sigma_y^*}{\sigma_x^*}\frac{4\pi}{\beta_y^*}\xi_y$$
$$\frac{4\pi}{\beta_y^*}\xi_y = \frac{2 r_o i_b}{e f_o \gamma \sigma_x^* \sigma_y^*\left(1+\sigma_y^*/\sigma_x^*\right)} \qquad (1)$$

where i_b is the current per bunch, γ is the relativistic factor, r_o the electron classical radius and f_o the revolution frequency. A star indicates quantities evaluated at the crossing point. Similar formulae can be written for coasting beams[3)].

The space charge parameters ξ_x, ξ_y measure the strength of the interaction, which produces betatron wavenumber shifts, $\delta Q_{x,y}$, given by:

$$2\pi\xi = \sin(2\pi\delta Q)\left(1+\mathrm{tg}(\pi\delta Q)\cot g\mu\right) \qquad (2)$$

μ being the phase shift between two consecutive interaction points (equation (2) is valid for a perfect machine in which all crossing points are exactly equivalent). In fact, the nonlinearity of the interaction produces a continuous Q distribution extending from the unperturbed Q value all the way up to $Q + \delta Q$.

For two identical gaussian beams in head-on collision, luminosity is given by:

$$L = \frac{b\, i_b^2}{e^2 f_o 4\pi\sigma_x\sigma_y} \qquad (3)$$

Equations (1) and (3) can be combined to give:

$$L = \frac{f_o b}{r_o^2}\gamma^2\varepsilon_x^* \frac{\xi_x \xi_y}{\beta_y^*}\left(1+\frac{\xi_x\beta_y}{\xi_y\beta_x}\right)^2 \qquad (4)$$

where b is the number of bunches and ε_x^* is defined by:

$$\varepsilon_x^* = \frac{\pi\left(\sigma_{x\beta}^{*2}+\sigma_{xS}^{*2}\right)}{\beta_x^*} \qquad (5)$$

$\sigma_{x\beta}^*$ and σ_{xS}^* are the betatron and the synchrotron contribution to the horizontal beam size respectively. Whenever the dispersion function at the crossing point is zero, ε_x^* coincides with the usual betatron emittance.

Equation (4) clearly exhibits all the requirements to be met by an optimum design:

a) lowest possible value of β_y^*;

b) largest possible emittance, and means of controlling its energy dependence;

c) coupling dictated by the maximum achievable values of ξ_x, ξ_y.

Lacking a complete theory of the beam-beam effect, conventional design wisdom was based on the assumption that beam dimensions are determined by the optics (including quantum fluctuations and damping), and that an energy independent upper limit on $\delta Q_{x,y}$ can be reached. If then ε_x^* has its natural energy dependence ($\propto E^2$), for any given machine luminosity will scale with energy like E^4 and the storable current like E^3. If ε_x^* is kept constant, so as to utilize the full machine aperture at all energies, luminosity will scale like E^2 and current like E. In the absence of dispersion at the crossing the optimum coupling factor is given by:

$$K_{opt} = \sqrt{\frac{\varepsilon_y}{\varepsilon_x}}\Bigg|_{opt} = \frac{\xi_{xM}}{\xi_{yM}}\sqrt{\frac{\beta_y^*}{\beta_x^*}} \qquad (6)$$

ξ_{xM} and ξ_{yM} being the limit values for ξ.

Not surprisingly the very simple model, while providing a valuable guideline, does not explain the details of the effect.

Luminosity

Peak luminosity as a function of energy is shown, for various electron storage rings, in Fig. 1. It is known that very long periods of operation are needed for a machine to reliably establish its peak luminosity at all energies. Data from recent machines (PETRA[4)], CESR[5)], PEP[6)]) are therefore dotted to indicate that they may not be final.

On average, the peak values achieved tend to cluster at around a few times $10^{30} cm^{-2} s^{-1}$, rather far below the hoped for goals ($\sim 10^{32}$ for high energy, low-β machines).

To evaluate accelerator performances it is useful to divide out, in eq. (4), the trivial factor $f_o \gamma^2/r_o^2$; an adimensional quality factor, λ, is obtained that contains all the relevant machine parameters:

$$\lambda = r_o^2 L / f_o \gamma^2 \qquad (7)$$

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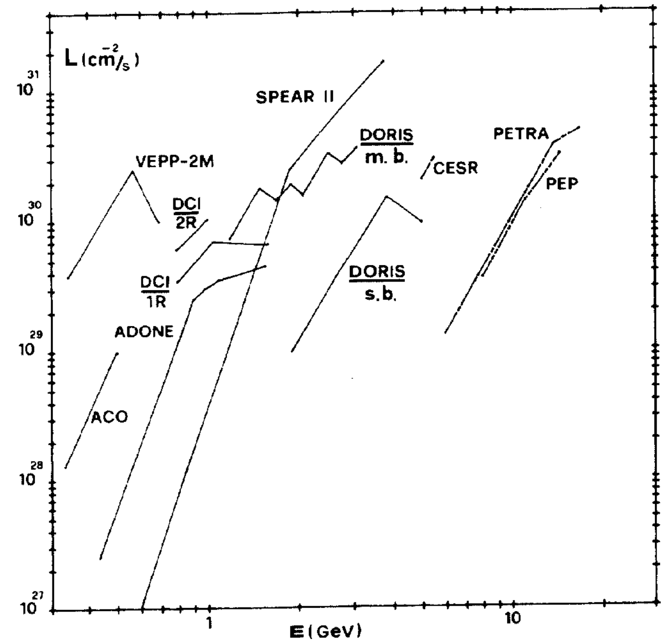

Fig. 1 - Electron ring luminosities.-

Measured values of L, λ and $\lambda \cdot \beta_y^*$, at the energy where λ peaks are listed, for various machines, in Table I.

If we compare the measured λ values to the corresponding design values (7÷13), we find that, for low-β machines, they are usually at least one order of magnitude lower. This is largely (but not only) due to unforeseen features of the beam beam interaction; it is interesting to examine what is known about the various factors appearing in λ.

Beam Emittance, ε_x^*

Most low energy storage rings, including DORIS (2 bunch operation) are operating at around their design emittance, or slightly higher. The SPEAR emittance is large compared to that of other machines but much lower than its design value: in the design a large dispersion at the crossing point, later to be proved incompatible with single beam instabilitites, was assumed.

PETRA, PEP and CESR operate at values of ε_x^* a factor between 2 and 6 lower than their respective design values. Low emittance is therefore an important accessory to lower-than-design luminosities, in my mind not connected with the beam-beam limit, but rather with difficulties encountered in tuning in the machine. Improved machine handling routines, diagnostics and control equipment, will hopefully allow the design values, and correspondingly higher luminosities, to be obtained.

As far as control techniques are concerned, wiggler magnets have worked as expected at SPEAR[14] and encouraging results have been obtained at PEP where a factor of ~ 2 in the beam cross section and of ~ 1.5 in luminosity has been gained at 8 GeV[15]. The variable optics approach certainly seems more difficult to implement, at least on large machines.

The Betatron Wavefunction, β_y^*

The principle of the low-β has been proved to work down to values of the order of the bunch length[16]. The resulting high values of the β function at the ends of long straight sections however, raise chromaticity correction and error sensitivity problems (in proportion to ℓ/β_y^*, with ℓ the length of the straight) that eventually limit the usefulness of the technique. This has been recognized since long and all low-β machines are operated (to within a factor of $\leqslant 1.5$), at their design values.

A very promising scheme, now being tested at PETRA[17] and DORIS[18], is the so called 'Mini beta' solution[19] where a low beta value is obtained over a short straight by means of small quadrupoles embedded in the experimental apparata.

It has also been suggested[20] that operation with round beams and equal low values of β in both planes would allow to overcome the limitation introduced by bunch length. I do not however know of any operable design incorporating this feature.

Space Charge Parameters, $\xi_x \xi_y$

The beam-beam effect exhibits a marked asymmetry between the radial and the vertical plane.

As a rule, when the charge density in the beam is increased, a threshold is reached where the vertical beam cross

Table I - Parameters of various storage rings at energy where λ is maximum.-

	f_o (MHz)	E (GeV)	$L \cdot 10^{-30}$ $cm^{-2} s^{-1}$	i_b (mA)	b	$\lambda \cdot 10^9$	$\lambda \beta_y^* \cdot 10^8$ (cm)	$\varepsilon_x^* \cdot 10^4$ (cm)	ξ_y	ξ_x	δQ_y	δQ_x	Comments
ACO	13.70	.51	~.1	~ 40	1	.58	23	~.4	.030	.021	~.035	~.023	
ADONE	2.86	.92	.28	19	3	2.4	81	.35	.058	.065	.024	.026	
DCI	3.17	1.05	.70	92	1	4.2	91	.8	.036	.036	.040	.040	One ring
DORIS m.b.	1.04	1.50	1.8	.5	480	16	45	-	.006	-	-	-	2 rings-m.b.
DORIS s.b.	1.04	3.80	1.5	~20	1	2.1	5.9	1.1	.021	.018	.020	.017	Single bunch
VEPP-2M	16.70	.58	2.5	~20	1	9.2	4.9	.52	~.055	~.018	.035	.015	
SPEAR II.....	1.28	3.70	14.0	35	1	16.6	16.2	1.98	.048	.016	.039	.015	
CESR	.40	5.50	3.0	8	1	5.1	5.5	.94	.034	.017	.029	.016	
PETRA	.13	14.00	3.6	3.3	2	2.9	6.2	.32	.036	.024	.037	.025	
PEP	.137	14.50	3.4	4.4	3	2.5	5.5	.32	.016	.029	.013	.023	

section starts blowing-up. No comparable blow-up is observed in the horizontal dimension, and it is usually assumed that σ_x has its computed current independent value. (Machines operating with round beams on coupling, like DCI, are an obvious exception). Concerning the threshold, it is interesting to recall that ADONE[21] and possibly SPEAR[16] exhibited the shrinking of σ_y, predicted by the linear theory[2] and by early one dimensional computer simulations[22], at low values of ξ_y ($\lesssim .02$), indicating that below threshold no unexpected phenomena were taking place. When the beams are blown-up, the shape is no longer gaussian (very characteristic shoulders and long tails develop[16)20)] that it would be interesting to see reproduced in the simulations). The blow-up, that it usually measured through luminosity and therefore refers to the core of the distribution, was not a dominant phenomenon at ACO and ADONE, while it seems to govern the space charge limit of all low-β machines.

The definition of the limit is rather unprecise and suggestive of two distinct regimes: a (resonance dominated?) regime where the limit on ξ_y (ξ_{yM}) is said to be reached when the lifetime becomes much shorter than for a single beam (typical of low-β machines over their whole energy range, but also observed at ACO and ADONE when operating at low energies) and a regime where the limit is very sharp, consistently observed at ADONE at the upper end of the operating energy range ($E \gtrsim .9$ GeV). In the latter case, just below the limit, any perturbation would cause the weak beam to 'flip' becoming a halo around the strong one without any decrease in lifetime[23].

In the strong blow-up regime the beam vertical size seems to adjust itself so as to keep the charge density (ξ_y) constant. At SPEAR[24] and DCI[25] the limit is reached at all energies at about the same value of σ_y, suggesting that amplitudes in the tails are proportional to those in the core and that the limit is imposed by the available aperture. In this regime, the beam cross section being dominated by the blow-up, eq.(6) becomes of little use.

Machines operating on resonance with roundish beams, like ACO, ADONE and DCI, necessarily have approximately equal values for ξ_x and ξ_y; values of ξ_x, much higher than ξ_{yM}, have been obtained in low-β machines. Horizontal space charge limited operating conditions have been reported only by the Novosibirsk group[20]. ξ_x does not therefore seem to impose practical limits at this stage. In eq. (4) ξ_{xM} becomes simply ξ_x: although determined by ξ_y through (1), it can have a different energy dependence.

Energy dependence

It has been observed at ACO[26], ADONE[23] and SPEAR[24] that ξ_{yM} is a function of energy. For ADONE and SPEAR the function is steep below a certain threshold energy and becomes a constant above. The measured energy dependence of various machine parameters is listed in Table II, for several machines. DORIS, operating below the space charge limit[18], is not included. It is very interesting (thinking of computer simulations) that at ACO[26] and ADONE[23] the same results have been obtained for strong-strong and for strong-weak beams.

Strong evidence for ξ being constant with energy (at the level of $.02 \div .03$) comes from PEP[6]; the same behaviour is reported from PETRA[27].

Various diffusion like models have been invoked[28) 29) 16)] to explain the energy dependence of ξ_y. They have to their credit that they naturally account for the dependence of ξ_{yM} on the number of bunches, discussed below, and for the dependence of the vertical cross section blow-up on the square root of the current observed at SPEAR[24] and PETRA[27]. Although the detailed behaviour and scaling from one machine to another are not well fitted, it could be predicted from the model that for long, high energy machines a constant ξ_y regime would be reached[30]. Results from PEP and PETRA would then confirm the prediction. The dependence on the number of bunches discussed below, if produced by the same mechanism, would then also have to disappear.

The values of ξ_{yM} and ξ_x obtained at various machines are plotted in Fig. 2 as functions of the diffusion parameter $\sqrt{\dot{n}\tau}$, $\dot{n}$ being the number of crossing per unit time and τ the radiation damping time. Data from experiments performed at the ISR, with coasting beams[31] and with a weak bunched beam colliding with a strong coasting one[32] are also shown, τ having been replaced by the oberved lifetime. A discussion of the meaningfulness of this comparison on theoretical ground is found in Ref. 33) 34).

Dependence of ξ_M on the operating tune and beam blow-up

On small machines a very clear dependence of ξ_M on the phase advance between neighbouring crossings has been observed[26)20)21)]. When approaching an integer tune, multiple of b, μ approaches π from above, and in the linear model higher values of ξ are obtained from eq. (2) for the same Q-shift. This was all nicely consistent with the data, that exibited the expected constant limit value for δQ. Higher values of luminosity were also obtained near the integer at SPEAR I, but δQ was now increasing with ξ[16]. Early one dimensional computer simulations of the strong-beam strong-beam interaction[22] were in qualitative agreement with observation and actually motivated the choice of the ADONE operating tune. The strong tune dependence was no longer observed at SPEAR II. On ACO

Table II – Exponents of a fit with $y = a\,E^x - L(E)$, $I(E)$ measured, $\sigma_x \propto E$. Other exponents computed from eq.(1),(3). All quantities measured at the space charge limit.–

y	L meas.	I meas.	σ_x^* ass.	$\sigma_x^* \cdot \sigma_y^*$	σ_y^*	ξ_y	ξ_x	Comments
ACO (.3÷.5 GeV)	~5	~3.5	1	2	1	.5	.5	
ADONE (.45÷.9 GeV) ...	~7	~4.5	1	2	1	1.5	1.5	
ADONE (≳ .9 GeV)	–	–	–	–	–	0	–	Current ltd.; size controlled by coupling.
DCI (.8 ÷ 1 GeV)	~2	~1	$\sigma_x \approx \sigma_y$	0	0	0	0	Round beams.
VEPP-2M (~.35÷.55 GeV)	~4	–	–	–	–	–	–	
SPEAR II (.6÷2.2 GeV)	~6.7	~3.6	1	.5	-.5	2.1	.6	
SPEAR II (2.2÷3.7 GeV)	~2.2	~1.6	1	1	0	-.4	-1.4	My fit to data presented in Ref. 2.

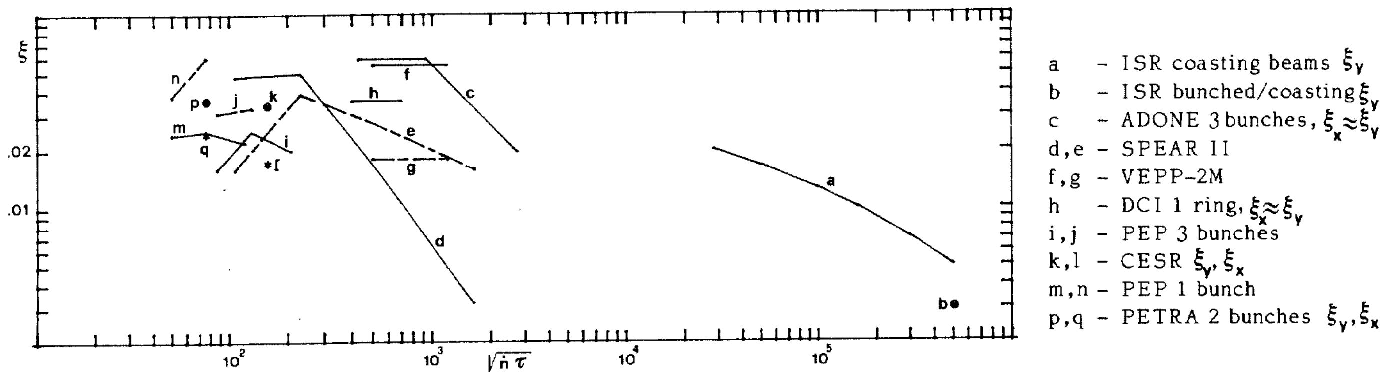

Fig. 2 – ξ_{yM}, ξ_x for various machines. ξ_{yM}: full line, ξ_x : dotted line.–

and DCI[25], operating in coupling below the integer, extensive studies on the contours of good lifetime areas as a function of current regions have been performed. Islands along the $Q_x = Q_y$ line shrinking with increasing ξ were found, consistent with the ξ values being limited by the distance from a nearby low order nonlinear resonance.

For the new large machines the situation is even more complex, and limitations due to machine imperfections can obscure the issues. This is best described through two examples.

Luminosity and lifetimes at several places in the tune diagram have been measured at CESR[5]: a complex pattern of synchrobetatron and other nonlinear resonance lines has been evidenced, along which lifetime becomes bad; large variations in the amount of blow-up are reported for small changes in tune. A strong-strong, three-dimensional computer simulation, based on a theoretical model has been developed[35] that computes luminosity and can identify bad lifetime regions. A complex pattern of good and bad regions has been predicted, and the experimental dependence of luminosity on current, including the lifetime limit, is rather well reproduced. The simulation evidences clustering of the best operating tunes near the integer and of the bad regions around calculated resonance lines.

The influence of machine imperfections is well illustrated by the measurement reported from PETRA in Ref. 4). At 14 GeV, two bunches per beam, and ~ 3.3 mA/bunch, luminosity was limited by strong blow-up to around $1\text{x}10^{30}$. By careful correction of the closed orbit, the dispersion function in the cavities and the horizontal and vertical dispersions at the crossing point, the blow-up was completely eliminated (as evidenced by the measured beam cross section being constant as a function of current) and luminosity increased by a factor of more than 3.5. The ξ_y value obtained without any blow-up and minimized vertical emittance is $\sim .034$. Strong-weak and strong-strong three-dimensional computer simulations, including the effect of errors, done for PETRA[36], show that resonant blow up, dependent on the value of ξ , tune and number of bunches, also observed in other recent simulations[37][38][39] and theoretically predicted[40] can be greatly enhanced by machine imperfections. The best results are predicted for tunes $\sim .1$ above the integer (in the particular computation not a multiple of b). A rather strong dependence of the blow-up on energy is found. Similar computer simulations for LEP are reported in Ref. 41).

Last, in the region of phase advance per crossing close to and above a multiple of b, the threshold for possible beam-beam induced coherent oscillations is highest[42].

Dependence on the number of bunches

At ACO and ADONE, the only low energy machines operated with different number of crossings, a clear dependence of the space charge limit on the number of bunches was observed. At ACO the limit value of ξ_y was proportional to $\sqrt{b}$[26] At ADONE δQ_M was proportional to $\sqrt{b}$ but, by virtue of eq.(2) and of the operating tune (3.05) being very close to the integer, the dependence of ξ_M on b was much less pronounced[23], again consistent with a limit on δQ rather than ξ.

At PETRA maximum achieved luminosity is independent of the number of bunches[27] (in a tune region where $\xi \approx \delta Q$) indicating that $\xi_{xM} \cdot \xi_{yM} \propto b$. A strong effect of the number of crossings on the vertical beam size blow-up is exhibited by the DESY computer simulation[36] when machine imperfections are included, indicating that a lower space charge limit in multi-bunch operation is to be expected from the model.

Data from PEP[6] at 14.5 GeV, with one and three bunches, at different tunes ($Q_y \approx 18.17$, 18.19, 18.76), show consistently lower values of δQ_M for three-bunch operation, the ratio of one to three bunches being rather close to $\sqrt{3}$. At 11 GeV the ratio between maximum achieved tune shift is instead close to 1 (a reverse energy dependence would have been expected on the basis of say a diffusion-like model). Given that operation with three bunches may not have been optimized[6], a firm conclusion should wait.

Concluding remarks

Lower than design emittance seems a common factor, limiting luminosity, on new high energy machines. This point certainly needs attention. Improved diagnostics and control equipment also seems vital to the achievement of better luminosities.

Computer simulations have come a long way and quantitative predictions are being made on how to improve performance. It would be interesting to see whether the programs can explain a wide set of data such as that available from simpler, lower energy machines.

New promising techniques like wigglers magnets and mini β sections, now under test, let us hope that improved performance of electron storage rings is on hand.

References

1) F.Amman, D.Ritson - Int.Conf.on H.E.Accel., Brookhaven, 471 (1961).
2) A.W.Chao - Proc.Symp. on Beam-Beam Interact., Brookhaven, 42 (1979).
3) J.C. Herrera - ibid.
4) A.Piwinski et al. - PETRA-Luminosity at 14 GeV - Preprint.
5) The CESR Operations Group - XI Int.Conf. on H.E.Accel., Geneva, 26 (1980).
6) H.Wiedemann, p.c.
7) F.Amman et al. - V Int.Conf. on H.E.Accel., Frascati, 703 (1965).
8) P.Marin - IX Int.Conf. on H.E.Accel., Stanford, 49 (1974).
9) The DORIS Group - Int.Report DESY 79/08 (1979).
10) The SLAC Storage Ring Group - VIII Int.Conf. on H.E.A. CERN, 145 (1971).
11) M.Tigner - Proc. IEEE Trans. NS-24, 1849 (1977).
12) Updated version of the PETRA Proposal, Int.Rep. DESY, (1976).
13) PEP - Int.Reports LBL 2688, SLAC 171 (1974).
14) M.Berndt et al. - Proc.IEEE Trans.NS-26, 3812 (1979).
15) J.M.Paterson - XI Int.Conf.on H.E.Accel., Geneva, 7, (1980).
16) H.Wiedemann - Proc.Symp.on Beam-Beam Interact., Brookhaven, 84 (1979).
17) J.Rossbach - This Conference.
18) N.Neseman et al. - Int.Report DESY M-80/09 (1980).
19) K.Steffen - Int.Report DESY M-79/23 (1979).
20) F.M.Iszailev et al. - Some Possibilities of Increasing the limiting Current Density in Storage Rings - Int.Rep. INP, Novosibirsk (1980).
21) F.Amman et al. - VIII Int.Conf. on H.E.Accel., CERN, 132 (1971).
22) M.Bassetti - V Int.Conf. on H.E.Accel., Frascati, 708 (1965).
23) S.Tazzari - Proc.Symp. on Beam-Beam Interact., Brookhaven, 128 (1979).
24) H.Wiedemann - XI Int.Conf.on H.E.Accel., Geneva, 744 (1980).
25) R. Chehab et al. - ibid. page 702.
26) M.Bergher et al. - IX Int.Conf.on H.E.Accel., Stanford, 410 (1974).
27) G.A.Voss - XI Int.Conf.on H.E.Accel., Geneva 748 (1980).
28) F.Amman - IEEE Trans. NS-20, 858 (1973).
29) S.A.Kheifets - XI Int.Conf. on H.E.Accel., Geneva, 718, (1980).
30) E.Keil - Int.Rep. CERN/ISR/TH/79-32 (1979).
31) B.Zotter - Int.Rep. CERN/ISR/TH/77-39 (1977).
32) A.Hofmann et al. - XI Int.Conf. on H.E.Accel., Geneva, 713 (1980).
33) E.Courant, ibid. page 763.
34) E.Courant et al. - Int.Rep. BNL 27973 (1979).
35) S.Peggs, R.Talman - XI Int.Conf.on H.E.Acc., Geneva, 754 (1980).
36) A.Piwinski - Int.Rep. DESY/80/131 (1980).
37) A.Piwinski - IEEE Trans. NS-24, 1408 (1977).
38) E.Close - Proc. Symp. on Beam-Beam Interact., Brookhaven, 210 (1979).
39) E. Keil - To be published.
40) B.Montague - Int.Rep. LEP Note 234 (1980).
41) S.Meyers. This Conference.
42) E.Keil - XI Int. Conf.on High Energy Accel., Geneva, 759 (1980).