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THE MULTIPACTOR EFFECT

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The present state of knowledge of multipactoring (secondary electron multiplier action) is limited to a considerable extent by experimental difficulties of obtaining data coherent with analyses.

This report presents a brief analysis of 'classical' multipactoring and of the anomalous single surface multipactor effect. The more difficult case of multipactoring in a microwave cavity is discussed and a specific case (TM-mode) is analyzed.

Secondary electron multiplier action (multipactoring) may exist between two opposed surfaces when a cyclic voltage is maintained between them. If a primary electron impinges on one of the two opposed surfaces, causing the emission of one or more secondary electrons, a sustained exchange of current will occur if at about the moment of emission the applied electric field reverses and accelerates the secondaries to the opposing surface where a similar circumstance occurs. It is only necessary that the primary electron energy at impingment results in a secondary emission coefficient (SEC) greater than unity.

Once initiated for whatever reason (field emission, ionization of residual gas, etc.) a multipactor 'discharge' then consists of a thin electron cloud that is driven back-and-forth between the opposed surfaces (or gap) in response to the applied RF field. The exchange current in each half-cycle will increase to a limit set by the perveance of the gap, reaching steady state in a number of half-cycles set by the secondary emission coefficient. This perveance limit may be interpreted that as the electron density increases mutual repulsion causes some electrons to fall out of step with the applied field thereby limiting the maximum electron density in the exchange cloud. (1)

The appropriate condition necessary to produce multipactoring may be derived from the equation of motion; the cyclic field in the gap $E = (V_0/d)\sin\omega t$,

$$\frac{d^2x}{dt^2} = \frac{eV_0}{md}\sin(\omega t+\varphi) \qquad (1)$$

where φ is the field phase at electron emission. Integrating, with $dx/dt = 0$, $t = 0$;

$$\frac{dx}{dt} = \frac{eV_0}{\omega md}\left(\cos\varphi - \cos(\omega t+\varphi)\right) \qquad (2)$$

Reintegrating, where $x = 0$, $t = 0$;

$$x = \frac{eV_0}{\omega^2 md}\left(\omega t\cos\varphi - \sin(\omega t+\varphi)+\sin\varphi\right) \qquad (3)$$

To achieve the condition stated, that the field reverses so as to produce a repetitive situation, the transit time must be an odd number of half-cycles, whence $\omega t = (2n+1)$ so that Eq(3) becomes

$$= \frac{eV_0}{\omega^2 md}\left((2n+1)\pi\cos\varphi + 2\sin\varphi\right) \qquad (4)$$

Further, for physical realism we must eliminate particles with initial or final retrograde motions, that is, to motion within the range $0 < x < d$. This latter condition restricts the phase span in Eq(4) to the range

$$0 \le \varphi \le \arctan\frac{2}{(2n+1)\pi} \qquad (5)$$

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Thus, the ballistic condition for producing multipactoring may be stated (in dimensionless form),

$$\left(\frac{d}{\lambda/2}\right)^2 = \frac{eV_0}{mc^2}\left(\frac{(2n+1)\pi\cos\varphi+2\sin\varphi}{\pi^2}\right) \qquad (6)$$

where φ is limited to the range indicated in Eq(5).

If one assumes an emission velocity for the secondary electrons, although a high SEC implies a low emission velocity, an appropriate specification of the boundary conditions will result in a slight change in Eq(6); the cosinusoidal coefficient $(2n+1)\pi$ becomes

$$\frac{k-1}{k+1}(2n+1)\pi$$

where $k = v_i/v$, the ratio of the primary impact velocity to the secondary emission velocity. The phase range, Eq(5), also becomes

$$0 \le \varphi \le \arctan\frac{2}{\frac{k-1}{k+1}(2n+1)\pi}$$

An extensive experimental investigation by Hatch and Williams has indicated good agreement with the above theory. (2) Obviously a good vacuum is essential to realization of the above theory since the effect of gas scattering was not included in the above analysis. (3)

Table I shows the extent of the phase range of multipactoring for different orders (number of half-cycles involved in the transit time).

TABLE I

n	phase range
0	32.48 deg
1	11.98
2	7.26
3	5.20
4	4.05
5	3.31
6	2.80

Clearly, this phase range may be interpreted as a voltage range (for a fixed geometry, or gap spacing) permitting multipactoring, or conversely the range of gap spacing for a fixed voltage. These regimes are plotted in Fig. 1.

When the primary electron strikes the opposing surface there are a number of competing processes through which it may become depleted of its incident energy. In the case of secondary emission clearly the incident energy must be greater than the work function for emission and less than that will result in energy loss principally by radiation and forward scattering deeper into the target material. The energy band, in order to eject secondaries is understandably characteristic of the material.[4] The energy of the primary at arrival on the secondary surface is given by Eq(2), where velocity is expressed as a corresponding voltage

$$\frac{V}{V_0} = \frac{1}{2\pi^2}\frac{eV_0}{mc^2}\left(\frac{\lambda}{d}\cos\varphi\right)^2 \qquad (7)$$

Examination of this relation shows that for the multipactor condition the arrival energy at the secondary surface varies from about .35 to .65 V_0 which indicates that multipactoring only occurs at low gap voltages (and small gap spacings) since the energy band of multipactoring occurs in the range of a few hundred volts. However, it is difficult to avoid this region during the transient response to a step function of input power.

Omitted in the above analysis is the effect of an applied magnetic field. In devices intended to produce a specified result it is usually desirable to

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straighten or contain the beam produced; the result is profoundly affected by the magnetic field applied. On the other hand, it has been shown in a special case that the SEC varied inversely with the cosine of the angle of incidence,

$$S(\vartheta) = S/\cos\vartheta \qquad (8)$$

Where S is the SEC for normal incidence ($\vartheta = 0$) (5). Hence, a cross field would greatly enhance secondary emission where that effect were desired.

Although multipactoring has been a shibboleth of microwave engineers for some time (6), the phenomenon is relatively rare; moreover, there have been several proposals to use this phenomenon as the operating principle of microwave devices (7). One of the principal defects of multipactor devices is pulse start jitter, owing, presumably, to the random nature of the initiating process. This imperfection can undoubtedly be remedied by imbedding a small piece of thorium or thoriated tungsten in one of the emission surfaces to provide starting electrons (as is done in heli-arc welding rods).

There is another sort of multipactoring which occurs under the condition where a cyclic field is maintained in, say, an evacuated cavity and an electron, emitted from a surface for whatever reason, is accelerated away from the emitting surface, but, upon field reversal is stopped and returned to the same surface, where on impact it causes the production of one or more secondary electrons. The equations of motion are precisely as stated earlier, Eq.(1), (2), and (3). But, to achieve a recurring phenomenon the transit time in orbit must be an integral number of cycles, $\omega t = n2\pi$, so that Eq.(3) becomes

$$x = \frac{eE_o}{\omega^2 m} \; 2\pi n \cos\varphi \qquad (9)$$

In this case, each orbit must begin when the electric field is nearly zero and end on the same surface after n RF cycles. As a result, in this case the particle will impact with the same energy as it departed (ie., essentially none) so that secondary production is unlikely. However, in reality the existence of an RF electric field in a cavity implies a magnetic field component which will affect the trajectory of the particle. For example, a cylindrical cavity supporting the TM-010 mode will have a circumferential magnetic field in quadrature with the electrid field, causing particles to move radially inward (or outward) depending upon the phase of the field.

In the microwave cavity the equations of motion are more complicated than in the simple condenser case discussed above. As each case depends upon the description of the oscillatory mode, only the TM-010 mode will be considered further; in this case the field is

$$\begin{aligned} &E = E_o J_o(kr)\sin\omega t \\ &H = (E_o/\eta) J_1(kr)\cos\omega t \\ &k = p_{01}/a \qquad J_o(p_{01}) = 0 \\ &E_o = V/h \qquad V^2 = 2RP \end{aligned} \qquad (10)$$

The non-relativistic electromagnetic equations of motion in cylindrical coordinates are

$$\begin{aligned} \frac{d^2z}{dt^2} &= \frac{e}{m}\left(E_z + \frac{dr}{dt}B_\vartheta - r\frac{d\vartheta}{dt}B_r\right) \\ \frac{d^2r}{dt^2} &= \frac{e}{m}\left(E_r + r\frac{d\vartheta}{dt}B_z - \frac{dz}{dt}B_\vartheta\right) + r\left(\frac{d\vartheta}{dt}\right)^2 \qquad (11) \\ \frac{d}{dt}\left(r^2\frac{d\vartheta}{dt}\right) &= \frac{e}{m}\left(E_\vartheta + \frac{dz}{dt}B_r - \frac{dr}{dt}B_z\right) \end{aligned}$$

Hence, where η is the impedance of free space and $\mu/\eta = 1/c$, (12)

$$\begin{aligned} \frac{d^2z}{dt^2} &= \frac{eE_o}{m}\left(J_o(kr)\sin\omega t + \frac{1}{c}\frac{dr}{dt}J_1(kr)\cos\omega t\right) \\ \frac{d^2r}{dt^2} &= \frac{-eE_o}{m}\left(\frac{1}{c}\frac{dz}{dt}J_1(kr)\cos\omega t\right) + r\left(\frac{d\vartheta}{dt}\right)^2 \end{aligned}$$

It is necessary, in order to produce single-surface multipactoring even if the particle returns to the same location where it was emitted, to have a non-conservative orbit; that is, there is an energy gain in transit. In any case, it is evident that the particle though it moves radially will not move far, but this will be sufficient to produce a different (greater) line integral on the return trip compared to the outward trip.

Integrating Eqs. (12), noting that $k/\omega = 1/c$, and with initial conditions $dx/dt = 0$, $d\vartheta/dt = 0$, $dr/dt = 0$, $t = 0$,

$$\frac{dz}{dt} = \frac{eE_o}{\omega m} J_o(kr)\left(\cos\varphi - \cos(\omega t + \varphi)\right) \qquad (13)$$

$$\int \frac{d\left(\frac{dr}{dt}\right)}{J_o(kr)J_1(kr)} = \left(\frac{eE_o}{\omega m}\right)^2 \frac{k}{c}\left(\cos\varphi\sin(\omega t + \varphi) + \frac{\sin 2(\omega t+\varphi)}{4} + \frac{(\omega t + \varphi)}{2}\right)$$

The radial equation is so hopelessly complicated that even if it were possible to integrate one could not foreseeably use the solution to gain an insight into details of particle trajectories over the surface of the cavity. Anticipating this, a multipactoring simulation computer program has been written for this mode, by means of which hypothetical particle orbits originating anywhere on the surface may be traced to impact (8). It may, of course, be easily determined then for a given SEC if the effect is self-quenching (by particles walking out of the originating region) or self-sustaining. A summary of results, presented by C. Lyneis et al. has shown the existence of a self-sustaining secondary emission on the side wall of a particular superconducting cavity geometry. The calculations are supported by some experimental corroboration.

While some supposed multipactoring has occurred in ambient temperature linacs (6), the problem has become acute in the superconducting cavity case (9). The several proposals to supress multipactoring include surface anodizing, vacuum firing, electro-polishing, modification of the cavity geometry to eliminate conditions which will enhance or support cyclic secondary emission and/or acceptance of the field gradient below which multipactoring will not occur (10). Another possibility, where it is applicable, is DC bias of the emitting surface with respect to an opposing surface.

This effect is also said to be remedied by RF conditioning of the cavity which presumably volatilizes surface contamination (11). However, those working with multipactoring have precisely the opposite opinion; that is, a fresh surface for which SEC is measured deteriorates when contaminated. Apparently the situation is more involved than is thought; "the many ramifications of multipacting are only now being discovered and explored" (12).

Historically, the phenomenon of multipactoring was apparently first described by Philo Farnsworth (13) and subsequently investigated, particularly for the condenser case, by a number of writers. This literature, while not particularly applicable to the contemporary problem, is quoted as being a summary of the work done in this area to date (14).

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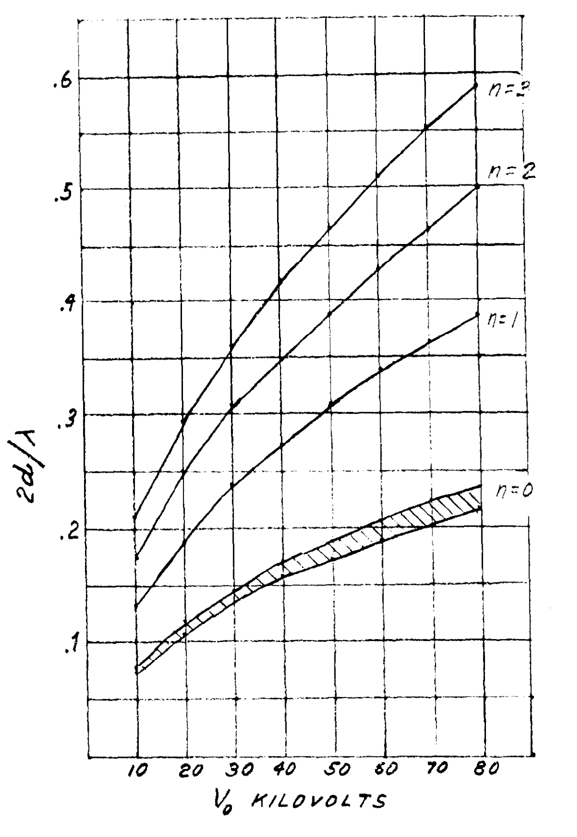

Fig. (1) Multipactor regimes from Eqs. (5) and (6).