

COLLECTIVE ION ACCELERATION WITH AN INTENSE REB IN A PERIODIC WAVEGUIDE

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Abstract

A scheme to collectively accelerate ions from 1.0 MeV to 30 MeV using a higher order spatial harmonic of a variable period slow wave structure is described.

Introduction

According to recent calculations it may be possible to collectively accelerate ions in the potential well of space charge waves to GeV energies with field strengths as large as 1 MeV/cm using a converging guide accelerator.¹

However, it is difficult to obtain very low initial phase velocities in such an accelerator, unless the current is very close to the limiting current for beam propagation. Hence, the converging guide accelerator requires an ion injection energy in excess of 30 MeV. In this paper the acceleration of ions from 1.0 MeV to 30 MeV using the $n = 1$ spatial harmonic of a variable length periodic slow wave structure is discussed. This accelerating configuration utilizes a periodically loaded waveguide to effectively reduce the phase velocity of a negative energy space charge wave. In the absence of the periodic structure the phase velocity of the space charge wave is close to the electron beam velocity. The periodic structure, however, "rectifies" the wave thus producing spatial harmonics. The first harmonic has a phase velocity of $v_{ph,1} = \omega / (k + 2\pi/L(z))$ where ω and k are the frequency and wave number of the fundamental wave and L is the period associated with the structure. For L sufficiently small the phase velocity, $v_{ph,1}$, can be made equal to the velocity of the background ions, thus trapping the ions. The trapped ions are now gradually accelerated by slowly increasing the period as a function of axial distance down the guide.

The relativistic electron beam excites and provides the energy to the waves which accelerates the ions. The zero order potential well due to the space charge provides radial confinement of the ions. The periodic wall accelerator can be designed so that protons of 1.0 MeV from an ion diode connected to a pulse line driven by a Marx generator are in resonance with the $n = 1$ spatial harmonic only. Hence, the ions see the other harmonics as rapidly varying fields.

In this paper the results from calculations of the dispersion relation, limiting current, maximum accelerating field and a discussion of an experimental configuration are presented.

Analysis of Periodic Wall Accelerator

The slow wave structure is shown in Fig. 1; in this model the electron beam is assumed to be infinitesimally thin, $\Delta x \rightarrow 0$, and constrained to move solely in the axial direction. All wave and beam quantities are taken to be independent of the spatial

variable y . We further assume that the wavelength of the fundamental wave on the beam is much greater than the opening of the slot, l_0 . The fields within the slots are approximated to be independent of z . From Maxwell's equations and the equation of motion we can determine the space and time dependence of the z component of the electric field in the three regions of figure 1 and the axial linear response current of the beam. Applying the boundary conditions we find the dispersion relation and the ratio of the field amplitudes. The dispersion relation for the system depicted in Fig. 1 is

$$\cot\left(\frac{\omega}{c} x_{21}\right) = \sum_{n=-\infty}^{\infty} \frac{\omega/c}{q_n} \alpha_n^2$$

$$\frac{\cosh(q_n X_1) - \Delta X q_n (p_n/q_n)^2 \sinh(q_n X_0) \cosh(q_n X_{10})}{\sinh(q_n X_1) - \Delta X q_n (p_n/q_n)^2 \sinh(q_n X_0) \sinh(q_n X_{10})} \quad (1)$$

where $\alpha_n^2 = (l_0/L) (\sin(k l_0/2) / (k_n l_0/2))^2$; $n = 0, \pm 1, \pm 2, \dots$; $k_m = k + 2m/L$, k is the fundamental wave number; $q_n^2 = k_n^2 - \omega^2/c^2$; $p_n^2 = -q_n^2 [1 - (\omega_b^2/\gamma_e^3) / (\omega - v_e k_n)^2]$; $\omega_b^2 = 4\pi |e|^2 N_0/M_0$, N_0 is the local beam density; $\gamma_e = (1 - \beta_e^2)^{-1/2}$, $\beta_e = v_e/c$.

To write the dispersion relation in a manageable form we make some simplifying assumptions. If we consider wavelengths much greater than the periodicity, i.e. $k \ll 2\pi/L$. This allows to neglect all but the $n = 0$ term on the right hand side of Eq. (1). This result can be used as an iteration to find $k_1 = k + 2\pi/L$.

It can be shown from limiting current arguments that

$$\Delta X X_0 \omega_b^2 / \gamma_e^3 c^2 = \xi X_2 / X_{20} \quad (2)$$

where $\xi = R \beta_e^2 (\beta_L \gamma_L / \beta_e \gamma_e)^3 \leq 1$, $R = I_e / I_L$ is the ratio of the beam current to the limiting current, $X_{20} = X_2 - X_0$ and $\gamma_L = (1 - \beta_L^2)^{-1/2}$ is the limiting beam gamma.

Using Eq. (2) and the low frequency approximations, i.e. $|q X_1| \ll 1$, $\omega X_{21}/c \ll 1$, $k l_0/2 \ll 1$. The dispersion relation can be written as

$$(\delta k - \delta k_1)(\delta k - \delta k_2) = -\delta k_1 \delta k_2 X_0 / X_{10} \quad (3)$$

where $\delta k = k - \omega/v_e$, $\delta k_2 = \frac{1}{2} \frac{l_0}{L} \frac{\omega}{c} \frac{X_{21}}{X_1}$ and $\delta k_1 = 2\xi \frac{\omega}{c} \frac{X_2}{X_{20}} \frac{X_{10}}{X_1}$. The dispersion relation in (3) is valid in the limit $v_e \approx c$, $|\delta k| \gg \omega / (2c\gamma_e^2)$ and $\omega_b^2 / \gamma_e^3 (\omega - v_e k)^2 \gg 1$.

With the above approximations we find the ratio of the first order harmonic to the zero order harmonic to be

$$\frac{E_{z,1}}{E_{z,0}} \approx \frac{X_1}{X_0} \frac{\sinh(2\pi X_0/L)}{\sinh(2\pi X_1/L)} \frac{\sin(\pi l_0/L)}{\pi l_0/L} (\delta k - \delta k_1) / \delta k \quad (4)$$

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The maximum accelerating electric field is reached when the electrons of the beam begin to trap in the fundamental ($n = 0$) electric field component of the wave. The onset of trapping occurs when the axial electric field reaches the value

$$E_{z,0} = \gamma_e^3 (\omega - v_e k)^2 \frac{M_0}{|e|k} \approx \gamma_e^3 \frac{\delta k^2 M_0 c^2}{|e|k} \quad (5)$$

As an example let us consider the parameter regime $\delta k_1 \ll \delta k_2$. This decouples the slow beam and electromagnetic mode, the dispersion relation then becomes

$$\delta k = \delta k_1 X_1 / X_{10} \quad (6)$$

where ξ must satisfy $X_{20} / 4\gamma_e X_2 \ll \xi \ll$

$\xi X_{21} X_{20} / 4L X_1 X_2$. Then Eqs. (4) and (5) become

$$\frac{E_{z,1}}{E_{z,0}} = \frac{\sin(\pi \ell_0 / L)}{(\pi \ell_0 / L)} \frac{\sinh(2\pi X_0 / L)}{\sinh(2\pi X_1 / L)} \quad (7)$$

and

$$E_{z,0} = 4\gamma_e^3 \xi^2 \frac{(\omega/c)^2 X_2}{X_{20}} \frac{M_0 c^2}{|e|} \quad (8)$$

As an example let $\ell_0 / L = \frac{1}{2}$, $X_0 = 1$ cm, $X_1 = 1.2$ cm, $X_2 = 3.0$ cm and $X_{20} = 2$, these values give $E_{z,1} / E_{z,0} = 0.28$. The maximum field due to trapping for $\gamma_e = 5$, $L = 1.5$ cm, $\xi = 1/10$, $v_{ph} / c = 1/20$ is ~ 1.2 MeV/cm. Hence, the accelerating wave has the magnitude, $E_{z,1} = 0.33$ MeV/cm.

The calculations have been carried out in cartesian coordinates; however, the results are approximately correct for a cylindrical system with a small aspect ratio. To accelerate ions from 1.0 MeV to 30 MeV in resonance with the first spatial harmonic using a frequency of 1 GHz requires an initial period of 1.5 cm and a final period of 8.0 cm. The rate of change of the structure and the total length are determined by the maximum E_z field where

the ions are trapped. As an example the accelerating field as determined from Eqs. (7) and (8) is about 0.33 MeV/cm and the minimum length would be $\sim 1M$. A more practical limit, based on electron linear accelerators is about 200 KV/cm for the fundamental. This would give a minimum length of $\sim 6M$.

1. P. Sprangle, A. T. Drobot, and W. M. Manheimer, *Phys. Rev. Lett.*, **36**, 1180 (1976).

Varying Period Slow Wave Structure

