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A PHASE MODULATED COLLECTIVE ION ACCELERATOR

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Abstract

The possibility of constructing compact, high-current traveling wave ion accelerators through employment of relativistic electron beam collective modes has been suggested often in recent years. A new mechanism based upon temporal modulation of the beam kinetic energy has been studied by us, using both analytic and numerical tools. Preliminary linear studies for the slow cyclotron beam mode indicate that such a mechanism can yield efficient acceleration to ion velocities of roughly 0.5 c in a simple, straight-walled waveguide. Numerical simulations have been performed to verify the theory and investigate nonlinear wave/particle interactions.

I. Introduction

Employment of natural collective modes of an intense relativistic electron beam is one of the most attractive concepts yet advanced for accelerating ions with strong self-fields of the beam. There are two fundamental requirements for any such collective traveling-wave accelerator. The first is simply that large amplitude waves must be stable and possess long coherence lengths. Numerical simulations have demonstrated the viability of this, at least under idealized conditions, in a variety of configurations.[1-3] Controllability of the wave phase velocity is the other requirement. Again, a number of methods have been proposed for accelerating the self-consistent beam modes.[5-8] We have linearly analyzed one such technique,[8] variation of the relativistic beam energy. This is found to give somewhat more efficient utilization of the wave fields, while simplifying waveguide design and beam equilibrium constraints. Although the previous analysis concentrated on slow cyclotron modes, the concept should also be applicable to space-charge waves. To investigate this mechanism further, accurate two-dimensional equilibria were needed. These fields are easily derived, but incorporation of them into the linear analysis makes the equations intractable. Numerical methods have, therefore, been used to study both nonlinear wave propagation and two-dimensional effects. Before describing simulation results, however, it is illustrative to review the theory.

The cyclotron mode is characterized as an oscillation in the r- and θ-directions. In a relativistic beam, there are two separate cyclotron waves. The slow wave can be accurately modeled by the equation

$$\frac{dp^+}{dt} - i\,\frac{\Omega}{\gamma}\,p^+ \cong \frac{e}{mc}\,\frac{dA^+}{dt} \qquad (1)$$

where $p^+ = \gamma(v_r + iv_\theta)$, $A^+ = A_r + iA_\theta$, $\Omega = eB_0/mc$, and $\gamma \cong [1 - (v_z/c)^2]^{-\frac{1}{2}}$. A similar equation exists for $p^- = \gamma(v_r - iv_\theta)$ but since the phase velocity for this wave is always greater than c, it has no utility for ion acceleration. Eq. (1) can be solved approximately by combining it with the equation for A^+. This has been done but the resulting expression is sufficiently complicated that certain simple physical results are obscured. A cleaner, but still moderately accurate expression can be derived by neglecting the RHS of Eq. (1) and assuming $p^+ = p_0 e^{i\psi(z,t)}$. The phase function can then be solved exactly. For beam energy which is only a function of $\eta = t - z/v_z$, with $(\Omega - v_\theta/r)$ constant in space, ψ can be obtained for arbitrary variations of γ, subject only to boundary conditions. If we impose the condition $\psi|_{z=0} = -\omega_0 t$, $\omega_0 \equiv k_0 v_z - \Omega_0/\gamma_0$, the phase function takes the particularly interesting form

$$\psi(z,t) = k_0 z - \omega_0 t - (\Omega_0/\gamma_0)[1 - \gamma_0/\gamma(\eta)](z/v_z) \; . \qquad (2)$$

The phase velocity is obtained by following the trajectory of a point of constant phase, i.e., $d\psi/dt = 0$. Numerical integration of ensembles of test ions in such traveling fields has indicated that efficient acceleration may be possible up to $v_{ion} \cong 0.5$ c with a variety of beam energy pulse forms.

Equations (1) and (2) have been very useful for studying qualitative features of this collective acceleration mechanism, but they are both one-dimensional. Strong, significant radial inhomogeneities exist in intense, unneutralized beam equilibrium, however. Space-charge fields, for instance, result in large radial variations in γ. For a solid, constant density beam of initial energy, $\varepsilon_0 = mc^2(\gamma_0 - 1)$ and radius, a, in a cylindrical guide of radius, R, we find

$$\gamma(r) = \gamma_0 + \frac{1}{4}\left(\frac{\omega_p}{c}\right)^2 \{r^2 - a^2\,[1 + 2\,\ell n(R/a)]\} \quad . \qquad (3)$$

Inserting an expression such as (3) into (2) gives a radial variation in the phase function which would tend to phase mix the cyclotron wave. This, in fact, occurs for continuum modes excited on the beam. Linear analysis of waves on self-consistent equilibria has been performed numerically, however, with the result that discrete, stable cyclotron waves are known to exist. These latter do not phase mix, at least in the small amplitude limit. Two dimensional effects may still introduce corrections into the dispersion relation, which in turn will modify the phase function. Even if (2) remains valid, it is not á priori obvious what effective value should be used for the quantity, $\Omega_0/v_z\gamma_0$. These problems, as well as questions of excitation efficiency and deleterious large amplitude nonlinearities, have motivated us to employ self-consistent two-dimensional particle simulations.

II. Simulation Results

Excitation of traveling cyclotron waves has been demonstrated in previous calculations.[1] This, though, involves simulation of a beam wave amplifier and is moderately complex. Since linear analysis

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of phase modulated acceleration, moveover, indicates an intrinsic weakening for phase velocities above $v_{ph} \tilde{>} 0.5$ c, we are not interested in "fast" traveling waves. Our initial studies have concentrated on waves with low phase velocities. In this class, it is almost trivial to excite a cyclotron wave with zero frequency. With $\omega = 0$, the phase velocity is also zero, and the wave envelope appears as a steady sausage modulation on the beam. This mode is excited by almost any discontinuity in the waveguide, and in fact, can only be avoided with great care in simulations. It is sufficiently ubiquitous that considerable effort has been made to suppress it.[9] To understand this more clearly, a brief discussion of the mode is appropriate.

Unneutralized beam propagation without radial motion is possible in a cylindrical waveguide if an external guide field, B_z, is imposed. The deconfining self-field, E_r, is almost balanced by a pinching force $v_z B_\theta$. This electric field is due primarily to space-charge while the magnetic one arises from unneutralized current flow. If a B_z field is present, an $\bar{E} \times \bar{B}$ drift in the θ-direction results,

$$v_\theta \cong -(E_r - v_z B_\theta)/B_z \quad , \tag{4}$$

and the radial forces are balanced. When a metallic obstruction such as an anode foil or iris is introduced into the guide, however, the E_r field is shorted, equilibrium is disrupted and the beam begins to pinch. The beam motion can be modeled by letting the grounding field act as a source term, while all other self-fields cancel. Thus, the net field which the beam experiences is

$$\frac{d\tilde{p}_r}{dt} = + \frac{e}{m} \tilde{E}_r(r,z) \quad . \tag{5}$$

For excitation sources such as the anode foil or an iris, the force is highly localized in the axial direction. While it is impulsive in nature, though, the field is still of finite extent, and the magnitude of radial perturbation must be calculated from exact spatial profiles. Since these are decidedly two-dimensional in a cylindrical waveguide, accurate estimates of the zero-frequency cyclotron wave amplitude are elusive. By assuming all quantities vary as $\exp(ikx - i\omega t)$, we can formally solve this problem. Thus, the perturbed density is

$$\tilde{n} \cong -in_0 \frac{1}{r(\omega - kv_z)} \frac{\partial}{\partial r} r \tilde{v}_r \tag{6}$$

and the associated potential is

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \tilde{\phi}}{\partial r} - k^2 \tilde{\phi} = -i\left(\frac{\omega_p}{c}\right)^2 \frac{1}{r(\omega - kv_z)} \frac{\partial}{\partial r} r \tilde{v}_r \ . \tag{7}$$

The quantity $\tilde{v}_r$ in turn comes from solution of Eq. (5). Note also that the magnitude of the modulation is

$$\Delta r = i \frac{\tilde{v}_r}{(\omega - kv_z)} \tag{8}$$

and if $\Delta r > R - a$, the beam hits the waveguide wall. This is a fundamental constraint on electric field amplitude in beam cyclotron waves. Equation (5) also suggests that most efficient coupling should be into the $\omega = 0$ mode, since there is no time dependence in the source.

Self-consistent numerical simulations have been conducted to evaluate this acceleration concept. In these calculations, beam particles are injected into an initially evacuated cylindrical waveguide through a grounded, "metallic" surface. An external B_z field of sufficient magnitude to maintain beam equilibrium is imposed. Since the metallic "anode" shorts radial self fields, zero-frequency cyclotron waves are generated. Wave acceleration is then achieved by linearly increasing the energy of the injected beam, i.e., $\gamma = \gamma_0(1 + t/\tau)$, where typically $\tau = 10^3$. In these preliminary calculations $\gamma_0 = 7$ (3 MeV) and the beam current is 30 kA. All distances are scaled to c/ω_p, $\omega_p = (4\pi e n_0/m)^{1/2}$, and fields to $(4\pi n_0 mc^2)^{1/2}$. For typical beam density, n_0, we find $c/\omega_p \cong 0.5$ cm and $(4\pi n_0 mc^2)^{1/2} \cong 1.0$ MeV/cm. This scaling further gives a waveguide length of 100 cm and radius, 1.9 cm in these simulations.

The principal conclusion from our linear analysis is that variation of beam energy, γ, in time will lead to corresponding changes in a long cyclotron wavetrain. Successful utilization for collective ion acceleration also requires that the wave phase changes occur approximately according to a predetermined formula and that no disruptive nonlinearities manifest themselves. Simulation results have verified both these characteristics. Figure 1, for instance, shows energy phase space plots of the beam (γ - 1 vs. z) at two different times, $\omega_p t = 280$ and 420. Injected beam energy has increased from $\varepsilon_b = 3.84$ to 4.26 MeV in time of 2.5 ns. During this period, the wave phases have been shifted differentially from about 1/4 initial wavelengths near the anode to almost 2 wavelengths near the exit plane. In Fig. 2 we directly compare the field measured at a fixed "probe" to that predicted by Eq. (2). We need an effective value of $k_0 \equiv \Omega_0/\gamma_0 v_z$ to evaluate (2), and this was found from <u>two-dimensional</u> linear theory. We emphasize here the importance of self-consistent linear theory, since the wavenumber, k_0, corresponds to an effective γ_0 <u>greater</u> than any value of γ <u>actually in the beam</u>. There is still a small phase discrepancy between simulation and simple theory, but the extreme sensitivity of phase to equilibrium conditions, such as k_0, leads us to believe that Eq. (2) is very satisfactory.

An axial profile of the E_z field is shown in Fig. 3. The importance of using space-charge dominated beams can be seen by noting the magnitude of field on the anode surface. This field may induce breakdown on the anode surface, but it is needed to produce the large amplitude cyclotron waves. The wave field here is roughly $E_z \sim 1.5\text{-}2.0 \times 10^5$ V/cm. Furthermore, virtually no cyclotron wave attenuation is observed in these simulations after the first wavelength.

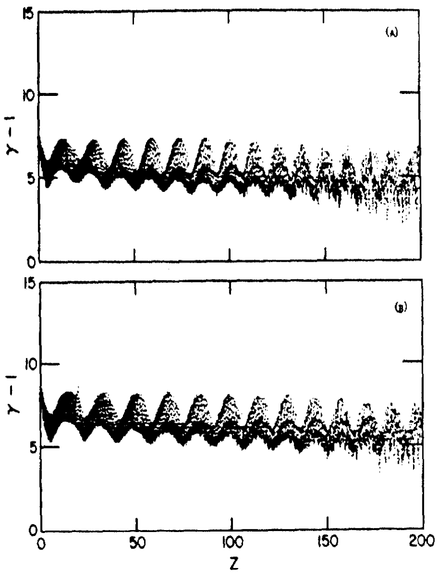

Fig. 1

Energy phase space plots showing $\omega = 0$ cyclotron modulation; $\gamma_0 = 7$, $\gamma = \gamma_0(1 + 10^{-3}t)$, (a) $\omega_p t = 280$, (b) $\omega_p t = 420$.

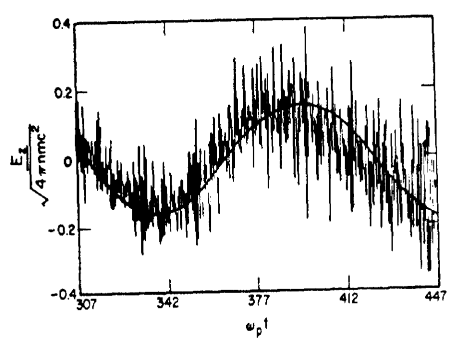

Fig. 2

Comparison of Eq. (2) (solid line) with E_z "probe" signal at z = 144, same parameters as Fig. 1.

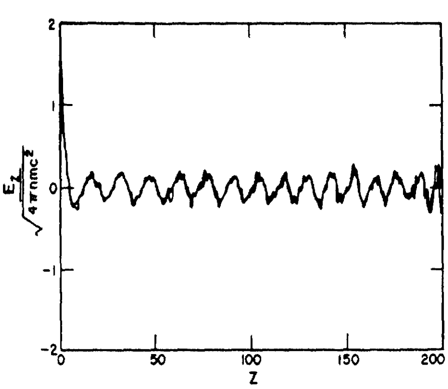

Fig. 3

Spatial distribution of E_z field for $\omega = 0$ cyclotron wave, same parameters as Fig. 1. For $n_0 = 10^{12}$ cm^{-3}, peak wave field $E_z \lesssim 2 \times 10^5$ V/cm.

III. Conclusions

Excitation and acceleration of a suitable cyclotron wave has, we believe, been successfully demonstrated in our numerical simulations. Our confidence is further strengthened by preliminary experimental results which also show $\omega = 0$ phase acceleration in time-varying relativistic electron beams.[10] The half of this problem, loading ions into the wave and accelerating them, is being pursued with test particle and full simulation calculations. Those results will be presented in a later article.

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