IEEE Transactions on Nuclear Science, Vol. NS-26, No. 3, June 1979

LASERS AND ACCELERATORS

J D Lawson *

ABSTRACT

Some features of accelerators, microwave tubes, and free electron lasers are discussed in such a way as to bring out essential similarities and differences. Two classes of wave-particle interaction, and two regimes of operation are distinguished.

1. INTRODUCTION

Lasers and radio frequency particle accelerators have the following features in common. First, there is resonant interaction between a harmonic electromagnetic wave and an ensemble of charged particles. Second, there is significant energy interchange between particles and wave. The average over all particles is from particles to wave in lasers, and from wave to particles in accelerators. At longer than optical wavelengths a wide variety of microwave tubes is available, and at wavelengths of the order 50 cm and above, triode or tetrode oscillators with an associated external resonant circuit are most convenient.

The viewpoints taken when analysing lasers (and the longer wavelength 'masers') are normally very different from those adopted in discussing microwave tubes. Furthermore, in many lasers a quantum mechanical treatment is necessary, whereas in microwave tubes and accelerators classical theory suffices. An important difference is that in lasers the charges emit and absorb real (or almost real) photons, so that descriptions in terms of scattering are useful, whereas in accelerators and most microwave tubes the photons are virtual. Papers exhibiting many different points of view have appeared in the literature. Several of these may be seen in the recent compilation entitled 'Novel Sources of Coherent Radiation'[1]

In 'conventional' lasers the charges responsible for radiation and absorption are constrained by atomic and molecular forces. In the free-electron laser and cyclotron maser the constraints are imposed by suitably shaped externally applied magnetic fields. This feature they share is common with most microwave tubes and accelerators. They form an interesting link between conventional lasers on the one hand, and microwave tubes and accelerators on the other.

2. CLASSIFICATION OF SYSTEMS AND MODES OF OPERATION

Devices which depend for their operation on the interaction between particle beams and harmonic electric fields can be classified in several ways. We distinguish first between those in which the interaction is localized in a small number of gaps, and those in which it is extended over a continuous distance of many wavelengths, an array of many gaps, or over many transits through the same gaps. In the first category are the triode oscillator and klystron; in the second are the travelling-wave tube, linac, magnetron, cyclotron maser, and free electron laser. Although in the synchrotron, cyclotron, and some linacs the field is localized, it can be considered as being composed of a number of fourier components, (space-harmonics), one of which has phase velocity approximately matching that of the particles. In nearly all devices, therefore, the interaction is effectively continuous over many cycles and wavelengths.

Two regimes of operation may be distinguished :

a) The electromagnetic wave interacts with individual charges, (eg. accelerators, Stanford free electron laser.[2])

b) The electromagnetic wave interacts with a space-charge wave propagating on the beam, (eg. travelling wave tube, magnetron, Columbia-NRL free electron laser.[3])

Two classes of interactions can be employed :

1) Synchronous, in which the phase velocity of the electromagnetic wave is very close to that of the particles (in regime a) or the beam wave (in regime b). (As in accelerators, travelling wave tube, magnetron, cyclotron maser).

2) Non-synchronous, in which these velocities are different. (As in the Adler tube,[4] ubitron,[5] and free electron laser).

Finally, some features only appear when the beam velocity is relativistic. As will be seen later, this is essential for obtaining sufficiently short wavelengths in the free electron laser.

3. SINGLE PARTICLE REGIME, SYNCHRONOUS INTERACTION

Since particles always move with less than light velocity c, synchronous interaction can only be obtained with a 'slow' electromagnetic wave. Such waves can only exist as space-harmonics of a periodic structure, or as evanescent waves outside a dielectric in which total internal reflection is occurring. A plane wave of this type (from which all other types may be obtained by suitable fourier synthesis) decays to 1/e of its amplitude at a distance $\gamma_z\lambda$ from the surface, where $\beta_z c$ is the phase velocity of the wave, and $\gamma_z^2 = (1-\beta_z^2)^{-1}$. The ratio of the longitudinal (accelerating) component of field E_z to the transverse component $E_\perp$ varies as $1/\beta\gamma$, which tends to zero at large γ. By suitably combining two or more waves in such a way that the transverse components cancel it is possible to ensure that $E_z/E_\perp$ remains greater than unity provided that two guiding surfaces spaced less than λ/π (or a tube of lesser diameter than this) are employed. This fact imposes a technical limit at short wavelengths in microwave tubes, and implies that an accelerator operating at this wavelength would have problems in obtaining high intensity.

4. SINGLE PARTICLE REGIME, NON-SYNCHRONOUS INTERACTION

Resonant interaction between wave and particle can be obtained even if their velocities are different, provided that the particle orbit is modulated in the transverse direction. This enables coupling with waves with phase velocity equal to or exceeding that

* Rutherford Laboratory, Chilton, Oxon, England.

of light to be achieved; the transverse dimensional limit may thereby be removed. This principle, described for example by Gorn[6], forms the basis of cyclotron wave amplifier tubes, the ubitron, and the free electron laser.

As a particular embodiment we describe the scheme used in the Stanford laser. The wave is a circularly polarized plane wave; a particle proceeding in the direction of the wave with a velocity component $\beta_z c$ experiences a force of frequency $\omega_r(1-\beta_z)$, where ω_r is the frequency of the wave. If now a static twisted transverse magnetic field, of wavelength λ_q is introduced, the particle moves in a spiral orbit, rotating with frequency $\beta_z c/\lambda_q$. If these two frequencies are equal,

$$\lambda_r/\lambda_q = (1/\beta_z - 1) \qquad (1)$$

and there will in general be a steady component of electric field along the helical particle orbit. This will be accelerating or decelerating according to the phase of the radius vector from the particle to the axis with respect to the direction of the rotating electric field. For a highly relativistic particle, with energy $\gamma m_o c^2$, the approximation $\beta \approx 1-1/2\gamma^2$ is good. Further, if the radius r of the helix is small, $\beta_z \approx \beta/(1 + r^2/2\bar{\lambda}_q^2)$.

Making use of these approximations and eqn. 1, we may write

$$\frac{\lambda_r}{\lambda_q} \approx \frac{1}{2\gamma_z^2} \approx \frac{1}{2\gamma^2}\left(1 + \frac{\gamma^2 r^2}{\bar{\lambda}_q^2}\right) = \frac{1}{2\gamma^2}\left(1 + \frac{\lambda_q^2 e^2 B_\perp^2}{m_o^2 c^2}\right) \qquad (2)$$

where $B_\perp$ is the transverse magnetic field. For large γ, λ_r can be made extremely small. Although the restriction to transverse dimensions of order $\bar{\lambda}_r$ no longer applies, this advantage is accompanied by the drawback that since the orbit and $E_\perp$ are almost orthogonal, the interaction is only a second order effect. The value of r/λ is limited if small λ_r, and realistic values of $B_\perp$ are required.

A slightly different arrangement is employed in the ubitron tube, where transverse modulation is provided by a Motz undulator, and the fast wave is guided with phase velocity exceeding that of light.

5. DYNAMICS OF A SINGLE PARTICLE

Representation of the motion of a particle interacting with a wave in the ϕ-$\dot{\phi}$ phase plane is familiar in the theory of linacs and synchrotrons, and has recently been applied to the free electron laser by Colson[7]. Indeed, it has also been applied to travelling wave tubes in the 'large signal' regime where collective interaction can be neglected,[8] and to the cyclotron maser.[9]

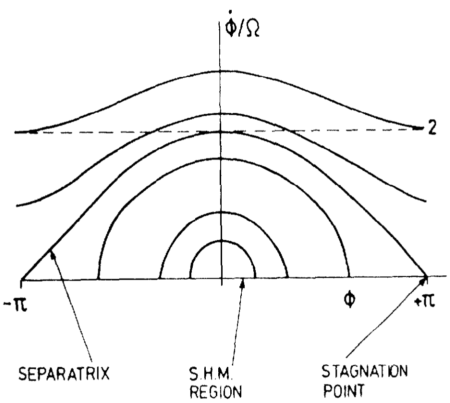
In all these devices ϕ represents the phase of a particle with respect to the wave (or an 'effective' wave with frequency ω_r and wavenumber $k_r + k_q$ in the case of non-synchronous interactions); $\dot{\phi}$ is proportional to the deviation in the particle energy from that of the synchronous particle for which $\dot{\phi} = 0$. We consider the limiting case of a particle accelerator in which, apart from the accelerating field, conditions are held constant. Under these circumstances a particle with ϕ and $\dot{\phi}$ both zero moves uniformly with no oscillation; in general the particles obey the pendulum equation

$$\ddot{\phi} + \Omega^2 \sin\phi = 0 . \qquad (3)$$

This may be integrated to

$$\dot{\phi}^2 - \dot{\phi}_o^2 = 2\Omega^2 \,(\cos\phi - \cos\phi_o) \qquad (4)$$

where the subscript zero denotes values at $t = 0$.



The well-known plot of $\dot{\phi}$ against ϕ is shown in the figure. Orbits inside the separatrix represent 'trapped' particles, those outside represent particles with continuously slipping phase. The quantity Ω^2 is proportional to the amplitude of the wave, and to the reciprocal of the effective mass m* of the particle; it depends also on the details of the orbit geometry. The effective mass in orbital machines is $\gamma m_o/(\gamma^{-2} - \gamma_t^{-2})$ where $\gamma_t m_o c^2$ is the transition energy; it is thus negative when $\gamma > \gamma_t$, and the phase stable particle shifts from $\phi = 0$ to $\phi = \pi$. An interesting special case is the cyclotron maser; in a uniform magnetic field $\gamma_t = 1$, and $m^* = -m_o/\beta^2$ for non-relativistic electrons. In the Stanford free electron laser, it is readily shown that $m^* = \gamma\gamma_z^2 m_o$, this tends to the longitudinal mass when $\gamma r \ll \lambda_q$, so that $\gamma \approx \gamma_z$, (eqn.2).

It has already been noted that $\dot{\phi}$ is proportional to the energy deviation $\Delta\gamma m_o c^2$ from that of the synchronous particle. The constant of proportionality depends on the particular device; for the laser $\dot{\phi}/\Delta\gamma = \omega_r/\beta_z^2\gamma_z^2\gamma$.

6. BEHAVIOUR OF A DISTRIBUTION OF PARTICLES

If a large number of particles, with some distribution in ϕ - $\dot{\phi}$ space is injected into the system, the distribution will evolve with time. In general some particles will gain and others lose energy. For N particles the net energy interchange can be found as a function of time by evaluating

$$\langle \dot{\phi}(t) - \dot{\phi}_o \rangle = \sum_{i}^{N} (\dot{\phi}_i(t) - \dot{\phi}_i(o))/N \qquad (5)$$

For the special case of a monoenergetic initial beam with single value of $\dot{\phi}_o$ but uniformly distributed in phase, ϕ_o, evaluation of this experession has been carried out for different values of $\dot{\phi}_o$ by Planner[10].

For small values of t, however, an analytical expression can be derived

$$\langle\dot\phi(t)-\dot\phi_o\rangle = \frac{\Omega^2}{\dot\phi_o^3}(\cos\dot\phi_o t-1-\tfrac{1}{2}\dot\phi_o t\sin\dot\phi_o t)$$

$$= \frac{\Omega^4 t^3}{4}\left\{\frac{d}{dx}\left(\frac{\sin x}{x}\right)^2\right\}, \qquad x = \tfrac{1}{2}\dot\phi_o t \qquad (6)$$

with range of validity [11] $t \ll 2\dot\phi_o/\Omega^2$. At values of ϕ_o for which there are values of $\dot\phi_o$ within the separatrix this represents a fraction of the phase oscillation period $2\pi/\Omega$; at large values of $\dot\phi/\Omega$, on the other hand, it represents many cycles of oscillation. For even shorter times, $t \ll 1/\dot\phi_o$, eqn. 6 simplifies to

$$\langle\dot\phi - \dot\phi_o\rangle = \Omega^4\dot\phi_o t^4/24 \qquad (7)$$

Asymmetry between particles gaining and losing energy only appears to fourth order in t. From eqn. 6 a formula for the gain can readily be found.[7]

These expressions show that if $\dot\phi_o$>o, that is , if the incident beam has higher energy than the synchronous particle; energy interchange is from the beam to the wave, and vice versa. Indeed, this fact is well known from the phenomenon of Landau damping, the formula for which can readily be found by integrating over a distribution f in $\dot\phi_o$ such that $\partial f/\partial\dot\phi_o$ at $\dot\phi_o$=0 is negative, and normalizing to the energy density of the wave.

The phenomenon described in this section, whereby charges move unsymmetrically in the phase-plane in such a way that net energy is transferred from the particles to the wave forms the basis of laser action. It has sometimes been referred to as 'inverse Landau damping'. It also forms the basis of 'beam break-up' in linacs,[12] the wave in this case having transverse electric field, and oppositely directed phase and group velocities. This latter property provides feedback as in a backward wave oscillator. In its most general form, a laser can be considered as an assembly of non-linear oscillators with distribution of amplitudes which shows a 'population inversion', (more with large amplitude than with small). Such a system, when interacting with a coherent wave of appropriate frequency, can transfer energy to it.[13] In a cyclotron maser, for example, the oscillators have frequency Ω_c/γ. The free electron laser is a limiting case in the sense that, as in the linac, the oscillator frequency is zero.

7. INTERACTION WITH COLLECTIVE OSCILLATIONS ON THE BEAM

When collective plasma or cyclotron oscillation on the beam are significant, the dynamical behaviour of the system is considerably more complex. The interaction may be described in terms of coupling between the positive energy electromagnetic wave, and a slow negative energy wave on the beam, as in a travelling wave tube. To achieve energy transfer from the beam to a wave with phase velocity equal to or exceeding that of light, it is again necessary to introduce modulation in the particle orbits in the beam. The system may be regarded as a parametric amplifier, with the static modulating field and beam wave playing the role of 'pump and 'idler'.[3] It is readily shown that eqn. 1 now becomes

$$\frac{\lambda_w}{\lambda_q} = \frac{v_w - v_p}{v_p - \Omega/\gamma k_q}, \qquad (8)$$

where Ω is the frequency of the beam wave.

Calculations of gain may be made in the usual way, by finding the dispersion relation and solving for complex k.

8.CONCLUSIONS

Particle accelerators, microwave tubes, and free electron lasers have many features in common, but also some interesting differences. These can be conveniently classified by assuming two types of wave-particle interaction, and two regimes of operation.

REFERENCES

1. S F Jacobs, M Sargent III, M O Scully, (eds.) "Novel Sources of Radiation", Reading (Mass) Addison-Wesley, 1978
2. D A G Deacon et al., "First Operation of a Free-Electron Laser", Phys. Rev. Lett. 38, 892-4 April 1977.
3. D B McDermott et al., "High-Power Free-Electron Laser Based on Stimulated Raman Backscattering", Phys. Rev. Lett. 41, 1368 -71 Nov. 1978.
4. G D Sims and I M Stephenson, "Microwave Tubes and Semiconductor Devices", London and Glasgow, Blackie and Son 1963, Chap.10.
5. R M Phillips, "The Ubitron, a High-Power Traveling-Wave Tube Based on a Periodic Beam Interaction in Unloaded Waveguide." IRE Trans. Elec. Devices ED7 231-241, 1960.
6. E J Gorn, "Traveling Wave Electron Reaction Device". U S Patent 2,591,350, Filed 1947, 1952.
7. W B Colson, "One-Body Analysis of Free Electron Lasers", Ref 1.p. 157-196.
8. S E Webber "Electron Bunching and Energy Exchange in a Traveling-Wave Tube", IRE Trans. Elec. Devices ED-4, 87-91, 1957.
9. P Sprangle and A T Drobot, "The Linear and Self-Consistent Nonlinear Theory of the Electron Cyclotron Maser Instability", IEE Trans. Microwave Devices, MTT 25, 528-544, June 1977.
10. C W Planner, "Numerical Solutions to the Non-Linear Phase Equation for the Free Electron Laser", Phys. Letts. 67A, 263-265, Aug. 1978.
11. J D Lawson, "Coherent Interaction between Waves and Particle Streams", Rutherford Lab. Report RL-78-092, 1978.
12. P B Wilson, "A Study of Beam Blow-up in Electron Linacs", Stanford memorandum HEPL-297, 1963.
13. A V.Gaponov, M I Petelin, V K Yulpatov "The Induced Radiation of Excited Classical Oscillators and its use in High Frequency Electronics," Izvestiya VUZ Radiofizika 10, 1414-1453, 1967.