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EMITTANCE GROWTH OF BEAMS CLOSE TO THE SPACE CHARGE LIMIT *

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Summary

In linear devices the effective phase space volume occupied by a beam can grow rapidly if the beam intensity is sufficiently close to the space charge limit and if a source of instability is available, like a periodic variation of the focusing force and/or considerable anisotropy between different phase planes. Results from analytic work and computer simulation are compared and shown to support each other. Regimes are defined in terms of tunes and tune depressions, where no emittance growth should occur; special emphasis is given to the coupling instability in case of considerably different energy content in two phase planes. The amount of transfer is found to depend critically on the strength of tune depression. The nonlinearity required for coupling is shown to arise from instability, which grows out of arbitrarily small initial fluctuations.

Introduction

We consider space charge induced instability of an intense beam, which is near the space charge limit in linear devices where the defocusing space charge force is only slightly smaller than the force due to the focusing magnets. Computer simulation of beams in proton linacs has shown that a considerable part of the total emittance growth is due to space charge effects.[1,2,3,4] One important observation was, for initially unequal emittances, growth of the smaller emittance, provided that the totally occupied phase space volume was sufficiently small.[1]

The influence of periodic focusing on emittance growth has been studied both analytically and numerically in connection with beam transport for heavy ion fusion.[5,6,7] Emittance growth found in computer simulation runs has been confirmed to a satisfactory degree by analytic theory, which was based on the K-V distribution.[8] Theory, however, has predicted numerous unstable situations [6] for which there was no strong indication in simulation work.

In this paper we present and compare results from analytic theory and computer simulation. We discuss the various mechanisms that give rise to space charge induced unstable behaviour and present thresholds and nonlinear saturation levels.

Analytic Theory and Simulation

Almost all of the analytic work done so far applies to an initial K-V distribution, which generates an unperturbed uniform density in space and makes the analysis of the Vlasov equation feasible. The value of analytic theory lies in the possibility of identifying in a straightforward way the eigenmode structure of the problem and recognizing the parameters that are essential for instability. This is particularly important for the coupling instability, where four independent parameters (in each phase plane the zero intensity tune σ_o and its space charge depressed value σ) describe a matched beam and computer simulation cannot cover the whole parameter space. We also take advantage of the fact that the K-V distribution in rectangular coordinates is the only known distribution that possesses an exactly matched stationary solution in periodic focusing so that onset of instability can be clearly identified and is not masked by an initial mismatch. Previous analytic studies on K-V instabilities of transport in a FODO channel [5] or coupling instabilities between two phase planes with anisotropic distribution [9] have shown that only very small regions are left in parameter space where not some eigenmode is unstable in the linearized theory. This has raised some doubt in the applicablility of the K-V distribution. Evaluating a large number of simulation runs based on initial K-V distributions but also waterbag distributions, we have found that K-V predictions are sufficiently reliable if one distinguishes between significant and insignificant instabilities (see section Results).

Mechanisms Responsible for Emittance Growth

Space charge induced growth of the emittance of an initially matched charge distribution in phase space depends on two factors:

1. Some eigenmode of oscillation of the beam is unstable so that it grows out of an arbitrarily small initial fluctuation and generates the nonlinear force necessary for emittance growth.
2. There is a sufficiently large reservoir of (free) energy to drive the unstable mode to significant amplitude so that emittance growth is observable in an r.m.s. sense.

In principle there are three sources of instability: in a continuously focused beam it can be a nonmonotonic distribution function, like the K-V distribution; in two or three dimensional beams it can be an anisotropic distribution with different emittance and/or energy in two phase planes; in addition, periodic focusing can act as a source if an eigenoscillation is in resonance with the period of focusing.

Results

Computer simulation runs have been performed with 8000 particles on a rectangular 64 × 64 mesh (for some test cases 128 × 128) using the particle-in-cell code SCOP2 developed at the IPP.[10] Initial sets - K-V or waterbag - are produced with a random number generator. Matching is performed by solving the envelope equation for the K-V case and by using the same data for r.m.s. quantities in the waterbag case.

I. Nonmonotonic distribution function

The theoretical prediction of instability of a continuously focused K-V beam [5,11] if $\sigma/\sigma_o \lesssim 0.4$ is confirmed by the observation of rapidly evolving wriggles on initially elliptic boundaries in $x-p_x$ and $y-p_y$ planes (Fig.1).

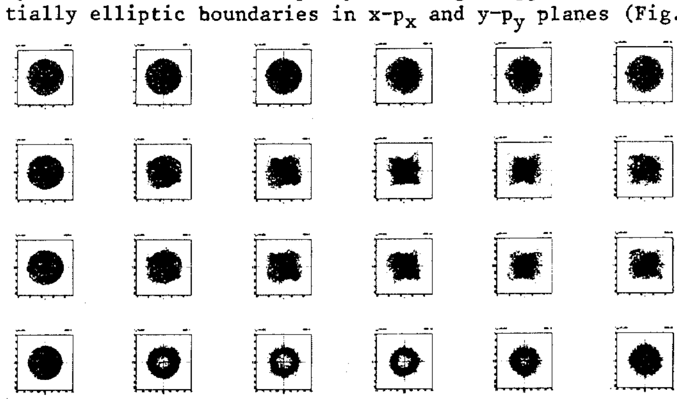

Fig.1 Time evolution of K-V instability of a continuously focused beam with $\sigma/\sigma_o = 0.16$. Shown are phase space projections into the $x - y$, $x - v_x$, $y - v_y$ and $v_x - v_y$ planes in equal time steps over 4 zero intensity wavelengths (from left to right). No growth of r.m.s. emittance.*

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*Note that emittances have been scaled arbitrarily; wavelengths for time scales refer to single particle oscill.

Remarkably, there is no growth of r.m.s. emittances, but apparently broadening of the distribution has occurred. This is in agreement with analytic theory, according to which only insignificant patches of instability remain, if the K-V distribution in continuous focusing is somewhat broadened, but still strongly nonmonotonic.[12] We conclude that the shape of the distribution function is insignificant from an r.m.s. stability point of view.

II. Resonance with periodic focusing

In a periodic transport system with $\sigma_o = 90^\circ$ and σ in the vicinity of 45° a strong "3rd order" (three arms in $x - p_x$ or $y - p_y$ planes) instability has been found.[13] This instability can be explained as a parametric resonance with the focusing structure (see also Fig.2). It has been suggested [8] that $\sigma_o \leqq 60^\circ$ and

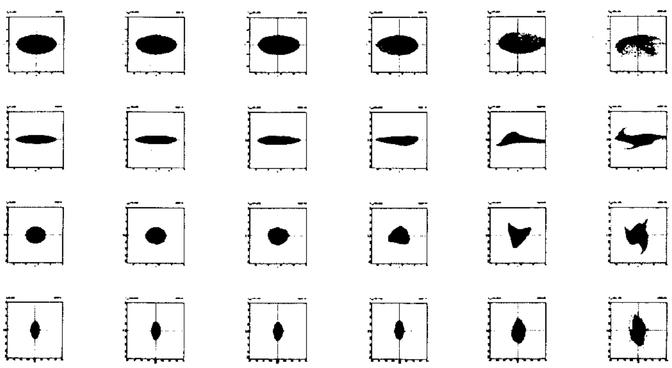

Fig.2 Time evolution of K-V distribution in periodic focusing with $\sigma_o = 90^\circ$, $\sigma = 45^\circ$ over 25 cells. R.m.s. emittance growth of 2.0 in x and 2.5 in y after 50 cells.

$\sigma/\sigma_o > 0.4$ could improve stability considerably by avoiding this resonance. Recently we have shown [14] - in accordance with our observations on nonmonotonic distribution functions - that no lower limit seems to exist for σ; for instance, $\sigma_o = 60^\circ$ and $\sigma = 13^\circ$ permits r.m.s. stable transport (see Fig.3).

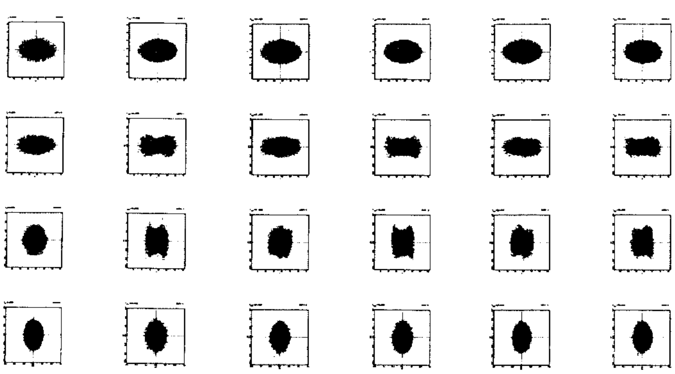

Fig.3 Time evolution of waterbag distribution in periodic focusing with $\sigma_o = 60^\circ$, $\sigma = 13^\circ$ over 25 cells. No growth of r.m.s. emittance over 100 cells.

III. Coupling due to anisotropic distribution

In a previous paper [9] we had presented frequencies of eigenoscillations of a beam with anisotropic K-V distribution function (in continuous focusing), where the ratio of emittances $\varepsilon_x/\varepsilon_y$ and energies E_x/E_y was allowed to differ arbitrarily from 1. We had found numerous unstable eigenfrequencies. Computer simulation can be used to decide whether they all affect r.m.s. stability. Our conclusion is that eigenmodes which are oscillatory (Re $\omega \neq 0$) are insignificant. Nonoscillatory (purely growing) instabilities contribute to r.m.s. unstable behavior if the energy ratio E_x/E_y differs sufficiently much from 1. From the analytic theory we have found a strong 3rd order instability (perturbed space charge potential a 3rd order polynomial in x, y) for $\varepsilon_x/\varepsilon_y = 3$, $E_x/E_y = 9$ and $\sigma_y/\sigma_{oy} \leqq 0.37$ ($\sigma_x/\sigma_{ox} \leqq 0.77$). Computer simulation shows in fact a rapid transfer of emittance and energy from the x-plane into the y-plane for $\sigma_y/\sigma_{oy} = 0.32$ ($\sigma_x/\sigma_{ox} = 0.71$) (see Fig.4),

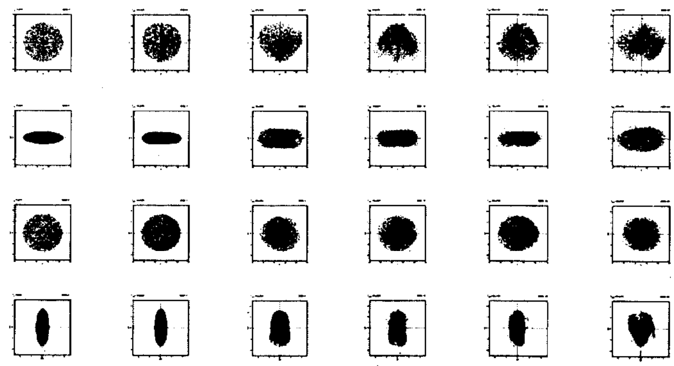

Fig.4 Time evolution of coupling instability (anisotropic K-V) over 10 zero intensity wavelengths in y. $\sigma_y/\sigma_{oy} = 0.32$, $\sigma_x/\sigma_{ox} = 0.71$, $\sigma_x/\sigma_y = 3$. R.m.s. emittances initial $\varepsilon_y = 1$, $\varepsilon_x = 3$ and final $\varepsilon_y = 2.3$, $\varepsilon_x = 2.3$.

whereas the case $\sigma_y/\sigma_{oy} = 0.38$ ($\sigma_x/\sigma_{ox} = 0.78$) remained stable.

The question may be asked, what the essential parameters for this coupling instability are. We found that the stability boundary of the 3rd order mode can be expressed conveniently in terms of tune depressions against the ratio of frequencies σ_x/σ_y. Fig.5 shows the stability limits evaluated for $\varepsilon_x/\varepsilon_y = 4$. We emphasize that these stability limits become independent from

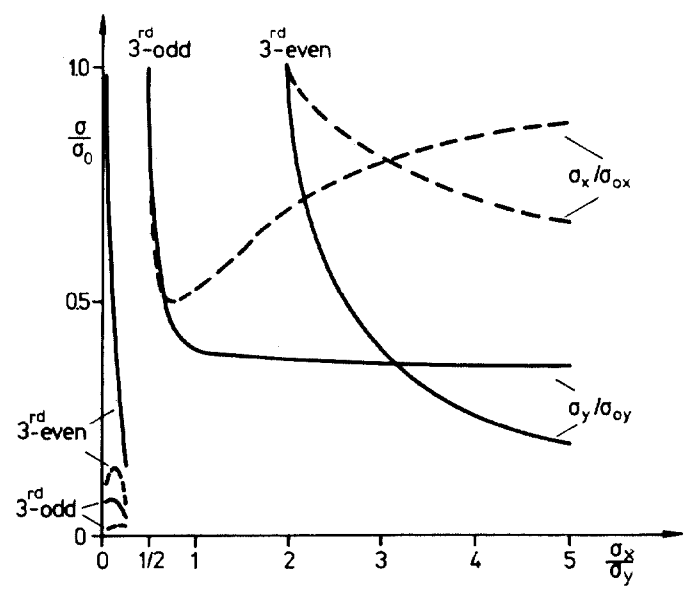

Fig.5 Stability limit of 3rd order coupling modes (even/odd symmetry) for $\varepsilon_x/\varepsilon_y = 4$ (approximately for all $\varepsilon_x/\varepsilon_y \geqq 3$). For given σ_y/σ_x these modes are only unstable if the σ_x/σ_{ox} and σ_y/σ_{oy} fall below the indicated lines.

$\varepsilon_x/\varepsilon_y$ if $\varepsilon_x/\varepsilon_y >> 1$ and Fig.5 can be applied to any $\varepsilon_x/\varepsilon_y \gtrsim 3$ with reasonable accuracy. Recalling that the ratio of energies follows from

$$E_x/E_y = \varepsilon_x/\varepsilon_y \cdot \sigma_x/\sigma_y \tag{1}$$

we can easily locate the two simulation runs mentioned above with $\varepsilon_x/\varepsilon_y = 3$, $E_x/E_y = 9$. Another simulation example inside the unstable regime of Fig.5 far from the stability boundary is shown in Fig.6, where complete isotropization occurs during less than half a single particle wavelength in y.

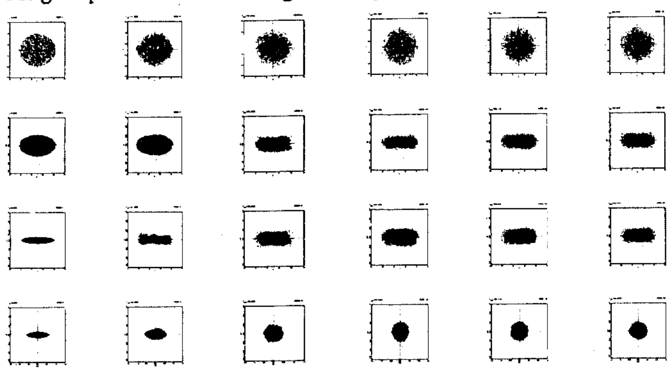

Fig.6 Same as Fig.4 with stronger tune depression: $\sigma_y/\sigma_{oy} = 0.11$, $\sigma_x/\sigma_{ox} = 0.36$, $\sigma_x/\sigma_y = 3.1$. R.m.s. emittance initial $\varepsilon_y = 1$, $\varepsilon_x = 4$ and final $\varepsilon_y = 2.8$, $\varepsilon_x = 2.8$. Note that in this case there is also isotropization of velocities.

Although Fig.5 only takes into account the 3rd order modes we found that its thresholds control very well the simulation runs, where certainly higher order modes could have been excited as well. Note that the two peaks at $\sigma_x/\sigma_y = 1/2$, 2 indicate single particle resonant behavior for vanishing intensity ($\sigma/\sigma_o \to 1$). Of course, growth rates also approach zero in this limit.

It may be of interest to note that according to Fig.5 and equ.(1) the beam is practically stable if (for $\varepsilon_x/\varepsilon_y >> 1$)

$$E_x/E_y \lesssim 1/2 \cdot \varepsilon_x/\varepsilon_y \tag{2}$$

(see Fig.7)

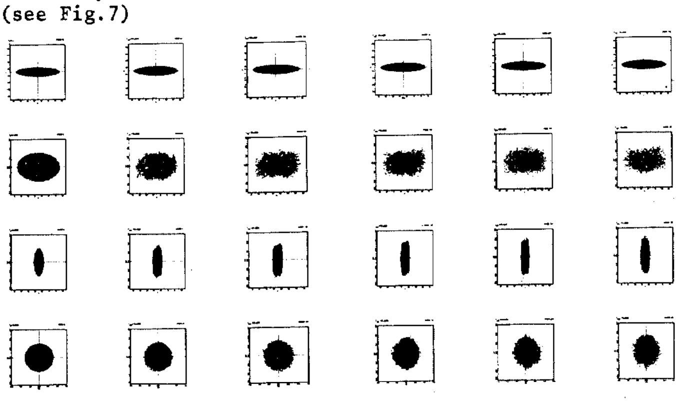

Fig.7 Case with strong tune depression but no energy anisotropy, over 12 zero intensity wavelengths in x. $\sigma_x/\sigma_{ox} = 0.10$, $\sigma_y/\sigma_{oy} = 0.21$, $\sigma_x/\sigma_y = 0.21$, $\varepsilon_x/\varepsilon_y = 5$, $E_x/E_y = 1.07$. No r.m.s. emittance growth.

Hence, instability requires energy anisotropy and not just different emittances (see also Fig.8).

Modifications by periodic focusing and other distribution functions

We have also simulated several of the anisotropy examples for a periodic focusing system with $\sigma_{ox}, \sigma_{oy} \leq 60^o$ and found practically the same results as in continuous focusing if the parameters σ_x/σ_{ox}, σ_y/σ_{oy} and σ_x/σ_y were chosen identically. A waterbag rather than

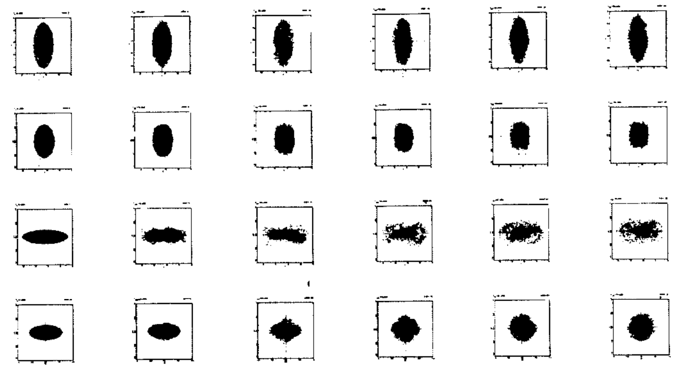

Fig.8 Case with equal initial emittances but different energies, over 12 zero intensity wavelengths in x. $\sigma_x/\sigma_{ox} = 0.36$, $\sigma_y/\sigma_{oy} = 0.114$, $\sigma_x/\sigma_y = 5.0$, $E_x/E_y = 5$. R.m.s. emittance initial $\varepsilon_x = 1.0$, $\varepsilon_y = 1.0$ and final $\varepsilon_x = 0.7$, $\varepsilon_y = 1.9$. Final velocities isotropic.

K-V distribution with the same r.m.s. properties has given quite similar results in cases of violent instabilities whereas weakly unstable K-V cases (close to the stability boundaries in Fig.5) were r.m.s. stable for a waterbag.

Conclusion

The following characteristics of space charge induced emittance growth have been found by comparing analytic theory based on K-V distributions with simulation results for beams in Cartesian geometry:

1. If the beam is in an unstable regime arbitrarily small initial fluctuations (nonlinearities) can grow until they saturate and fill a larger effective phase space.
2. Growth of r.m.s. emittances can occur if a beam oscillation mode is in resonance with the focusing period. The regime $\sigma_o \lesssim 60^o$ has been found r.m.s. stable even for small σ (i.e. high order mode resonances are of no concern).
3. A coupling instability (as expected in bunched beams in Linacs) may lead to phase space dilution if there is sufficient anisotropy of energy and the tune depressions in the two degrees of freedom are below certain threshold values, which depend only on σ_x/σ_y if $\varepsilon_x/\varepsilon_y >> 1$. Isotropic velocities are achieved only if the tune depressions are far below these thresholds. Periodic focusing with σ_{ox}, $\sigma_{oy} \lesssim 60^o$ does not change these results.

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