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BEAM ENERGY MEASUREMENTS AT THE BEVALAC*

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I. Summary

There is an increasing need for experimenters to have a more exact value for the kinetic energy of the particles used in their experiments. Values for kinetic energy can be calculated within about two percent for particles in the Bevalac using nominal values for the magnetic field and the radial position of the circulating beam.

In the Bevalac there are several problems that make it very difficult to determine a more precise value for magnetic field. The radial field shape enclosed within the B dot integrating loop on the poletips changes as a function of field strength. The effective magnetic quadrant length also changes as a function of field strength. This causes a major perturbation in the radial position of the equilibrium orbit as well as some uncertainty in the value of the magnetic field. The details of these effects are discussed in an internal report.[1]

In addition to the magnetic field value, we must have adequate information about the radial position of the closed orbit to determine a precise value for the kinetic energy. If we have sufficient information about the closed orbit, we have a known effective path length for the particle. If we can measure the transit time of the particle on that path we have a time-of-flight measurement. In a circular machine, this time measurement is a frequency measurement which is one of the most precise measurements we can make. A Hewlett Packard 5360A frequency counter can read to 1 part in 10^6 for a 0.1 msec read time. This can be extended to 1 part in 10^{10} for longer read time. With time measured to this precision, the error in the kinetic energy is then determined by the error in the determination of the closed orbit. At a kinetic energy of 500 MeV/amu, we can determine the energy to ± 1/4% at the Bevalac.

II. Closed Orbit

If the closed orbit is known at many points around the accelerator then the problem is straight forward. In a machine such as the Bevatron we only have access for orbit measurements ninety degrees apart at the four straight sections. We must therefore determine from a limited number of measurements how well we know the closed orbit.

Any closed orbit can be described by Fourier analysis in the form:

$$R = R_0 + \sum_i (a_i \cos i\theta + b_i \sin i\theta) \qquad (1)$$

That is, the closed orbit can be represented by a fixed radial position R_0 plus a series of sinusoidal oscillations that are all an integral multiple of 2π once around the accelerator.

Let us now examine the path length of a sinusoidal oscillation around a uniform orbit R_0 for one cycle of oscillation.

$$R = R_0 + a \sin i\theta$$

$$L = \int d\ell = \int_0^{2\pi/i} R d\theta = \int_0^{2\pi/i} (R_0 + a \sin i\theta)\, d\theta \qquad (2)$$

$$L = 2\pi R_0/i$$

The path length traveled by the sinusoidal oscillation just cancels and the path length is equal to the fixed orbit path length along R_0. Looking at Eq. 1 describing the closed orbit, we can see that the sinusoidal parts average to zero in each cycle of oscillation and the path length around the machine is just equal to $2\pi R_0$ plus the straight sections.

At the Bevatron, the closed orbit is measured using radial probes. A U shaped target is flipped up into the beam. The target is adjusted radially to give maximum beam survival. The center of the U target is the radial position of the center of the beam. This can be repeated at each target position. An alternate method and the one used at the Bevatron is to use single finger targets at each target station and to record relative survival as a function of radial probe position. This gives the orbit eccentricity relative to the azimuthal location of each probe. One U target measurement must be made to get the actual radial beam position at one of the azimuthal locations. These measurements are made at the four straight sections yielding four measurements 90 degrees apart.

If we substitute in Eq. 1 for four measurements and sum them, we have:

$$\sum_{j=1}^{4} R_j = 4 R_0 + \sum_{j=1}^{4} \sum_i (a_i \cos i\theta_j + b_i \sin i\theta_j)$$

This reduces to:

$$\sum_{j=1}^{4} R_j = 4 R_0 + 4 (a_4 + a_8 + a_{12} + \dots)$$

Solving for R_0:

$$R_0 = \frac{1}{4} \sum_{j=1}^{4} R_j - (a_4 + a_8 + a_{12} + \dots) \qquad (3)$$

We know the value for R_0 to within the error of the amplitude of the 4th, 8th harmonics for four azimuthally equidistant measurements of radial position. The question now is, can we put an upper limit on the amplitude of these fourth harmonic oscillations without knowing the exact closed orbit?

An upper limit can be calculated for the case of a systematic perturbation in the field value of each sector. The coefficients for the jth harmonic are derived by substituting Eq. 1 into the second order differential equation for radial betatron oscillations.

$$\frac{d^2R}{d\theta^2} = - k^2(1-n)\,(R-A_j) \qquad (4)$$

This yields:

$$\begin{matrix} a_i \\ b_i \end{matrix} = \frac{k^2(1-n)}{k^2(1-n)-i^2}\, \frac{1}{\pi} \int_0^{2\pi} A_j \begin{matrix} \cos i\theta \\ \sin i\theta \end{matrix} d\theta \qquad (5)$$

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where: $k = 1 + 4L/2\pi R_0$; L is the length of the straight sections; R_0 is the radius of the curved section; A_j is the shift in equilibrium orbit position in the jth sector, corresponding to a change in the magnetic field in the jth sector from the nominal value.

The integral is over both the curved and straight sections. As we are interested in the approximate value of a fourth-harmonic distortion, we can ignore the slight change in form by integrating over the straight section. The major correction to the amplitude from the straight section is included in the k term.

Set A_j equal to A_0 in each sector with a sign change to always be additive for the sin θ_j term. Therefore, the cosine term sums to zero and we have:

$$b_4 = \frac{k^2(1-n)}{k^2(1-n)-16} \quad \frac{4A_0}{\pi}$$

Substituting for $k = 1.255$, $n = 0.66$, $A_0 = 18"$ (where $A_0 = 18"$ corresponds to a 1 percent magnetic field change) gives $b_4 = 0.79$ inches.

A highly systematic perturbation of one percent at each sector only produces a fourth harmonic distortion in the closed orbit of about 0.8 of an inch.

If we assume we could have a value of 20 percent of this undetected in our closed orbit measurement, we would have an uncertainty in R_0 of ± 0.16 inches from the undetected 4th harmonic.

III. Time-of-Flight Measurement

As we are working with particles in the relativistic region, we must determine the relationship between changes in kinetic energy and changes in β. We can derive the relationship from $E = E_0(1-\beta)^{-1/2}$ and $E = E_0 + KE$. Where E is the total energy, E_0 is the rest energy, KE is the kinetic energy and β is the particle velocity divided by C, the velocity of light. The kinetic energy is given by:

$$KE = E_0\left[(1-\beta)^{-1/2}-1\right] \tag{6}$$

Taking the derivative and dividing by KE yields:

$$\frac{dKE}{KE} = \left[\left(\frac{KE}{E_0}\right)^2 + \frac{3KE}{E_0} + 2\right]\frac{d\beta}{\beta} \tag{7}$$

For a time-of-flight measurement in the Bevatron, the velocity is given by:

$$v = Sf \tag{8}$$

where S is the equivalent path length and f is the frequency of the accelerating system (1st harmonic acceleration). The error in v for errors in S and v is given by:

$$\frac{dv}{v} = \sqrt{\left(\frac{dS}{S}\right)^2 + \left(\frac{df}{f}\right)^2}$$

As the frequency can be measured to from 1 part in 10^6 to 1 part in 10^{10} we have:

$$\frac{dv}{v} = \frac{d\beta}{\beta} \approx \frac{dS}{S} \tag{9}$$

The path length S is given by $S = 2\pi R_0 + 4L$ where R_0 is the mean value of the closed orbit and L is the length of a straight section. For L equal 20 feet and R_0 equal 604 inches we have:

$$\frac{dS}{S} = \frac{dR}{R_0 + 4L/2\pi} = 0.789\,\frac{dR}{R_0} \tag{10}$$

Substituting Eq. 9 in Eq. 7 gives:

$$\frac{dKE}{KE} = \left[\left(\frac{KE}{E_0}\right)^2 + \frac{3KE}{E_0} + 2\right]\frac{dS}{S} \tag{11}$$

The values of dS/S as a function of KE for a 1 percent error in KE is shown in Table I.

Table 1. Uncertainty in path length vs. kinetic energy for 1% error in KE.

KE (MeV)	dS/S
100	4.29×10^{-3}
500	2.56×10^{-3}
1000	1.57×10^{-3}
2000	7.66×10^{-4}
3000	4.54×10^{-4}
4000	3.00×10^{-4}
5000	2.13×10^{-4}
6000	1.59×10^{-4}
7000	1.20×10^{-4}
8000	9.80×10^{-5}

The radial beam position measurements are made as follows. The beam is placed at the correct radial position for beam extraction by setting the appropriate value of radio frequency(rf) of the accelerating system. The rf is adjusted to remain constant over an extended period of constant magnetic field (flattop). The intensity of the circulating beam is monitored by a capacitive pick up system (BIE). The intensity of the circulating beam is read and then a finger target is flipped into the edge of the circulating beam. The intensity is read again and the ratio of the two readings is taken. The value of magnetic field and the rf are also read and recorded. These three values are recorded for ten Bevatron pulses. The radius of the finger target is changed and another set of data points is taken. A series of these runs is taken at each of the four probe positions. These measurements give the relative orbit positions around the Bevatron. A similar run is made with a U target at the east straight section to give an absolute radial measurement. These are shown in Fig. 1.

The relative radial position for a fixed value of ratio can be read from these plots to ± 1/16 inch. The radial position can be read to ± 1/8 inch. The shaft end play on the probes is about ± 1/16 inch and the absolute radial position calibration is about ± 1/16 inch. From section II we have an estimate of ± 0.16 inches for a possible undetected fourth harmonics. This gives an error in radius of:

$$\Delta R = \pm\sqrt{(.062)^2 + (.125)^2 + (.062)^2 + (.062)^2 + (.16)^2}$$

$$\Delta R = \pm\ 0.24 \text{ inch}$$

From Eq. 3, ignoring the 4th, 8th, etc. harmonics, we have:

$$R_0 = \frac{1}{4}\sum_{j=1}^{4} R_j \tag{12}$$

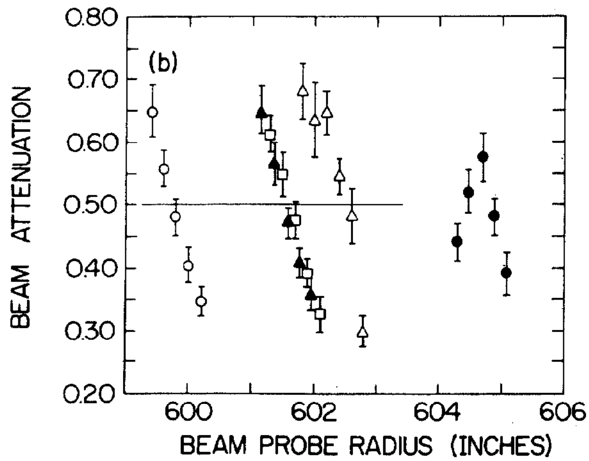

Fig. 1. Radial beam position measurements. Ratio BIE signal to radial position of probe. Finger probes location: North (○), West (▲), East (□), South (△). U Target measurement in East (●).

Therefore we have:

$$\left(\frac{\Delta R}{R}\right)_{Rms} = \sqrt{\left(\frac{1}{4}\right)^2\left(\frac{\Delta R}{R_0}\right)^2 4}$$

$$\left(\frac{\Delta R}{R}\right)_{Rms} = \pm\ 2.02 \times 10^{-4}$$

for R_0 = 604 and ΔR = 0.24.

Substituting this value in Eq. 10 we have:

$$\frac{dS}{S} = \pm\ 1.61 \times 10^{-4} \tag{13}$$

The change in effective magnetic quadrant length causes a maximum shift in closed orbit of about 13 inches. It also puts a scallop in the orbit through the quadrant as shown in Fig. 2. This can be calculated from a normal betatron oscillation with a radial kick at the end of the quadrant.[1] This gives a path length correction of

$$\frac{\Delta p}{p} = -\ 3.51 \times 10^{-4}\ DL \tag{14}$$

where DL is the extra length of effective magnetic field at each end of the quadrant. The maximum value at the Bevatron is 7.5"/2 = 3.75". For an uncertainty of a factor of 2 in this value, $\Delta p/p = -3.51 \times 10^{-4} \times 3.75/2) = 6.58 \times 10^{-4}$.

The error in the path length for this error plus the error from R_0 measurements is given by:

$$\left(\frac{\Delta S}{S}\right)_{Rms} = \pm\ 6.88 \times 10^{-4}$$

Substituting in Eq.11 for KE = 500 MeV/amu gives ΔKE/KE of ± 1/4%. The major part of the error comes from the quadrant length uncertainty not from the radial measurements of closed orbit. This uncertainty is probably too high, but until some measurements of beam radius in the quadrants can be made, I feel that a factor of two possible error is reasonable.

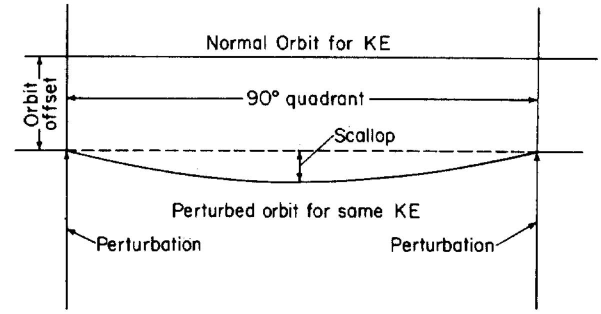

Fig. 2. Perturbed orbit from quadrant length variation. Maximum orbit offset in Bevatron is about 13 inches.

For the actual energy measurement R_0 is determined from the probe runs (Fig. 1). The radius is measured in the field free region of the straight sections so a radial correction must be made for the orbit drifting outward when passing through the fringe field at the end of the sectors (see Fig. 3). This is about a 1/4 inch in the Bevatron. R_0 is therefore given by $R_0 = 1/4\ \Sigma\ R_i - 1/4$. The path length is then calculated from $S = 2\pi\ R_0 + 4L$. A correction to the path length must be made because of the quadrant length correction using Eq. 14.

$$Sc = (2\pi\ R_0 + 4L)\ [1 - 3.5 \times 10^{-4}\ DL]$$

The velocity β is then calculated from Eq. 8:

$$\beta = v/c = S_c f/c$$

$$\beta = (2\pi\ R_0 + 4L)\ (1 - 3.51 \times 10^{-4}\ DL)\ f/c$$

This value of β is then substituted in Eq. 6 to determine the kinetic energy of the particles.

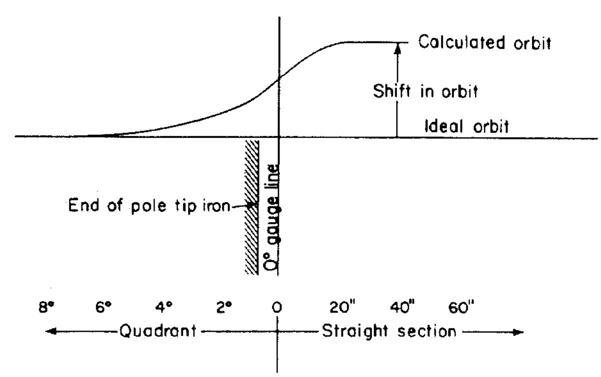

Fig. 3. Fringe field trajectory.

Reference

1. Kenneth C. Crebbin, "Magnetic Field, Closed Orbit, and Energy Measurements in the Bevatron," Lawrence Berkeley Laboratory Report LBL-11669 (in preparation).