IEEE Transactions on Nuclear Science, Vol. NS-28, No. 3, June 1981

A TRANSVERSE SCHOTTKY NOISE DETECTOR FOR BUNCHED PROTON BEAMS

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Summary

Transverse Schottky noise analysis is a powerful standard technique for continuous proton beam diagnosis[1]. The extension to bunched proton beams of very low intensity, as will be used in $p\bar{p}$ colliders, requires new technical features to enhance the detector sensitivity and to reduce the coherent signal of the parasitic common mode. During SPS $p\bar{p}$ collider experiments, we met this requirement by using a resonating transverse pick-up three meters long, the two plates being movable to match the electrode gap to the beam width at high energy. The parasitic sum signal was drastically reduced by centering the plates around the beam and by filtering the output signal. Clean betatron and longitudinal Schottky bands have been observed for a 0.3 mA continuous beam and for a 0.01 mA single bunch beam. Betatron satellites have also been detected for a 6 mA multi-bunched beam. As a future development the coil inductance in parallel to the electrode plates will be varied by a movable ferrite core, to allow continuous modification of the pick-up transverse acceptance through a change of the gap distance without changing the resonant frequency, the band-width or the filtering device.

Theory

The transverse Schottky noise of a bunched proton beam can be derived from the position signal of a single particle performing betatron and synchrotron oscillations. Detected by a transverse pick-up station at azimuth θ, such a signal can be represented in the time domain as follows :

$$S(t,\theta) = e \sum_{n=-\infty}^{+\infty} \delta(t-\tau + \frac{\theta T_o}{2\pi} - nT_o) * \hat{x} e^{i\phi(t)} \qquad (1)$$

with $T_o = 2\pi/\omega_o$ = revolution period, τ = time delay with respect to the synchronous particle, and

$x(t) = \hat{x}\, e^{i\phi(t)} = \hat{x}\, e^{i(\omega_\beta t + \phi_o)}$ = betatron motion equation. Assuming simple harmonic motion for the synchrotron oscillations, with $\tau(t) = \hat{\tau} \sin(\omega_s t + \psi_o)$, the spectral form of S becomes :

$$S(t,\theta) = \frac{e\hat{x}}{T_o} \sum_{n,p=-\infty}^{+\infty} J_p(\Omega_n \hat{\tau})\, e^{i\left[\omega_{np} t - n\theta + \phi_o - p(\frac{\pi}{2} - \psi_o)\right]} \qquad (2)$$

with $\Omega_n = (n+\nu_o)\,\omega_o - \omega_\xi$, $\omega_\xi = \omega_o \nu_o \frac{\xi}{\eta}$, $\nu_o = \frac{\omega_\beta}{\omega_o}$ = transverse tuning, ξ = chromaticity, $\eta = \gamma_{tr}^{-2} - \gamma^{-2}$,

$\omega_{np} = (n+\nu_o)\,\omega_o + p\omega_s$, and J_p = Bessel function of order p.

The usual form for unbunched particles can be derived from (2) in the limit as $\tau \to 0$

$$S(t,\theta) = \frac{e\hat{x}}{T_o} \sum_{n=-\infty}^{+\infty} e^{i(\omega_n t - n\theta + \phi_o)} \qquad (3)$$

with $\omega_n = (n + \nu_o)\,\omega_o$.

From (3) we can see that the Schottky spectrum of an unbunched beam contains an infinity of betatron lines each having a central frequency ω_n and a spread, due to momentum and transverse tuning dispersion, given by

$$\Delta\omega_n = (n\eta + \nu_o \xi)\,\omega_o \frac{\Delta p}{p} \qquad (4)$$

The additional synchrotron movement of a bunched beam, considered in (2), transforms each of these bands into a central betatron line associated with an infinite set of synchrotron satellites, spaced at the synchrotron frequency and intensity modulated by Bessel functions of increasing order. In a linear machine without ripple, the central betatron lines are sharp while the synchrotron satellites reproduce the momentum distribution of the beam. Information on chromaticity now lies in the relative height of the satellites compared to the central line.

A particular feature of the Schottky noise spectrum is that the total power per band is proportional to N, the number of circulating particles, due to the incoherency of the particle motion. More explicitly we find :

$$\langle S^2 \rangle = \frac{N}{2} \frac{e^2 x_{rms}^2}{T_o^2} \qquad (5)$$

per betatron band of an unbunched beam.

$$\langle S^2 \rangle = \frac{N}{2} \frac{e^2 x_{rms}^2}{T_o^2} F_{n,p} \qquad (6)$$

per betatron (p=0) or synchrotron (p≠0) band of a bunched beam. The form factor $F_{n,p}$ is the integral along the bunch of the Bessel function squared $J_p^2(\Omega_n \hat{\tau})$ weighted by the normalized momentum distribution.

Considerations for the $p\bar{p}$ collider

Two particular features of $p\bar{p}$ colliders, the low d.c. current and the tight bunching, are relevant to the Schottky noise receiver project. The low d.c. current, due to the low production rate of antiprotons and to the big dimensions of colliders, implies a low total power per spectrum line as shown in (5) and (6). The tight bunching, required to increase the luminosity, produces a coherent enhancement of the parasitic longitudinal component in the transverse pick-up as well as a decrease in width of the betatron lines. The latter effect increases the spectral power density of the Schottky signal compared to an unbunched beam and reduces the total power of the superimposed thermal noise. Therefore, in a Schottky detector system, the most critical parameters are the sensitivity of the pick-up, the noise factor of the electronics, and the rejection level of the parasitic common mode signal. In particular the order of magnitude of this last parameter, required to avoid the risk of amplifier saturation, is

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fixed by the ratio of the power in the coherent longitudinal line to that in the incoherent transverse Schottky line and corresponds roughly to N, the number of circulating particles.

To evaluate explicitly the change in the signal to noise ratio in going from a tightly bunched to an unbunched beam of the same intensity, note that in both cases the total power per betatron line of the Schottky signal is practically the same. This is because for short bunches, and for a large range of n, including normally the revolution frequency harmonic tuned by the Schottky receiver, the form factor $F_{n,p=0}$ in (6) becomes approximately equal to 1. Therefore the change in signal to noise ratio depends only on the superimposed noise power, determined by the transverse line width ratio. As already mentioned the transverse frequency spread of a bunched beam is strongly reduced compared to that given by equation (4), and reflects only the non-linear tune spread coming from the machine octupole component, the space charge and the second order effect of sextupoles. A rough computation of the line width ratio for the SPS collider gives a value between 10 and 100, depending on the operating conditions, and shows good agreement with the measurements.

The receiver for the SPS collider

To observe the Schottky noise during single beam experiments in the SPS collider, we have built a special horizontal pick-up station, having three meter long electrodes, both movable radially to match the interelectrode gap to the beam width at high energy. For maximum sensitivity we have chosen to resonate the electrodes with an air cored coil and to extract the information with a one turn loop. The central frequency of the resonator is a compromise between the line width to be observed on unbunched beams and the ease of making a high Q coil. A broad optimum has been found between 1 and 20 MHz. The final choice of 10.7 MHz is due to the availability of low cost, narrow band filters optimised for rejection of the common mode longitudinal component during bunched beam operation. A 3 dB resonator band-width of 50 kHz has been implemented, by adjusting the coupling between the coil and the loop, to cover the full range between two revolution frequency harmonics.

The final system used is shown in Fig. 1. The signal detected by the resonating plates is transmitted to a sharp crystal filter and then strongly amplified by a low noise amplifier chain. A high quality, three kilometer long, cable sends the information to the Main Control Room, where a second crystal filter removes out of band added noise. After further amplification the signal is mixed down to between d.c. and 100 kHz and finally analysed by an averaging fast Fourier transform spectrum analyser. The equivalent diagram for the electrode circuit is given in Fig. 2, together with the response dependence on the resonator frequency, electrode length and plate capacitance.

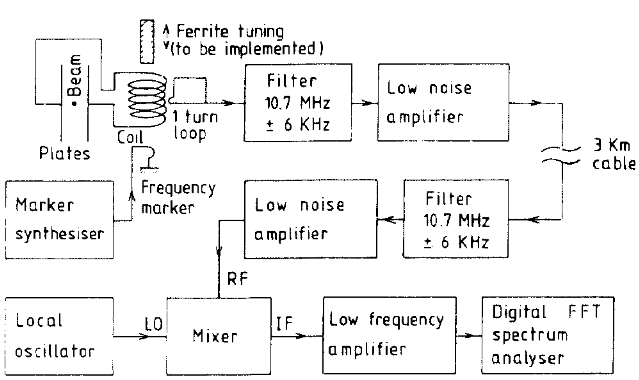
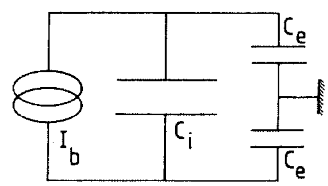


_Fig.1_Block diagram of the Schottky receiver



$I_b = I \frac{\omega \ell}{v}$ = current on the plates

I = Schottky current

ω = central frequency of the resonator

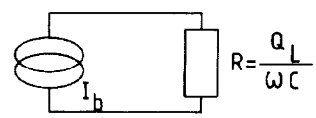
ℓ = electrode length

v = particle velocity

C_i = interelectrode capacitance

C_e = electrode to earth capacitance

a) Block diagram of the electrodes



Q_L = loaded Q

$C = C_i + \frac{C_e}{2}$ = total capacitance

b) Block diagram at resonance

Fig. 2 Electrode equivalent circuit

An example of the estimation of the signal to noise ratio per Schottky band is computed as follows :

- $\langle I^2 \rangle$ developed in the plates per Schottky band

  $$\langle I^2 \rangle = \frac{1}{2}\frac{e}{T_o} N \left|\frac{x_{rms}}{\frac{1}{2}G}\right|^2$$ with G = interelectrode gap,

  x_{rms} = rms betatron amplitude. This shows how the gap reduction by movable plates enhances the pick-up response.

- Signal power in the external load

  $$P = \langle I^2 \rangle \frac{\omega^2 \ell^2}{v^2} R \left|1 - \frac{Q_L}{Q_u}\right|$$

  with Q_u = unloaded Q.

- Numerical computation of P : for a gap of 10 mm we have measured C = 115 pF, Q_L = 155 and Q_u = 350. For N = 10^{12} and x_{rms} = 1 mm one obtains P = 3 10^{-15} W.

- Thermal noise computation : for a noise figure of 5 dB at 300° K in a 100 Hz band we find P_N = 1.2 10^{-18} W.

- Signal to noise ratio : $\frac{P}{P_N}$ = 1.7 10^3 = 32.3 dB.

- Coherent longitudinal bunched signal :

  $$\langle I_L^{\;2} \rangle = \frac{4\,e^2}{T_o^{\;2}} N^2$$

- Rejection level to reduce the coherent longitudinal signal to the level of the incoherent transverse signal :

$$A = \frac{\langle I_L^2 \rangle}{\langle I^2 \rangle} = 8 \left| \frac{x_{rms}}{\frac{1}{2} G} \right|^{-2} N$$

Following the previous numerical computation :

$A = 200\,N = 2\ 10^{14} = 143$ dB.

The parasitic longitudinal signal has been reduced by 110 dB; 70 dB is obtained by sharp filtering and 40 dB by carefully centering the p.u. plates around the beam. This last value requires a precision of .1 mm over a gap of 10 mm. With the crystal filter used, B.W. of $\sim$ 11 kHz, the tune was measurable in the range 0.545/0.454 to 0.798/0.202.

Examples of experimental results [2]

Fig. 3 shows a typical scan at 240 GeV/c of $\sim 10^{12}$ protons. The lower trace gives the horizontal signal of the bunched beam, where the synchrotron satellites of order 1 and 2 are visible. The upper trace shows the same betatron line for the unbunched beam.

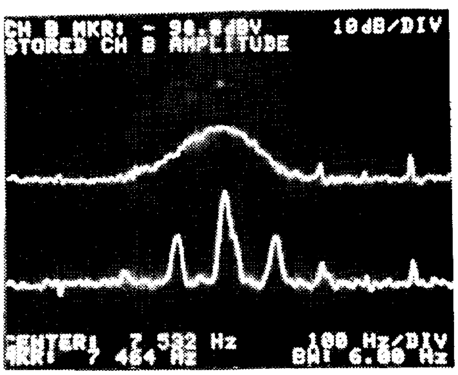


Fig. 3

Unbunched beam signals have been observed with a P/P_N ratio of 10 dB for $3\ 10^{10}$ circulating protons, whereas detectable signals are obtained with a single bunch of 10^9 protons.

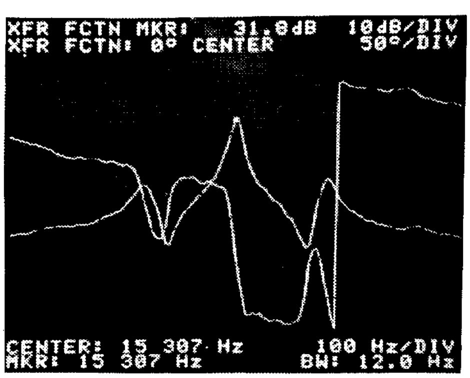


Fig. 4

A strong increase in the unbunched beam signal has been produced by a small transverse excitation of the particles. Fig. 4 shows the amplitude and phase response of a 10^{10} bunched proton beam to a noise excitation of 10^{-8} mrad rms-deflection-angle per revolution centred at 10.7 MHz. This type of response allows the stability diagram to be computed.

Further improvements

In certain experiments, the 10 mm interelectrode gap can be an unwanted limitation in the transverse acceptance. A movable ferrite core placed in the resonating coil allows continuous variation of the gap from 10 to 25 mm, without changing the central frequency of the resonator and hence the filtering device. Power losses in the ferrite core are negligible so the 3 dB band width is also unchanged. Changing the resonating coil (remotely) will allow monitoring of the large low energy beam ($\sim$ 50 mm, 26 GeV/c).

The limited range of explorable tune will be extended by using a set of narrow band crystal filters each with a slightly different central frequency. Present filter technology allows a ν_0 range of between 0.12 and 0.88 at low as well as at high energy operation to be measured.

Separate receivers and pick-ups for horizontal and vertical planes will be provided and a synthesised frequency marker signal will be superimposed on the Schottky spectrum to allow jitter monitoring and to simplify initial pick-up tuning.

As a last improvement, we intend to selectively excite protons or antiprotons with a special directional coupler used as a kicker. Therefore excited Schottky lines as well as direct and crossed transfer functions for each beam will become observable.

Acknowledgement

It is impossible to name individually here all the people of the mechanical group and extraction section in the SPS Division who give engineering support in the construction of the Schottky receiver, but we thank them all. In addition, we thank D. Boussard and C. Graziani for the suggestions and the strong support during the measurements.

References

1) J. Borer et al., Non destructive diagnostics of coasting beams with Schottky noise, IXth Int. Conf. on High Energy Accel., SLAC, 1974.

2) SPS Improvement Reports Nos. 167 and 175 (unpublished), CERN, SPS Division.