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MATHEMATICAL MODELS FOR THE CONTROL PROGRAM OF THE SLAC LINEAR COLLIDER*

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Abstract

The operation of the SLAC two-mile linear accelerator in the single pass collider mode will be computer controlled. Mathematical models will be used in the control program to set up and restore the beam optics and to correct orbits. Some of the requirements imposed upon the on-line model calculations and the ways to satisfy these requirements will be described in this paper.

Introduction

The proposed single pass SLAC Linear Collider (SLC)[1] will be operated under computer control. One of the main components in SLC that needs computer control will be the SLAC linear accelerator (linac). The computer control of the linac involves the following steps: choice by the user of the lattice-parameters to be controlled; on-line calculation of the control-parameters corresponding to desired values of the lattice-parameters; setting of the power supply currents automatically to the set-point values. These steps are to be used also for the computer control of other SLC components such as the damping storage rings and the transport beam lines. In general, the control-parameter values are to be computed from mathematical models which have been made to represent the various components; they are solutions to a set of equations which are usually nonlinear and are solved numerically on-line during machine operation. The machine operator can enter the values of the machine-parameters desired and the control program will solve these equations for the control-parameter values corresponding to these machine-parameters. Since the actual machine-parameters are not directly controlled by this procedure, the performance of the machine is strongly influenced by the precision of the models. Hence, model making is a very important part of the design of a control program.

We have developed models for determining the control-parameter values which match the envelope of the beam at injection, focus the beams transversely along the linac, correct the central orbits and compensate the effects due to the failure of a klystron. We will need many more models for the other SLC components besides the linac. In this paper, we will describe some of the linac operating conditions required for SLC, how these requirements are incorporated into the models and how the models are integrated into the control program. This paper may facilitate the work required in the model development for other SLC components.

The Beam Matrix

We consider the basic elements in the linac lattice to be drift spaces, accelerator sections and quadrupole magnets. As a first approximation, the behavior of the beam is represented by the (symmetric) beam sigma matrix, σ, which satisfies the condition[2] $\sigma = R\,\sigma_o\,\tilde{R}$, where R is the transfer matrix from point o to the observation point and $\tilde{R}$ is the transpose of R. The particle motion is given by the vector (x,x',y,y'). For the case of uncoupled x and y motions, we have two symmetric 2x2 matrices, σ_x and σ_y, and the beam emittances, ε_x and ε_y, are given by $\sqrt{\sigma_{11}\sigma_{22} - \sigma_{21}^2}$ where σ_{ij} means σ_{xij} or σ_{yij}; we omit the subscript x or y for convenience. The beam matrix is related to the betatron functions, β, α and γ, by

$$\sigma = \varepsilon \begin{bmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{bmatrix} . \qquad (1)$$

The value of the beam emittance varies as 1/E so that in the presence of acceleration Eq. (1) serves as a definition of β, α and γ provided that we take $\varepsilon = \varepsilon_o E_o / E$.

The Linac Lattice

There are 30 sectors in the linac lattice. The rf power for each sector is supplied by 8 klystrons, with each klystron supplying power to a 12.34-m girder which supports four ~3-m accelerator sections. At the beginning of each girder is a F or D quadrupole magnet to focus the beam horizontally or vertically. At the end of a sector is a 2.8-m drift space in which a quadrupole doublet has been placed. This doublet and the two nearest singlets form a matching insertion which matches the beam matrix between adjacent sectors. In this lattice, which is shown in Fig. 1, one sector begins with a D quadrupole and the next begins with an F quadrupole since there is an odd number (nine) of quadrupoles in each sector. A half-superperiod consists of two and one-half cells and one matching insertion.

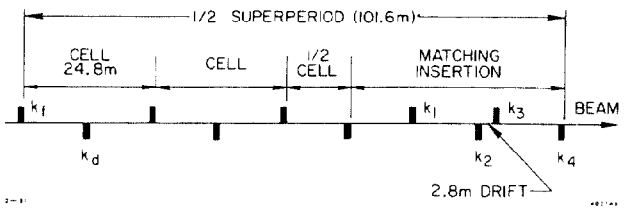

Fig. 1. Half-superperiod of the linac quadrupole lattice.

A procedure for setting up the lattice configuration will now be described. This procedure will be valid so long as the relative energy change per cell is small. The lattice configuration is defined by the betatron phase shifts per cell, $\Delta\psi_x$ and $\Delta\psi_y$. Let k denote the strength of a quadrupole (integrated gradient/magnetic rigidity) and let the beginning of a given sector be denoted by o. For given values of $\Delta\psi_x$ and $\Delta\psi_y$, the strength of the cell quadrupoles, k_f and k_d, are computed as well as the values of the periodic betatron functions at the entrance of the cell (β_o, α_o, γ_o). We impose the condition that

$$\sigma_o = \frac{1}{E_o} \begin{bmatrix} \beta_o(\Delta\psi_x, \Delta\psi_y) & -\alpha_o(\Delta\psi_x, \Delta\psi_y) \\ -\alpha_o(\Delta\psi_x, \Delta\psi_y) & \gamma_o(\Delta\psi_x, \Delta\psi_y) \end{bmatrix} \qquad (2)$$

where the invariant emittance εE has been normalized arbitrarily to unity. Matching is achieved by adjusting the strength of the four quadrupoles k_1, k_2, k_3, and k_4 in the matching cell such that the σ matrix at the end of this sector is equal to the σ matrix at the beginning of the next sector, i.e.,

$$\sigma = R\sigma_o\tilde{R} = \frac{1}{E_o} \begin{bmatrix} \beta_o(\Delta\bar{\psi}_x, \Delta\bar{\psi}_y) & -\alpha_o(\Delta\bar{\psi}_x, \Delta\bar{\psi}_y) \\ -\alpha_o(\Delta\bar{\psi}_x, \Delta\bar{\psi}_y) & \gamma_o(\Delta\bar{\psi}_x, \Delta\bar{\psi}_y) \end{bmatrix} \qquad (3)$$

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where R is the transfer matrix of the given sector, $\bar{E}_o$ is the energy at the beginning of the next sector and $\Delta\bar{\psi}_x$ and $\Delta\bar{\psi}_y$ are the desired phase shifts per cell in the next sector. Since not all of the matrix elements of σ are independent, we need to satisfy the following equations:

$$\sigma_{x11}(k_1,k_2,k_3,k_4) = \frac{\beta_{xo}(\Delta\bar{\psi}_x,\Delta\bar{\psi}_y)}{\bar{E}_o}$$
$$\sigma_{x21}(k_1,k_2,k_3,k_4) = -\frac{\alpha_{xo}(\Delta\bar{\psi}_x,\Delta\bar{\psi}_y)}{\bar{E}_o} \quad (4)$$

and the same equations with subscript y. The inversion of these equations is done on-line by the control program. In this model, the lattice-parameters are $\Delta\psi_x$, $\Delta\psi_y$, $\Delta\bar{\psi}_x$ and $\Delta\bar{\psi}_y$ and the control-parameters are k_f, k_d, k_1, k_2, k_3, and k_4. The control-parameters are found numerically for given values of E_o and ΔE_i with $i = 1,2,..\ 8$ denoting the energy gain from the ith linac girder within the sector.

The main consideration that influenced our choice of the linac lattice has been the need to minimize the β-function in order to reduce the disruption caused by transverse wakefields. Using the proposed scheme, it is possible to keep the maximum β values in the matching cells smaller than those in the cells in the adjacent sectors as shown in Fig. 2. The phase advance of

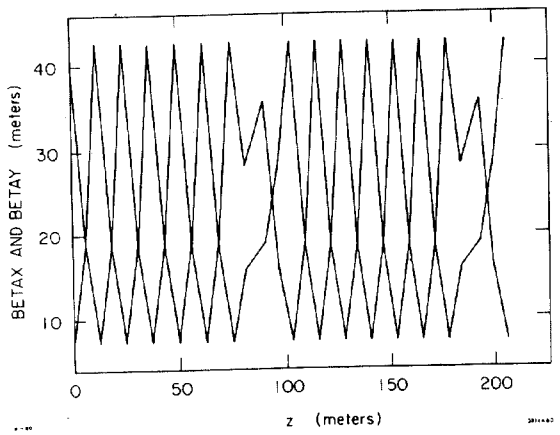

Fig. 2. The betatron functions for one superperiod (two sectors) of the linac lattice.

90 degrees/cell is maintained up to a beam energy of 26 GeV. Between 26 and 50 GeV the phase advance decreases to 40 degrees/cell at the end of the linac since we will be limited by the maximum current of the magnet power supplies.

A special scheme has been developed to be used for launching the beam from the linac injector into sector 2, where the injection system and extraction system for the damping rings are to be located. Tests to measure the wakefield effects in sector 1 are being conducted. For these tests the beam matrix is measured at the beginning of sector 1 and we adjust the strength of the quadrupoles in the first half of the sector 1 such that the beam matrix at the end of the sector is equal to the desired value at the beginning of sector 2. This scheme has been incorporated into the control program for the sector 1 test.

Orbit Correction

In order to suppress the generation of transverse wakefields it is important that the trajectories of simultaneous beams of electrons and positrons passing through the lattice be well centered. Within each quadrupole there is a beam-position monitor to measure the positions of the electron beam and of the positron beam separately. Each quadrupole has trim windings that can be used to make a magnetic dipole field in any direction perpendicular to the linac axis. There are two possible causes for trajectory errors: static fields (due to quadrupole misalignments) which deflect positrons and electrons in opposite directions; rf fields (due to a tilted linac girder or to an asymmetry in the waveguide-end couplers) which deflect positrons and electrons in the same direction. It is possible to simultaneously correct the trajectory distortions caused by both sources of error with the static dipole windings on the quadrupoles.

We have made a computer model of the linac lattice with thin-lens elements according to the scheme described in the preceding section. This model has been used to study the trajectory correction problem. We model the rf error deflections by giving the beam a single random kick at the center of each 12.34 meter girder, the same point at which the acceleration has been introduced. The static error problem has been modeled by introducing a random displacement of each thin-lens quadrupole, and another random displacement of each position monitor. We have chosen the following distributions for random errors for our study. Quadrupole displacements and beam-position-monitor errors have been uniformly distributed between -.1 and +.1 mm. The rf kicks are uniformly distributed between -3×10^{-5} and $+3 \times 10^{-5}$ GeV/c and are assumed to be centered in each girder. This distribution is consistent with measurements which have been made on the linac.

Our correction scheme is as follows. Let the correctors be specified by index j (j=1,2,.. N) and the monitors by index i (i=1,2,.. M). Let x_i^+ and x_i^- be the measured positions of the e^+ beam and e^- beam at the i-th monitor before corrections are applied. If we turn on the j-th corrector, the orbit at the i-th monitor will change by $C_{ij}^+\theta_j$ for the e^+ beam and $-C_{ij}^-\theta_j$ for the e^- beam. $\pm\theta_i$ are the kicking angles for the e^+ and e^-. $C_{ij}^\pm$ are the response matrices determined by the linac lattice; $C_{ij}^\pm$ will be zero if the j-th corrector is downstream of the i-th monitor. We wish to find a solution θ_j (j=1,2,..,N) which minimizes the sums of squares of residuals (S) of the orbits after correction:

$$S = \sum_{i=1}^{M}\left[\left(x_i^+ + \sum_{j=1}^{N} C_{ij}^+\theta_j\right)^2 + \left(x_i^- - \sum_{j=1}^{N} C_{ij}^-\theta_j\right)^2\right] \quad (5)$$

If we group the constants C_{ij}^+ and C_{ij}^- to form two M×N matrices $\underset{\sim}{C}^+$ and $\underset{\sim}{C}^-$, and group x_i^+ and x_i^- to form two M-dimensional vectors $\underset{\sim}{x}^+$ and $\underset{\sim}{x}^-$, the solution for θ which minimizes S is given by the following expression:

$$\underset{\sim}{\theta} = -(\tilde{\underset{\sim}{C}}^+\underset{\sim}{C}^+ + \tilde{\underset{\sim}{C}}^-\underset{\sim}{C}^-)^{-1}(\tilde{\underset{\sim}{C}}^+\underset{\sim}{x}^+ - \tilde{\underset{\sim}{C}}^-\underset{\sim}{x}^-) \quad (6)$$

$\tilde{\underset{\sim}{C}}$ is the transpose of $\underset{\sim}{C}$. This solution can be readily computed on-line.

The trajectories of the electron and positron beams before and after correction calculated from Eq. (6) are shown in Figs. 3 and 4 for a typical case having the error values given above. It can be seen that after correction the rms displacement error for each beam is less than .05 mm. This orbit correction scheme will be implemented into the SLC control program. In this model the input-parameters are the measured orbit values at the position monitors $x_i^\pm$. The control-parameters, θ_j, are calculated on-line and the results are stored for use by a separate program module which controls the power supplies. The same procedure is used for y positions.

Klystron on/off Compensation

There will be a total of about 240 klystrons in the linac. Each klystron will supply pulsed rf power to one of the 240 linac girders. During the operation of the linac, a few of the klystrons will be operated in the standby mode such that the rf accelerating fields in the corresponding girders are delayed in time relative to the beam pulse. In order to keep the beam energy at the end of the linac constant, whenever a klystron failure

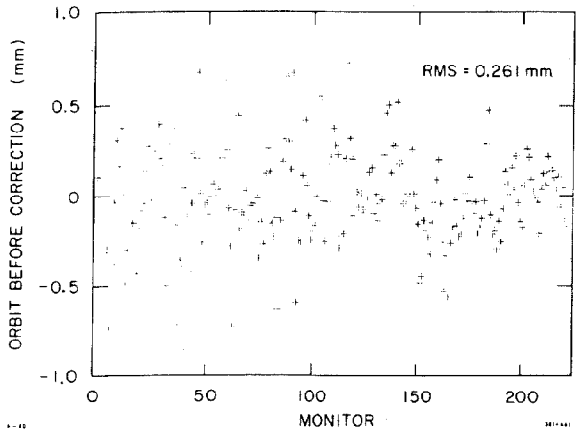

Fig. 3. Distribution of the positron beam position before correction. The distribution of the electron beam position is similar and has an rms value of 0.245 mm.

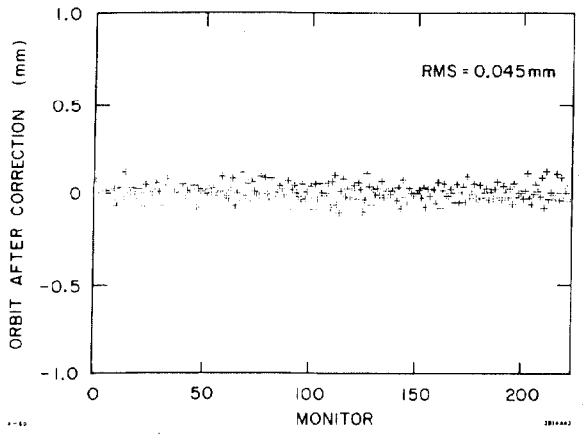

Fig. 4. Distribution of the positron beam position after correction. The distribution of the electron beam position is similar and has the same rms value as e^+.

occurs one of the standby klystrons will be used as a replacement and be switched into the accelerate mode. At the same time the klystron which failed will be placed on standby. The replacement of klystron A by another klystron B will cause the energy of the beam along the lattice between the corresponding sectors A and B to be either raised or lowered. Because of this change in the beam energy, the strength of the quadrupoles at these sectors between A and B has to be adjusted to compensate for the mismatching of the lattice resulting from the energy change.

We have also studied this rematching problem with the thin-lens linac model. It has been found that the computation time to rematch a sector is about 20 msec (VAX computer). In order to keep the computation time to a minimum, in practice, only the strength of the quadrupoles in sectors A and B will be recalculated using the model, while the strength of the quadrupoles in sectors between A and B will be obtained by scaling their values appropriately with the updated beam energy value. To further reduce the computation time by well over an order of magnitude the technique of Chebyshev representation can be used. (See appendix.)

The beam orbit values are also affected by adding or subtracting a klystron due to the addition or subtraction of the errors in the corresponding linac girder. In principle, the least squares global correction scheme could be used to correct this effect. However, the time required to remeasure the orbit may be too long and the orbit after correction would not be restored to the orbit before klystron failure. A better scheme would be to correct the orbit effect locally and to restore the orbit globally. We have studied a local correction scheme with the thin-lens linac model. In this scheme, we assumed the rf field errors are known at girder A or B. To correct either an x or y orbit we compute the strength of the four orbit correctors which are nearest to girder A or B such that the orbit change they introduce is opposite to the orbit change caused by the rf field error. The result has been found to be satisfactory, i.e., the strength of the correctors and the orbit error between the four correctors are within acceptable limits. The orbit outside of the correctors is also restored if the strength of the orbit correctors outside of this region is scaled inversely proportional to the new beam energy.

Appendix

We consider a continuous function $F(x,y,z)$ which can be evaluated somehow for any set of (x,y,z). This evaluation may be slow, for example, if lengthy iterative approximations are needed. We choose, however, specific values of (x,y,z) and use the resulting function values to generate N Chebyshev coefficients. These coefficients can subsequently be used efficiently to find F for any new set of (x,y,z). The computation time required to generate F is then essentially only the time needed to do N multiplications.[3] Moreover, since Chebyshev expansions are known to have, in general, the fastest convergence, there are usually surprisingly few terms in the Chebyshev series needed to achieve a required precision.

This method has been applied to solve for the values of k_1, k_2, k_3 and k_4 from Eq. (4). We used a computer program[4] from CERN which was modified to our needs of three variables E_o, $\Delta\psi$ and $\Delta\bar{\psi}$, for the case of equal phase shift/cell in both x and y and a given set of values of ΔE_i. This program, when given the (slow) function generator , Eq. (4), produces explicitly the fast algorithm with the correct numerical values. We have four functions to be evaluated rapidly, i.e., $F(E_o,\Delta\psi,\Delta\bar{\psi}) = k_1$, k_2, k_3 or k_4. We find that using 15 coefficients gave a precision of 1%. The time required to compute these functions is typically less than 1 msec.

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