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BUNCH LENGTH COMPRESSION INSIDE A SEPARATED SECTOR ISOCHRONOUS CYCLOTRON AND RELATED PROBLEMS

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Bunch compression inside an isochronous cyclotron can be achieved by means of a field perturbation. In order to reduce the energy spread this perturbation is applied at injection over less than one tenth of the accelerated orbits. Injection phase has then to be displaced and energy gain over the first turn is reduced by about the same ratio as the bunch compression one aims at. If a precessional injection is therefore used, for a relativistic cyclotron with a large number of turns the radial beam emittance has been found to exhibit a large increase. A simple method to overcome this difficulty by a slight additional field perturbation over the remainder of the acceleration has been found and successfully tested by numerical simulation. It is intended to use this possibility on the GANIL SSC's.

1. INTRODUCTION

In a cyclotron, the energy spread depends on the length of the accelerated bunches. In an SSC with a large energy gain, if one is using an r.m.s. definition of energy spread $\pm\Delta W$ and of bunch length $\pm\Delta\phi$ (in radian) as twice the square root of the centered second moment (which is not far, for practical distributions, from half maximum width) |1|, one has :

$$\pm\Delta W_{out} \simeq \pm\sqrt{(\Delta W_{in})^2 + W_{out}^2\cdot(\Delta\phi/2)^4} \qquad (1)$$

where the subscripts refer to input and output and the phase width is supposed constant during acceleration.

In order to achieve a small energy spread, $\Delta\phi$ has to be small. Instead of cutting in $\Delta\phi$ which reduces the intensity, $\Delta\phi$ can be minimized by matching in longitudinal phase space $\Delta\phi$, ΔW. Such a matching is normally made by a proper use of drift spaces and rebunchers i.e. beam transport focusing devices and additionnal RF cavities, both elements which exist inside a cyclotron. What is presented here is a way to achieve optimum matching in longitudinal phase space in a cyclotron for minimizing energy spread, a description of some of the difficulties encountered and a solution to overcome them.

2. COMPRESSION MECHANISM

Let us consider a cyclotron where the magnetic field B_0 departs by ΔB_0 from isochronism at injection and over some distance in radius (or energy).
In the ϕ,W plane the longitudinal motion of a particle in a cyclotron obeys the equations |2|, |3|

$$d\phi/dN = -(2\pi h\cdot\Delta B_0/B_0 + \sin\phi\ d(eV)/dW) \qquad (2)$$

$$dW/dN = eV.\cos\phi \qquad (3)$$

from which one has :

$$eV\cdot\sin\phi + \int 2\pi h\cdot(\Delta B_0(W)/B_0)\,dW = C^t \qquad (4)$$

for each particle. Here V is the RF accelerating voltage of harmonic number h and N the number of turns.

Considering two particles separated by $\Delta\phi$, ΔW and of central phase ϕ_c, a comparison between input and output, gives :

$$(eV\cdot\cos\phi_c\,\Delta\phi + 2\pi h\cdot(\Delta B_0/B_0)\cdot\Delta W)_{out} =$$
$$(eV.\cos\phi_c\,\Delta\phi + 2\pi h.(\Delta B_0/B_0)\cdot\Delta W)_{in} \qquad (5)$$

This relation offers a method for compressing the bunch length inside an SSC : assuming $\Delta B_{out} = 0$ and for instance, two particles of the same initial energy ($\Delta W_{in} = 0$), one gets

$$(V\cos\phi_c\,\Delta\phi)_{out} = (V\cos\phi_c\,\Delta\phi)_{in} \qquad (6)$$

In order to change $\Delta\phi$ one may change V as already suggested by Muller and Mahrt |3| ; it is also possible to change ϕ_c. This is obtained from a field perturbation.

In practice the compression is made quickly after injection (over one tenth of the number of turns) in order that acceleration be made mostly with a small bunch length.

Neglecting the spread in energy occuring during compression by a factor C, while the bunch length is not yet fully reduced, expression (1) for output energy spread can be replaced by :

$$\pm\Delta W_{out} \simeq \pm\sqrt{(C\cdot\Delta W_{in})^2 + W_{out}^2\,(\Delta\phi_{in}/2C)^4} \qquad (7)$$

Even if one keeps in mind that this value has to be slightly increased to include the growth during compression, relation (7) shows how to optimize the compression factor C in order to minimize energy spread :

$$C_{opt}^3 = (W_{out}/\Delta W_{in})\cdot(\Delta\phi_{in}^2/2\sqrt{2})) \qquad (8)$$

For such a value, longitudinal emittance grows by 30 % (plus the additional effect mentioned above) ; one may be tempted to go in compression slightly over the optimum in order to reduce this growth while not increasing very much ΔW_{out} over the minimum which is quite flat, but the general behaviour is well described by (7).

3. PRECESSIONAL INJECTION

According to (6) the product of energy gain per turn by bunch length is the same at injection as at the end of the compression process. In order to reach a reasonable compression factor, the energy gain and hence the turn to turn separation must be appreciably reduced at injection. Special schemes may have to be introduced.

For GANIL, a precessional method with an amplitude of about 2.5cm was found satisfactory for injection. Such a device, without compression had already been considered previously as an help for extraction |4| and had not exhibited in computer simulation any parasistic difficulty.
Similarly, simulation studies of an operation slightly out of isochronism but without precession in order to make possible a sharp cut in bunch length had shown no problem |1|.

The test of compression with precessional injection on the contrary, if satisfactory in terms of compression was desastrous in terms of radial emittance at the end of acceleration when minimizing energy spread.

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4. MECHANISM OF EMITTANCE GROWTH |5|

The emittance of a bunch in the ϕ,W space moves during compression and acceleration as shown on curve 1, fig 1 The central phase ϕ_C starting at injection from, say, - 70° shifts due to the lack of synchronism towards 0°. In practice it goes further, onto a small positive value, in such a way that head and tail particles (A and B) reach at extraction an equal energy : the emittance is then symmetrical and energy spread is minimized. During all the process, however, W_A is larger than W_B.

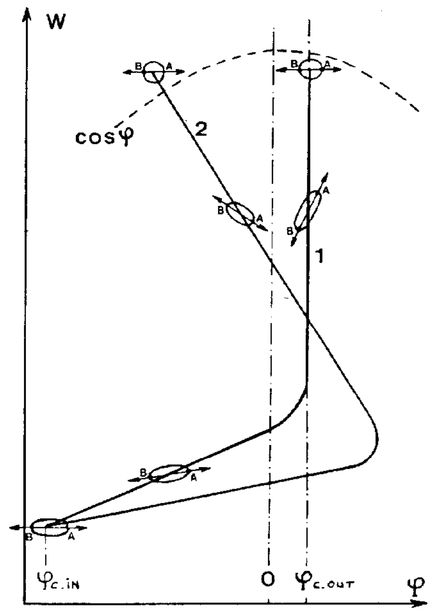

Fig. 1

Compression mechanism. Path of the bunch emittance in longitudinal phase space.
Curve 1 : standard scheme when no precession is used
Curve 2 : scheme for radial emittance growth compensation when precession is used.

The case studied was a high energy SSC (100 MeV/A output) for which relative energy gain and turn separation were small, thus requiring precessional injection. In the range of energy covered during acceleration, relativistic effects increase regularly the radial betatron frequency ν_r from injection to extraction by a little more than 0.1. Particle A previously considered has then always a ν_r larger than particle B and it performs more precessional betatron oscillations.

In radial phase space, the position of the emittances relative to head or tail of a bunch are at output as shown on fig. 2. Such a situation can lead to a large apparent emittance growth.

Let us call 2 $\Delta\psi$ the separation in phase of precessional oscillation of particles A and B, distant in longitudinal phase space during acceleration by (2 $\Delta\phi$,2 ΔW). One has, rotating with the azimuth in the cyclotron :

$$\Delta\psi = \int (d\nu_r/dW)\cdot\Delta W\cdot d\theta \tag{9}$$

and if, as it is approximately the case, $d\nu_r/dW$ is supposed constant during the main part of acceleration

$$\Delta\psi = 2\pi N \frac{\nu_{r.out} - \nu_{r.in}}{(W_{out} - W_{in})^2} \int_{W_{in}}^{W_{out}} \Delta W \, dW \tag{10}$$

As said, when ΔW is always positive, the integral cannot be null.

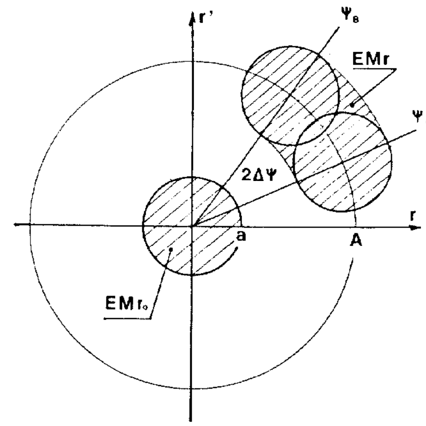

Fig. 2

Radial emittance growth. Particle A makes slightly more precessional oscillations than particle B.

In the case of fig. 2 where the injected beam is matched for betatron oscillations, with a radius a and an amplitude A of precession, the growth in r.m.s. emittance for a separation $\pm\,\Delta\psi$ in betatron phase space is approximately :

$$EM_r/EM_{ro} \simeq \sqrt{1 + (2\ A\ \Delta\psi/a)^2/3} \tag{11}$$

In order to limit the growth, one has to make sure that

$$\Delta\psi < k.\ a/A \tag{12}$$

with k = 1 for 50 % growth and k = 0,5 for 15 %.

5. METHOD OF COMPENSATION

In the compression and acceleration scheme shown on fig. 1, if one lets the central phase ϕ_C drift to a slightly higher positive value, ΔW will reach 0 before the end of acceleration and then take negative values. Integral (10) will go down. It is clear that there exists a value of ϕ_C for which it goes through zero. There is then no emittance growth. The output energy spread, however, is large. If one finds a way to bring back ΔW to zero at the end of acceleration while (10) is also null, this situation will be satisfactory both for energy spread and radial emittance.

This can be obtained with the help of a very small change δB_O of the magnetic field during the full acceleration precess, change of a sign opposite to the one used for compression. The corresponding path in longitudinal phase space is shown on curve 2 fig. 1.

Let us call ϕ_{C1} and $\pm\Delta W_1$ the central phase and energy difference between particles A and B at the end of compression, at energy W_1. According to relations

$$d\phi_C/dW = -\ (2\pi\ hN/\ (W_{out} - W_{in}))\ \delta B_O/B_O \tag{13}$$

$$d\ (\Delta W)/dW = -\ \Delta\phi.\ \sin\phi_C \simeq -\ \Delta\phi.\ \phi_C \tag{14}$$

one obtains by integration :

$$\Delta W_{ex} \simeq \Delta W_1 + \left[\pi\ N\ h\ (W_{out} - W_1)/(W_{out} - W_{in})\ \delta B_O/B_O) - \phi_{C1}\right]\cdot(W_{out} - W_1)\ \Delta\phi_{out} \tag{15}$$

$$\int_{W_{in}}^{W_{out}} \Delta W\ dW = \int_{W_{in}}^{W_1} \Delta W\ dW + (W_{out}-W_1)\cdot\Big[\Delta W_1 - \Delta\phi_{out}\ \phi_{C1}\cdot (W_{out}-W_1)/2 + \Delta\phi_{out}\cdot\pi hN/3.\delta B_O/B_O\ (W_{out}-W_1)^2/(W_{out}-W_{in})\Big] \tag{16}$$

Assuming as a simple approximation, that one can adjust φc_1 and $\delta B_0/B_0$ without changing ΔW_1 and $\int_{W_{in}}^{W_1} \Delta W \, dW$ it becomes possible to make ΔW_{out} and $\Delta\psi$ simultaneously equal to 0 and to minimize energy spread and radial emittance. In practice ϕ_{c_1} is directly dependant upon ϕ_{cin} even if the compression reduces its excursion, but ΔW_1 and the small integral (16) also change with ϕcin. Nevertheless, the general behaviour is not modified : fig. 3 and 4 illustrate the compensation mechanism.

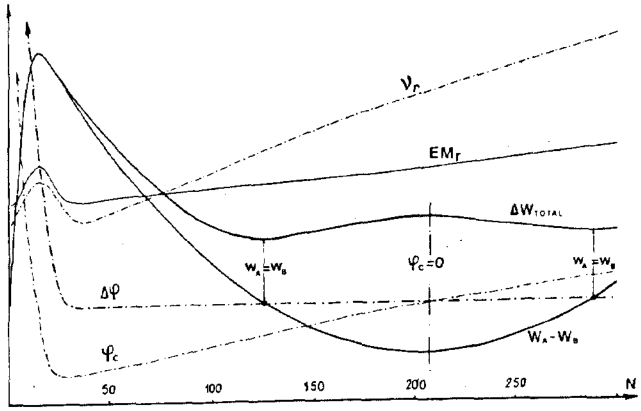

Fig. 3

Mechanism of radial emittance growth compensation.
One may comment on the various curves :
ν_r increases linearly with the number N of turns except at the beginning of acceleration due to compression field perturbation.
ϕ_c follows a law of the type shown on fig. 1 curve 2.
$\Delta\phi$ is compressed fast and stays constant during the rest of acceleration.
$W_A - W_B$ comes to a large value during compression ; it then goes through zero, becomes negative and comes back to zero at the end of acceleration.
Radial normalized emittance increases slowly as explained on paragraph 6.

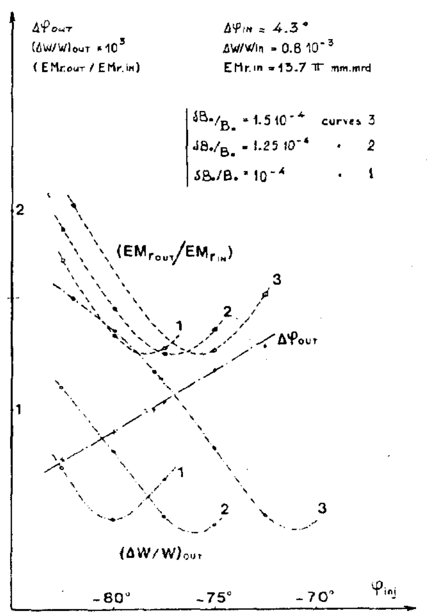

Fig. 4

Effect on radial normalized emittance and on energy spread of a change in injection central phase and in compensation field δB_0 : $\phi inj = -77°$ on curve 2 gives a good operation, corresponding to fig. 3. Tolerances adjustment appear clearly.

Fig. 4 also indicates the required tolerances. Injection phase ϕcin adjustment accuracy is mainly required for minimizing energy spread : it is the same as without compression and compensation. Field correction δB_0 has to be quite tightly adjusted but a change of 10^{-5} moves the output phase by 2.5° and phase mesurements make such an accuracy accessible.

6. EMITTANCE GROWTH DUE TO INITIAL ENERGY SPREAD

As shown by SCHULTE |6|, even without any phase wandering during acceleration, there is a slight growth in radial emittance when a precessional oscillation is established. This author studied its incidence on a precessional extraction mechanism. Such an effect also exists in the case studied here, with the same amplitude and same properties. It should be included in the reference emittance EMr_0 of (11). It is responsible for the slow growth seen on Fig. 3.

7. CONCLUSION

A method of matching longitudinal phase space during the early part of acceleration inside an SSC has been successfully tested on numerical simulation for GANIL SSC's. Tolerances required are not tighter than for normal operation. The effect of field errors has also been studied |5| and no special effect has been found with them.

Such a method ecomonizes the use of rebuncher and should offer the possibility of a better matching. It is foreseen for the operation of GANIL.

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