IEEE Transactions on Nuclear Science, Vol. NS-26, No. 3, June 1979

Q-MEASUREMENTS OF NIOBIUM SUPERCONDUCTING CAVITIES

V.V. Androsov, V.M. Svetlov, and I.S. Shchedrin
Moscow Engineering Physics Institute

The possibility of getting high electrical fields in superconducting microwave cavities has stimulated the development of methods of experimental investigation of superconducting cavities. Loaded quality factor measurements in the range of values exceeding 10^5 require special instruments. So the development of an instrument for superconducting cavities quality factor measurements with automatic readings appears to be essential.

The instrument's principle of operation is based on the scheme which analyzes the transient response of a cavity, i.e., the decrement method.[1]

Two ways of cavity exitation have been used; that of slow sweeping, and that of fast sweeping of the master-generator frequency. In the first one[2] the signal-generator is switched off by a modulating pulse when signal frequency coincides with the resonant frequency of a cavity. The second state is used for loosening of requirements to the master-generator stability.[3] In the range of values 10^5-10^7, Q-factors have been measured with an oscilloscope (slow sweeping methods). Calculations have been carried out according to the following equation:[4]

$$Q_L = \pi f_o \frac{\tau}{\ln \frac{V_o}{V_1}} \qquad (1)$$

where Q_L = loaded quality factor of a cavity;
V_o = signal voltage amplitude;
f_o = resonant frequency of a cavity;
V_1 = the chosen voltage level; and
τ = the time interval corresponding to V_1.

The fast sweeping method has been used for higher values of Q_L measurements ($Q_L > 10^7$). The output signal was registered on the oscilloscope screen and the automatic reading was indicated on the measuring instrument scale. The decremental signal can be expressed as

$$V(t) = V_o e^{-\frac{\omega_o}{2Q_L}\cdot t} \quad . \qquad (2)$$

By integrating Eq. (2) and multiplying the integral by $\frac{\omega_o}{2V_o}$, one can get Q_L value of a cavity

$$Q_L = \frac{\omega_o}{2V_o} \int_{t_o}^{\infty} V(t)dt \quad . \qquad (3)$$

In practice the integral is being taken over the finite time interval. The latter is chosen on the basis of required accuracy.

The block diagram of the instrument for superconducting cavities Q-measurement is shown in Fig. 1. In Fig. 2 the photograph of the instrumentation is shown.

The main parameters of the system are listed below:

measurement range: $Q_L = 10^5$-10^9;
frequency range of the signal generator: 2400-3300 MHz;
power output of the signal generator: up to 70 mW;
power level at the VHF amplifier output at $f = (2783 \pm 3)$ MHz: up to 20 W.

The measurement errors are: less than 11% in the Q range 10^5-10^7, and less than 5% in the Q range 10^7-10^9.

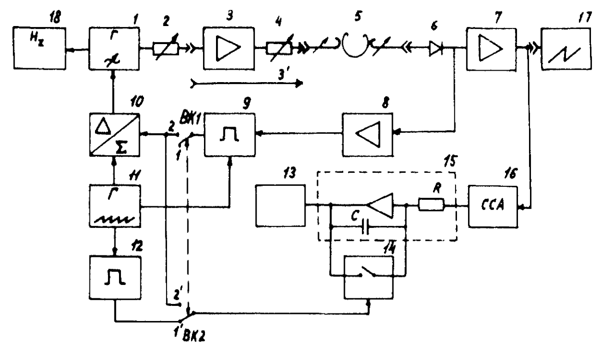

Fig. 1. Block diagram of the instrumentation.
1 - microwave signal-generator; 2,4 - attenuators; 3 - microwave amplifier; 5 - superconducting cavity under investigation; 6 - detector; 7,8 - amplifiers; 9 - trigger; 10 - amplifier-integrator; 11 - sweep-generator; 12 - trigger; 13 - indicator; 14 - discharge element; 15 - integrator; 16 - amplitude stabilizing network; 17 - oscilloscope; 18 - frequency meter.

Fig. 2. Photograph of the instrumentation.

0018-9499/79/0600-4271$00.75 © 1979 IEEE

The calibration of the instrument has been carried out with the help of a reference signal which corresponds to $Q = 10^7$ at $f = 2783$ MHz.

Consider the problem of power transmission through a cavity with two couplings. The equivalent circuit for such a cavity is shown in Fig. 3. In most practical cases signal-generators and loads are well matched to the cavities or there are isolators at the input and output of a cavity, so R_G is considered to be equal to Z_1, where Z_1 is the characteristic impedance of the input transmission line and $Z_L = Z_2$, where Z_2 is the characteristic impedance of the output transmission line. Power transmission from generator through the cavity into the load can be expressed as[1]

$$K(\omega) = \frac{4\beta_1\beta_2}{(1 + \beta_1 + \beta_2)^2} \cdot \frac{1}{1 + 4Q_L^2\delta^2} , \tag{4}$$

where β_1 is the coupling between the cavity and the generator and β_2 is the coupling between the cavity and the load, Q_L is the loaded quality factor, and δ is the respective frequency shift.

$$\beta_1 = \frac{n_1^2 Z_1}{R} ; \qquad \beta_2 = \frac{n_2^2 Z_2}{R} ;$$

$$\delta = \frac{\Delta\omega}{\omega_o} = \frac{\omega - \omega_o}{\omega_o} ; \qquad Q_L = \frac{Q_o}{1 + \beta_1 + \beta_2} .$$

At resonant frequency ($\omega = \omega_o$) we have

$$K(\omega_o) = \frac{4\beta_1\beta_2}{(1 + \beta_1 + \beta_2)^2} . \tag{5}$$

Coefficient $K(\omega_o)$ is the function of two variables β_1 and β_2. It has maximums at the following conditions:

1) if β_2 = constant, $K(\omega_o)$ has maximum at

$$\beta_1 = 1 + \beta_2 , \tag{6}$$

$$\text{so } K(\omega_o)_{max} = \frac{\beta_2}{1 + \beta_2} = \frac{\beta_2}{\beta_1} . \tag{7}$$

2) if β_1 = constant, $K(\omega_o)$ has maximum at

$$\beta_2 = 1 + \beta_1 \tag{8}$$

$$\text{so } K(\omega_o)_{max} = \frac{\beta_1}{1 + \beta_1} = \frac{\beta_1}{\beta_2} . \tag{9}$$

The fulfillment of the above equations can be interpreted as matching of the cavity as seen from the generator, Eq. (6), and matching of the cavity as seen from the load, Eq. (8).

We now define the equations for reflection coefficients at the input and output of the cavity when $\omega = \omega_o$.

As seen by the generator at $\omega = \omega_o$,

$$\Gamma_1 = \frac{1 + \beta_2 - \beta_1}{1 + \beta_1 + \beta_2} \tag{10}$$

and $\Gamma_1 = 0$ if Eq. (6) is valid.

As seen by the load at $\omega = \omega_o$,

$$\Gamma_2 = \frac{1 + \beta_1 - \beta_2}{1 + \beta_1 + \beta_2} \tag{11}$$

and $\Gamma_2 = 0$, if Eq. (8) is fulfilled. It should be noted that by Eqs. (10) and (11) Q_o and Q_L can be expressed in terms of Γ_1 and Γ_2,

$$Q_o = \frac{2Q_L}{\Gamma_1 + \Gamma_2} . \tag{12}$$

For Eq. (6) and $\Gamma_1 = 0$ one gets

$$Q_o = \frac{2Q_L}{\Gamma_2} . \tag{13}$$

For Eq. (8) and $\Gamma_2 = 0$ one gets

$$Q_o = \frac{2Q_L}{\Gamma_1} . \tag{14}$$

If the second coupling doesn't exist, $\beta_2 = 0$ and $\Gamma_2 = 1$, so Eq. (12) can be rewritten

$$Q_o = \frac{2Q_L}{1 + \Gamma_1} . \tag{15}$$

Equations (12) and (15) are valid for the series equivalent circuit (Fig. 3). The latter corresponds to the reference planes of standing wave mimimums in the transmission lines at cavity detuning. It can be shown that in the case of a parallel scheme, which corresponds to the reference planes of standing wave maximums at cavity detuning, a similar equation can be obtained. For a cavity with two ports

$$Q_o = \frac{2Q_L}{-(\Gamma_1 + \Gamma_2)} , \tag{16}$$

and for a cavity with one port

$$Q_o = \frac{2Q_L}{1 - \Gamma_1} . \tag{17}$$

It should be noted that equations similar to (15) and (17) are obtained in Ref. 5.

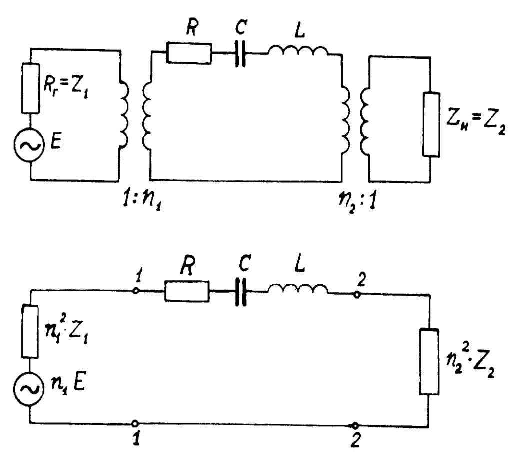

Fig. 3. Equivalent circuits of a two-port cavity.

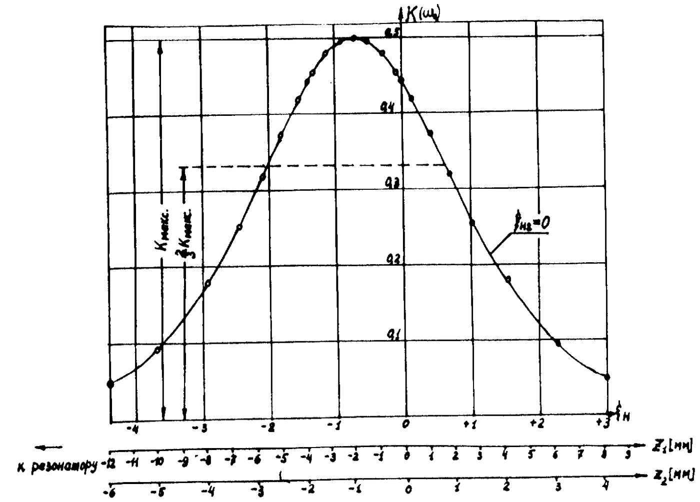

Fig. 4. Power transmission coefficient vs position of coupling loop.

In most cases superconducting cavities are coupled to transmission lines by a movable loop placed in a cutoff circular guide. Consider the case when inside the cutoff guide H_{11}- mode is exited. It can be shown that for practical diameters of the cutoff circular guides and for proper choice of the reference planes ($x_i = 0$ when $\beta_i = 1$), the coupling coefficient can be expressed as

$$\beta_i \simeq e^{-\xi_{Ni}} , \tag{18}$$

where $\xi_{Ni} = 3682 \frac{\xi_i}{\eta}$, $\xi_i = \frac{x_i}{\lambda}$, $\eta = \frac{\tau}{x}$, and τ is the radius of the cutoff circular guide. Taking Eq. (18) into account, it can be seen that Eq. (5) is optimum over the position of one coupling loop at the fixed position of the other coupling loop.

In Fig. 4 power transmission coefficient versus the position of the first loop (ξ_i) at a fixed position of the second loop is shown. In the bottom of this figure two scales of x_i are shown. They correspond to diameters 20 mm and 10 mm of cutoff circular guides. Using this figure one can readjust the loop position for maximum signal. Similar curves can be obtained for other values of β_i (or ξ_{Ni}).

The developed instrumentation enables us to carry out a number of experimental investigations:

- Measure Q_0 and Q_L of a cavity.
- Choose the range of coupling loops adjustment.
- Measure coupling coefficients β_i.
- Test superconducting cavities at different power levels.

For confirmation of the method, Q_0 and Q_L measurements were carried out. The niobium cavity with diameter 81 mm and height 81 mm was exited in E_{101}- mode at the frequency 2783 MHz. The surface of the cavity was not finished. At $\beta_1 = 2$ and $\beta_2 = 1$ the experimental values were $Q_L = 2.6 \times 10^5$ and 1.4×10^6.

Acknowledgments

The authors wish to thank Dr. V. G. Andreeve for the support he gave, which made possible the realization of this work. They particularly appreciate the stimulating discussion with Prof. Valdner, O.A.

References

1. E. L. Ginzton, Microwave Measurements, McGraw-Hill Book Co. (1957).

2. F. F. Mende, V. M. Dmitriev, E. V. Khristenko, and G. E. Churilov, "Measurement of Superconducting Cavities Q-factors", Prib. Tekn. Eksper. (USSR), 4 (1968), p. 134 (in Russian).

3. H. J. Schmitt and H. Zimmer, "Fast Sweep Measurements of Relaxation Times in Superconducting Cavities", IEEE Trans. Microwave Theory Tech. 14 (1966), pp. 206-207.

4. E. I. Kulakov, "Q-measurements of Cavities with Small Losses", Izmerit. Tekn. (USSR), 6 (1959), p. 55 (in Russian).

5. A. N. Didenko, "Superconducting Wave Guides and Cavities", Sovetskoe Radio, Moskva, 1973 (in Russian).