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DESIGN OF MUON STORAGE RINGS FOR NEUTRINO OSCILLATIONS EXPERIMENTS

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Abstract

A muon storage ring can provide electron and muon neutrino ($\nu_e, \bar{\nu}_\mu$) beams of precisely knowable flux which are excellent probes for ν oscillations. Constraints on storage ring and injection design are described. Sample muon storage rings are presented and compared with parasitic use of the Fermilab $\bar{p}$ precooler as a μ storage ring.[1] "Stochastic injection" with injected pion beam decaying to circulating muon beam is favored. A practical possibility may be a low-energy (1 GeV) ring matched to a medium energy proton injector (8 GeV Booster).

The Precooler-μ Storage Ring

The operation of a muon storage ring to provide neutrinos has been described previously by Cline and Neuffer[1], and is illustrated in Figure 1A. A proton beam is focussed with high intensity on a target of ~1-2 interaction lengths, producing large number of secondary pions. The pions are separated from the non-interacting protons and transported to injection in a storage ring. Pion decay within the injection transport and after injection provides muons ($\pi \to \mu\nu_\mu$), which are stored within the ring for the muon lifetime. Muon decay in the ring straight sections ($\mu \to e\nu_e\bar{\nu}_\mu$) provides collimated ν_e and $\bar{\nu}_\mu$ beams. A detector is placed along a beam line at some distance L from the ring to observe ν-interactions.

The Fermilab $\bar{p}$ precooler[2] inescapably functions as a ~4.5 GeV μ storage ring during the first ms of its cycle and its large acceptance, designed for $\bar{p}$ acceptance, makes it a candidate for the first experiment of this type. The production target, transport line and precooler are shown in Figure 1B. Pulses of 1.8×10^{13} 80 GeV protons are focussed on the target producing secondaries (π,K,$\bar{p}$, etc.) which follow the transport line to injection in the ring. The production is dominated by π's which decay ($\pi \to \mu\nu_\mu$), producing many μ's which circulate in the ring. Calculations of π production and decay indicate ~5×10^8 ν per beam with parasitic use.

The precooler is, however, not an optimal μ storage ring. It is designed for $\bar{p}$ acceleration to 9 GeV/c which limits its acceptance at 4.5 GeV. The straight sections are shorter than desirable for ν beam intensity. The energy is set by $\bar{p}$ production and cooling requirements. The final precooler design may have the transport length shortened so that fewer μ's will be accepted (see below). In the following sections, the possibility of designing a ring specifically for μ storage is explored.

Choice of Storage Ring Energy

We first comment on the constraints governing the choice of an optimum storage ring energy. Neutrino oscillations are supposed to occur at a rate given by

$$P(\nu_1 \to \nu_2) \cong \delta_{12} - \sum_{i,j} 4\, U_{1i}U^*_{2i}U^*_{1j}U_{2j} \cdot \sin^2(\Delta_{ij}) \qquad (1)$$

where the U_{ij} are ν - mixing matrix elements and

$$\Delta_{ij} = 1.27\, M^2_{ij}(eV^2) \cdot \frac{L}{E}\frac{(m)}{(MeV)}$$

where M_{ij} is the neutrino mass difference, L is the distance from source to detector, and E is the neutrino energy. A sensitive search for ν-oscillation requires L/E large.

The transverse momentum of neutrinos in the decay $\mu \to e\nu_e\bar{\nu}_\mu$ has an average value $\bar{p}$ = 30 MeV, so the mean angle of ν production is $\bar{p}/E$ and the ν intensity at the detector is proportional to $(L\theta)^{-2}$ or E^2/L^2. The ν-N cross section increases linearly with E, so the detector event rate varies as E^3/L^2. These factors favor higher energy and small distance, opposing the constraints of the previous paragraph. Cost of storage ring is roughly proportional to radius and therefore to energy E; this provides a counterfactor favoring small E.

ν-intensity also depends directly on the number of capturable π's produced at the target. Following empirical formulae of Wang[3] this production has a broad maximum at $E_\pi \cong 0.1\, E_{proton}$. Our design strategy is to choose $E_\pi \cong E_\mu$ (ring energy) within this maximum. We design below sample storage rings with E_μ = 8 GeV, 4.5 GeV and 1.5 GeV, and note that these can be matched to several possible proton beams such as the Fermilab 80-GeV line, the CERN PS (30 GeV) or the Fermilab Booster (8 GeV).

Storage Ring Injection

In the precooler/storage ring most of the π decays occur within the transport from target to ring injection. This is favored because: (1) the momentum acceptance of the $\bar{p}$ precooler (±2%) is much smaller than the injection transport (±10%), (2) single turn injection is demanded for $\bar{p}$ accumulation, (3) the initial precooler design contains sufficient transport length.

In a redesigned muon storage ring the momentum acceptance can be more directly matched to the injection acceptance, and to the pion decay momentum width, so π decays can occur within the storage ring. We suggest that better injection schemes are shown in Figure 2. The injection optics is provided by a short section containing a strong focusing element (lithium lens) and matching elements which place the π beam in the desired orbit. Only a few meters of magnets are necessary for matching.[4]

π decay permits use of what we call "stochastic injection" in which injected π's decay to circulating muon orbits, which is similar to a previous scheme for collecting $\bar{p}$'s from decaying $\bar{\Lambda}$'s.[5] Injected pions naturally decay to lower energy muons within ~1 turn in the ring, and the muons within the ring acceptance will be stored. "Stochastic injection" can continue indefinitely without changing the injection optics; the proton pulse length need not be fitted to the storage ring length as in the $\bar{p}$ precooler. Designs for stochastic injection are shown in Figures 2A and 2B.

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In Figure 2A the injection optics matches injected π beam (at a higher momentum $p_\mu+\Delta p$ and circulating μ beam to identical orbits in an $\eta = 0$ (zero dispersion) straight section. The surviving π beam separates from the circulating beam in the curved section ($\eta = 0$) and is lost. π decays in the straight section contribute to the circulating beam. Separate injection and circulating orbits need only be provided in the injection area.

In Figure 2B, the pions are injected into an off-momentum orbit centered at $p_o+\delta p/2$ with a circulating muon orbit at $p_o-\delta p/2$ where p_o is the central ring momentum and $\pm\delta p$ is the acceptance. Pions circulate for a full turn of the ring, but separate π and μ orbits must be provided for the full ring circumference which limits μ acceptance. A first appraisal indicates similar muon current accumulated by both designs and we have followed design B below.

The important advantages of "stochastic injection" are:

1. A separate transport for π-decay is not necessary.
2. The proton pulse length need not be shortened to the storage ring circumference but can be the full proton synchrotron length. This allows greater proton intensity.
3. Stochastic injection is particularly attractive because the storage ring circumference is naturally matched to the pion decay length (see below).

Muon Storage Ring Design Constraints

In this section we outline some general design requirements of a μ storage ring. In Figure 1 we show a model μ ring with two long straight section of length S and two half circle sections of length πR. The possible values of R are limited by the bending requirement $R = B\rho/\bar{B}$ where $B\rho$ is the magnetic rigidity, which can be found from the formula: $B\rho(T\text{-}m) = 3.3$ P (GeV/c) and P is the momentum. $\bar{B}$ is the mean bending field which is limited to $\lesssim 2$ T for conventional magnets. The mean path length L_π for relativistic pions of momentum P_π before decay is $L_\pi = 53.6\ P_\pi$ (GeV/c) meters.

Stochastic injection demands that $L_\pi \lesssim 2\pi R + 2S$ since it is desired that π's decay within one turn. If we choose $S \gtrsim \pi R$ we find a requirement $\bar{B} \lesssim .8$T. This is reasonably well matched to the field strength limit on $\bar{B}$ and will be used in our sample designs.

The mean path length of relativistic muons is $L_\mu \simeq 6000\ P_\mu$ (GeV/c) meters or ~110 turns if the ring circumference be L_π. To separate π decay ν_μ's from μ decay neutrinos we require that injection occur within a fraction of this lifetime, say $\lesssim 10$ turns of the μ ring. The proton synchrotron is ~10 times as large as the storage ring so this requirement is naturally fulfilled by single turn injection.

ν beam intensity depends directly on the straight section length S, which should be chosen large relative to R. We will typically choose $S \simeq 4R$. The ring should be designed with a large transverse and momentum acceptance. This seems to require a strong focusing ring lattice (FODO) with a relatively short FODO period.

Sample μ-Storage Ring Designs

In this section we present possible parameters for μ rings with $E_\mu = 8$, 4.5, 1.5 GeV. The general ring design is a FODO lattice and we assume that beam acceptances are dominated by the lattice parameters. In designing the lattices we follow general design constraints outlined elsewhere by Collins.[6] For the 4.5 and 8 GeV rings we will use precooler magnets.[2] For the 1.5 GeV ring these magnets are shortened.

As noted by Collins,[6] there is a broad optimum in lattice design at ~90° particle phase shift per FODO period, and we have conservatively chosen maximum fields of ~1 T. The aperture limitations are set by the maximum beta function value $\beta_{max} \cong 3.4\ L_{FO}$ and the maximum dispersion function $\eta_{max} \cong 2.7\ L_{FO}\cdot\theta_0$. For the 8 GeV case we use one quad and two dipoles per half period. For the 4.5 GeV design we use one precooler quad and one bend per half period (2.2 m). For the 1.5 GeV case we scale to a 1 m length, envisaging a bending magnet of ~.7 m length and a .3 m quad. Acceptance parameters are shown in Table 1.

To calculate neutrino beam intensities we have used the formula of Wang[3] to calculate pion production. We have assumed transverse acceptance is dominated by the lens aperture immediately following the target, which we assume to be a Li lens with the precooler design parameters. This sets the acceptance angle in Wang's formula:[3]

$$\frac{d^2N_\pi}{d\ P_\pi\ d\Omega} = A\ P_p\ x\ (1-x)\ e^{-Bx^C\ -DP_\pi\theta}$$

$$\frac{\text{pions}}{\text{GeV/c - interacting proton}}$$

with A = 1.57, B = 5.73, C = 1.33, D = 4.25, P_p the incident proton momentum, P_π the product pion momentum, $x = P_\pi/P_p$ and θ is the production angle with θ_{max} = 25 mR at 8 GeV P_π/c and 50 mR at 4.5 GeV, 100 mR at 1.5 GeV.

The acceptance with stochastic injection is limited by the necessity of separate orbits for both injected pions and circulating muons. We assume the momentum acceptance for both orbits is given by momentum half-width $\Delta p/p$.

We finally obtain the number of ν_e and $\bar{\nu}_\mu$ per proton on target by multiplying by the target efficiency (0.4), a π-decay efficiency (0.75) and μ decay efficiency (0.3 per beam line). The results are shown in Table 2. We have considered the possibility of 80 GeV, 30 GeV and 8 GeV proton beams. We have also calculated the number of ν events per day which would be observed by a 200 ton detector 2 km from the μ-ring.

In a two component ν model, Eq. (1) for the ν oscillation probability can be written as:

$$P_{1\to 2} = 1 - \sin^2\alpha\ \sin^2\ \Delta_{12}.$$

In Figure 3 we indicate limits on values of $\sin^2\alpha$ and Δ_{12} that can be measured by a simple six month event counting experiment by the precooler and by a new storage ring, and compare with oscillations measured by Reines et al.[7] Both cases can provide useful measurements, competitive with other existing and future ν experiments. Greater precision will be pos- if the final state leptons be identified.

Discussion of the Test Cases

The above design cases are very conservative possibilities which have not been optimized for maximum neutrino production. Some possible improvements are:

1. An improved target lens with greater acceptance than the precooler lens is probably possible.
2. Higher field quads and somewhat shorter FODO

periods are probably possible.

3. Protons on target can probably be increased from ~10^{18}/day to 2-4x10^{18}/day at some accelerators.

4. Using positives ($\pi^+ \rightarrow \mu^+ \bar{\nu}_\mu \rightarrow e^+ \nu_e \nu_\mu$) will double flux.

5. A larger detector is also possible.

Of the cases considered, the 4.5 GeV is preferred, provided a 30-80 GeV proton accelerator is available, and can provide ~10 times more ν's per proton pulse than the parasitic precooler use of reference 1. The ~1.5 GeV storage ring has nearly as great a ν-oscillation sensitivity, and is half as long and therefore half the expense. The 8-GeV Fermilab Booster may be available as an injector. This, or a still lower energy, storage ring may be a more practical possibility.

Summary

We have considered the possibility of constructing a muon storage ring to be used as a source of ν_e and $\bar{\nu}_\mu$ beams for neutrino oscillation experiments. We have found it possible to measure oscillations with $M_{ij}^2 \gtrsim 0.1$ (eV)^2. We summarize some of the major advantages of a μ storage ring as a ν source:

1. The decaying muons can be monitored so that the neutrino intensities can be precisely known.
2. There are no impurities from π-decay, K-decay or stray charged particles.
3. The beam pulse is localized in time to a muon lifetime after the p pulse, so cosmic rays and other noise effects can be removed.

Both oscillations of the "first kind" ($\nu_e \rightarrow \nu_\mu, \nu_\tau$) and of the "second kind" ($\nu_e \rightarrow M$ where M is a "moron", a non-interacting particle) are observable.

Acknowledgements

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References

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Table II

ν FLUXES IN SAMPLE CASES

ν ring energy	8 GeV	4.5	4.5	1.5	1.5
Proton injector energy	80 GeV	80	30	30	8
Angle Acceptance	25 mR	50	50	100	100
$\frac{dN_\pi}{dp}$ (pions/Gev/c -proton)	3.9×10^{-3}	2.2×10^{-2}	1.4×10^{-2}	1.7×10^{-2}	0.9×10^{-2}
ν_e and $\bar{\nu}_\mu$ per 3×10^{13} proton pulse per beam line	6×10^{8}	5.7×10^{9}	3.6×10^{9}	3.9×10^{9}	1.8×10^{9}
Events per day (10^{18} p/day, 200 ton detector, L = 2 km)	30	54	34	1.3	0.6

Table I

SAMPLE MUON STORAGE RINGS

Parameter	Symbol	8 GeV Ring	4.5 GeV Ring	1.5 GeV Ring
FODO Half Cell Length	L_{FO}	3.8 m	2.2	1.0
Bend Per Half Cell	θ	0.1 R	0.1	0.1
Maximum Beta-Function Value	β_{max}	13.0 m	7.5	3.4
Magnet Aperture (~1 T field)	a_{max}	4.8 cm	4.8	4.0
Maximum Dispersion Function	η_{max}	1.03 m	0.60	0.27
Momentum Acceptance $\frac{\Delta p}{p} \doteq .75 \frac{a_{max}}{\eta_{max}}$	$\pm\frac{\Delta p}{p}$	$\pm$3.5%	$\pm$6.0	$\pm$10
Ring Circumference	$2(\pi R + S) \doteq 5\pi R$	600 m	350	160

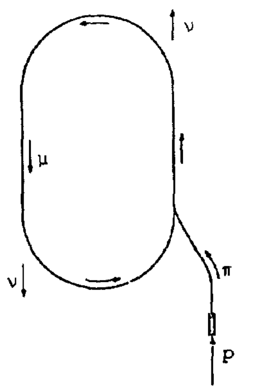

Fig. 1A A μ storage ring

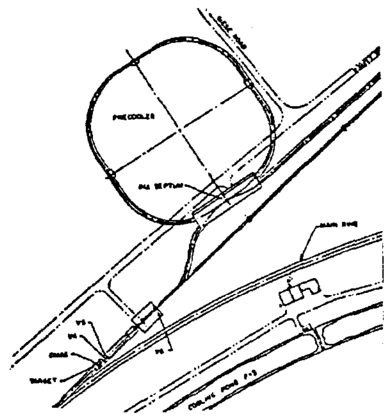

Fig. 1B The $\bar{p}$ precooler/ μ storage ring

FIGURE 2A "STOCHASTIC INJECTION" WITH NON-CIRCULATING π BEAM.

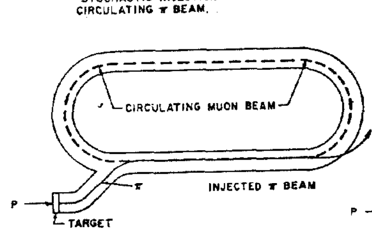

FIGURE 2B "STOCHASTIC INJECTION" WITH SEPARATE π AND μ ORBITS.

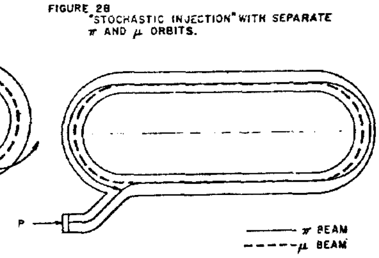

Fig. 3: