

A COMPUTATIONAL STUDY OF ONE ASPECT OF AUTORESONANT ACCELERATION

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Abstract

Excitation and suppression of zero-frequency cyclotron waves on a relativistic electron beam in a parameter regime appropriate to autoresonant collective ion acceleration are examined through numerical simulation. The excitation mechanism considered is that of anode shorting, while suppression is achieved through a combination of nonadiabatic change in the guide magnetic field and the introduction of a finite radial velocity divergence in the beam. Some brief remarks are included concerning the effect of a scattered beam distribution.

Introduction

Collective acceleration schemes employing large-amplitude traveling slow-cyclotron¹ or Langmuir waves² excited on an intense relativistic electron beam offer the possibility, with present or foreseeable pulsed high voltage technology, of providing significant currents of ions with single particle energies of up to a Bev. In such proposed devices, ions are trapped in the trough of the wave and the ion-wave system is accelerated by suitable spatial^{1,2} or temporal³ variation of system parameters. One such method of collective acceleration which has received particularly thorough theoretical investigation is that employing the principal of autoresonant acceleration,¹ in which ions placed in a slow-cyclotron wave are accelerated by permitting the wave to propagate into a region of spatially decreasing magnetic field. Previous analyses generally have assumed that the electron beam is free of waves other than the slow-cyclotron mode of desired frequency. The presence of such extraneous waves may lead to serious degradation of performance, both through complications of the several steps to which the beam-wave system must be subjected prior to ion-loading and by subsequent detrapping of ions, or, less drastically, the destruction of conditions for optimum phase stability. One such troublesome wave which seems to be particularly ubiquitous is the zero-frequency cyclotron wave, which can be established by static perturbations present in the system. We here present a computer simulation study of excitation and subsequent suppression of this mode.

Investigations of the physics of nonneutral relativistic electron beams generally involve a combination of intense self-fields, inhomogeneity and non-linearity. Any one of these factors can render the application of the methods of classical mathematical physics either difficult or impossible. To meet this problem, one of the present authors⁴ has developed a two-dimensional, electromagnetic, fully relativistic particle simulation code employing a novel Galerkin-type algorithm and capable of treating an arbitrary curvilinear coordinate system. This code has previously been found to be indispensable in theoretical investigations of other phases of the autoresonant acceleration process.^{5,6} Examination of

the results of running this code in configurations relevant to zero-frequency wave excitation and suppression form the core of the present report.

Cold-Beam Results

Although zero-frequency cyclotron waves can conceivably be established by a multitude of static perturbations, here we shall be concerned only with that provided by anode shorting of the beam's self-electric field. A nonneutral electron beam propagating along an external magnetic field is maintained in equilibrium by interplay amongst the forces arising from the external field, the rotation of the beam and its self-electric and -magnetic fields. Upon passing through the metallic anode, the self-electric field of the beam is shorted out, permitting the pinching effect of the self-magnetic field to establish a radial modulation of the beam. Since the conditions at the anode establishing the modulation are stationary, the zero-frequency nature of the disturbance is readily understood. As a cyclotron wave is basically a radial modulation of the beam, the fact that the disturbance is a discrete cyclotron mode might also be thought to be obvious. This, however, is not an a priori evident point. Investigations of the linear theory of a cold, nonneutral, relativistic electron beam⁷ demonstrate that the inhomogeneity of the relativistic gamma factor induced by the self-field of the beam leads to the appearance of a broad band of continua. Without a detailed analysis, one might be tempted to conclude that much of the disturbance would be enveloped by this region of mode space. If this were to prove to be the case, it would be extremely fortunate, for such disturbances phase-mix away in a reasonably short time. Unfortunately, the simulation results exhibited below indicate that the discrete mode also is strongly excited.

Amongst the factors controlling equilibrium, the external magnetic guide field is most readily amenable to experimental control. Accordingly, our investigation of methods of suppression has focused upon finding an appropriate profile for this field.⁸ Examples of the family of profiles employed are depicted in Fig. 1. These fields are simply those generated by two solenoids of differing radii placed end-to-end, supplemented by a flattening magnet. The quantity Γ measures the strength of the flattening and is a critical quantity for effectiveness of suppression. That profile labeled by $\Gamma = 100$ proved to be the best in that regard, and is employed in the simulations described below. In Fig. 1 and throughout this discussion, units natural to the numerical code are employed. Lengths are measured in units of c/ω_b , where ω_b is the plasma frequency of the beam, and magnetic field strengths are measured in terms of gyrofrequency normalized to ω_b . For a 3-MeV, 30-kA electron beam, corresponding to that previously suggested for a proof-of-principle experiment,⁹ the unit of length is, in fact, roughly a centimeter, and the profiles depicted in Fig. 1 correspond approximately to a jump in field strength from 2.5 kg to 3.4 kg.

An initial simulation run was made with the point of inflection near the center of the computational box rather than near the boundary of beam

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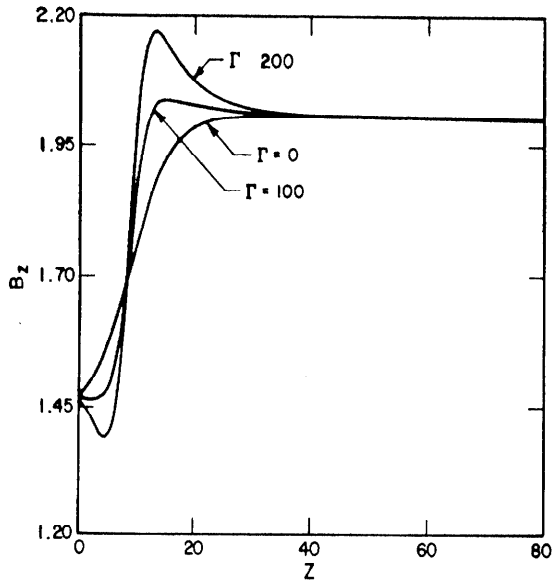


Fig. 1. Magnetic field profiles employed with various values of the flattening factor γ . Units are those described in text.

entrance as depicted in Fig. 1. The information to be gained from this run is three-fold. Firstly, since the first half of the box is filled with an essentially uniform magnetic field, one may readily determine the extent to which the wave is excited. Secondly, by rough measurement of the wavelength, it may be ascertained to what extent the excitation obeys the linear dispersion relation $\omega \cong kv_0 - \Omega/\gamma$.

Lastly, the effectiveness of a more or less random positioning of the point of field inflection can be determined. The results of this run are depicted in the r-z particle plot presented in Fig. 2. Strong excitation is indeed observed with an initial radial modulation of at least 50%. The initial amplitude is in fact so large that particles are initially lost to the outer wall, and it is this effect, rather than any due to the magnetic field, which leads to the subsequent apparent damping. Measurement of crest-to-crest distances shows that the linear dispersion is approximately satisfied. Examination of field plots shows that the excitation is qualitatively similar to the linear mode, a result emphasized earlier for finite-frequency waves. No significant suppression is observable.

Having clearly demonstrated that the excitation of this mode is a problem, we next moved the point of inflection to a point near the point of beam injection. A series of simulations were effected with the beam injected normally to the anode surface as above. Parameters of the profile were varied to obtain optimum suppression. No strong dependence on the detailed positioning of the point of inflection was observed, but, as noted previously, that profile with $\Gamma = 100$ seems to yield best results. Even with this optimum case, however, radial modulations of 25-30% were observed, clearly an unacceptable result.

In the actual proof-of-principle experiment, the diode is so configured that the electron beam emerges in a diverging cone such that particles a finite distance r from the center of the beam have positive radial momentum, given by $P_r = P_0 r \sin \alpha / R_0$, where P_0 is the initial momentum of the beam particles, R_0

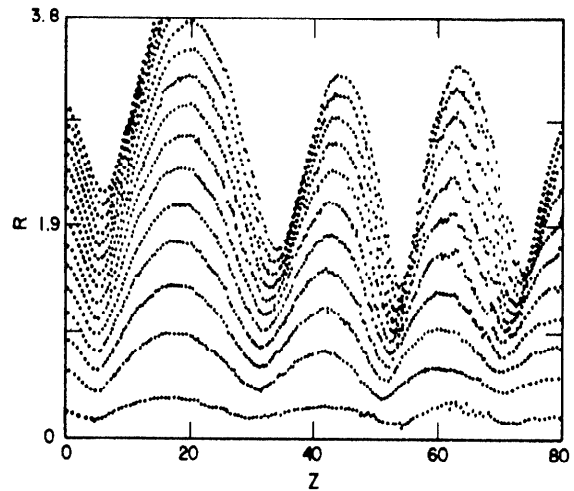


Fig. 2. R-Z particle plots for normal injection without effective suppression.

is the initial beam radius and α is the angle of divergence. Such divergence should aid in suppression, since the positive radial momentum serves to counteract the pinching effect of the self-magnetic field. To determine the extent to which this is the case, we made a number of simulation runs in which α was varied from 4° to 13° . A minimum of residual modulation was observed in the range 9° - 10° . An r-z particle plot taken from a case with $\alpha = 9.5^\circ$ is shown in Fig. 3. Here $\delta r/R_0 \cong 0.046$. A useful overview of this series of simulations may be obtained by plotting against α the root-mean-square of the radial velocity of all the particles, averaged over the entire box, at a point when full current is achieved at the end of a run. Such a plot is presented in Fig. 4. The effect of divergence is decisive.

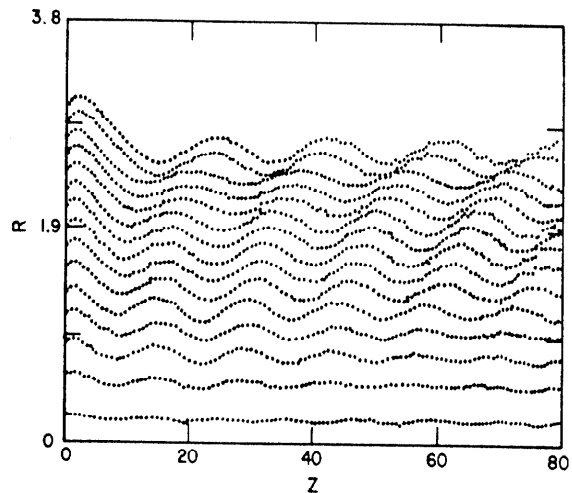


Fig. 3. R-Z particle plots for beam injection with $\alpha = 9.5^\circ$.

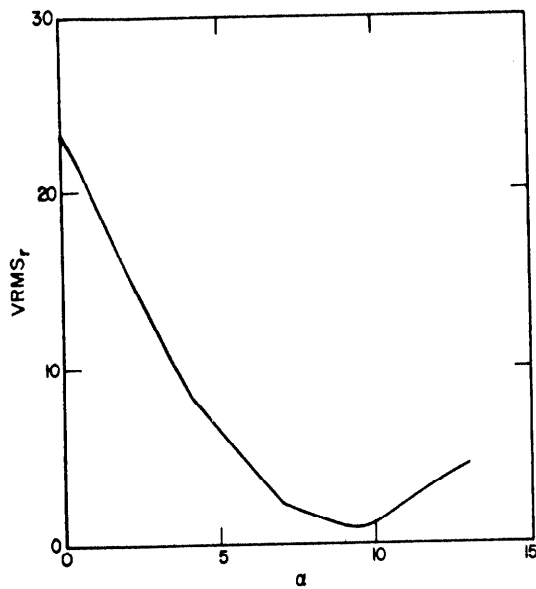


Fig. 4. Summary of simulations of zero-frequency cyclotron wave suppression in cold electron beams.

Scattered Beam Results

As part of our continuing numerical investigation of autoresonant acceleration, we have initiated a study of the effect of a scattered beam distribution on various phases of this process. Only preliminary results are available, but we can already make some qualitative statements with regard to excitation of zero-frequency cyclotron waves. Firstly, for the case of normal beam injection, no dramatic decrease in the initial amplitude of the excitation was observed, although some such effect was apparently present, since proportionately fewer particles were lost from the box. This was observed to be the case, even though the relatively large mean angle of scattering of 12° was employed. This observation perhaps renders ambiguous conclusions regarding correlations between the presence of zero-frequency cyclotron waves and the absence of significant beam scattering. However, upon doubling the length of the computational box, some phase-mixing was observed for such a large angle of beam scatter. Lastly, no significant changes were observed for a scattered diverging beam.

Should the residual modulation of 4.5% obtained above prove prohibitive for practical achievement of autoresonant acceleration, additional measurements such as further electrode shaping,¹⁰ parametric de-excitation,¹¹ or running in foilless diode mode will have to be examined. Need for such additional

efforts can be ascertained numerically only through further self-consistent calculations in which the effects of zero-frequency cyclotron waves on other phases of the autoresonant acceleration process are determined.

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