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# HIGH FLUX ION, AND ODD MULTI-HALF WAVE LENGTH RESONANT TRANSFORMER ACCELERATORS IN THE SPACE AND TIME DOMAINS

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## Abstract

This paper carried out analytical work for identification of beam instabilities in the high flux ion collective accelerator generated by MHD effects. Density as well as beam velocity deviations are observed with the effect of incremental change in electron temperature. Also for the resonant transformer collective accelerator, dynamic model has been derived indicating the dependence of the accelerating potential with respect to various system parameters, establishing steady state stability limit and optimum tuning ratio at resonance.

## Introduction

This paper presents supplemental work in the field of collective accelerators with initial excitation generated from a Blumlein circuit and resonant transformer equivalent to an odd multi-half wave length respectively. The Blumlein energized accelerator could produce a large flux of medium energy ions of the order of 10kJ, 10 MeV and equivalent current density beyond $10kA/cm^2$, created by the electron separated electric field with the application of an axial magnetic field. Electron streaming instabilities are responsible for both rapid heating of the original plasma beam and impedance matching with other compatible power sources. Current research efforts point to the creation of hydrogen plasma flow of 10 MeV by the application of 1 MeV electric potential. Results also indicate that geometrical dimensions of the plasma tube have important and profound effect on the ion flux density and velocity of the accelerated beam. However in those results, effects such as radial motion, thermal conductivity and MHD instabilities have been neglected.

Work is also progressing for the resonant transformer accelerator for gated as well as un-gated pulsed and average power ion beams. Beam formation is accomplished by gating a gridded thermionic cathode, and then accelerated through agraded acceleration tube. In the case of un-gated beam, the transformer secondary capacitor could be modeled as a transmission line with deionized water acting as the dielectric.

Current results point to compromise operational spectrum for the transformer effective coupling up to the value of 0.6, optimum criterion for resonance in terms of turning ratio as well as limitation on tuning ratio. Also it has been pointed out that the system accelerating efficiency could be at higher limits for low tuning ratios and high quality factor.

## Statement of the Problem

A. With respect to the high flux ion collective accelerator energized by a Blumlein circuit and controlled by electrons instabilties with an axial magnetic field, required:

To investigate the order of magnitude variations in electron temperature, and pressure beam velocities and MHD instabilities in time and space domains.

B. For a resonant transformer accelerator for gated and un-gated beam characterized with certain resonance criterion, tuning ratio and coefficient of effective coupling, required:

To develop a dynamic model for transient energy extraction under an odd multi-half wave equivalent transformer, ultimate tuning ratio at resonance and stability of reponse in the steady state.

## Analysis of the High Flux Ion Accelerator

The two fluid-dyanmic equation for electrons and ions are:

$$\frac{D\rho}{Dt} = \rho\ \nabla\cdot v = 0$$

The momentum equation,

$$\rho(v\cdot\nabla)\ v = -\ \nabla\rho + j \times B$$

and the energy equation

$$\rho(v\cdot\nabla)\left[\ \frac{1}{2}\ v^2 + \frac{P}{\rho} + C_v\ T\right] = E.J \qquad (1)$$

where

D = convective derivative.
$C_v$, T = specific heat at constant volume and absolute temperature respectively.
P = total plasma pressure
v,ρ,J = beam velocity, mass density and current density respectively.

The conventional Maxwell field equations are:

$$\nabla\cdot J + \frac{\partial\rho_e}{\partial t} = 0$$

$$\nabla \times B = \mu_o J$$

$$\nabla \times E = \frac{\partial B}{\partial t}$$

and

$$J = \sigma[E_a \times v \times B] \qquad (2)$$

where

$E_a$, $\rho_e$ = applied electric field and charge density respectively.

The magneto-fluid equations are applied with respect to a charge neutral plasma in an axial magnetic field generated by N turns solenoid whose mean diameter is 2r and axial length is 2L,

$$B_Z = \frac{NLi^2\mu_o}{2r\sqrt{r^2+L^2}} \qquad (3)$$

From 1, 2, and 3 and the following constraints:

a. ion pressure and electron interia are very small
b. $\Delta T_e \approx J^2\rho\ \Delta t/n$
c. MHD instability is assumed to exist when the magnetic Reynolds number $R_m \geqslant 0.10$,

i.e. $$B_o/B_Z \gtrsim 0.10 \qquad (4)$$

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d. electron instability is expressed in the relation

$$0.01\ W_e^{-1} \lesssim \rho < 0.1\ W_e^{-1} \quad (5)$$

$W_e$ = electron plasma frequency.

These results have been established:

A.

$$\frac{\Delta V}{\Delta X} \approx \frac{k}{T_e} \frac{i^2}{n^{3/2}} \quad (6)$$

$$k = \frac{2.5 \times 10^6}{\pi} \left(\frac{1}{r^4} - 3\right)$$

B.

$$\frac{\Delta v}{v} \approx k' \frac{\Delta T_e}{T_e} \quad (7)$$

$$k' = \frac{\pi k}{2.5 \times 10^6}$$

C.

$$n = 2 \sqrt{k_1^3}\ 2 \sqrt{T_e^3 \left(\frac{\Delta v}{\Delta X}\right)^3} \quad (8)$$

$$k_1 = \frac{2.5 \times 10^6 i^2}{\pi k'}$$

D. MHD instability imposed geometrical as well as well as physical constraints on the ion beam, namely:

a. $r \geqslant 0.318L$

b. $\Delta P_e$ as the electron presure attains a constant level at any value of r, and accelerates several folds as r increases.

c. $\Delta v$ as the beam velocity perturbation attains a constant level as r increases.

E. Velocity distribution of the ion flux beam will be influenced by MHD instability such that with any sizable increase in r and L will lead to flattening of the beam velocity besides to a reduction in the intensity of momentum.

F. Electron temperature gradient is extremely small across the beam, as $T_e$ maintains constant limit.

<u>Anyalysis of the Resonant Transformer Accelerator</u>

Figure 1 shows a schematic for this type of accelerator with the gated beam i(t) formed by gating a gridded cathode and then accelerated down a graded electron channel. While for un-gated beam the secondary capacitance is modeled as a line with $R_2$ shunting $C_2$.

A. Modeling in the frequency domain

$$V_2(s) = V_1(s) + G(s) - H(s)\ G(s) \quad (9)$$

$$V_1(s) = \frac{k_e}{1-k_e^2} G_v E_o \frac{W_2^2 S}{(S^2+2\alpha S+W_a^2)(S^2+2\beta S+W_b^2)} \quad (10)$$

$$G(s) = \frac{I(s)}{SC_2}$$

$$H(s) = \frac{W_2^2}{1-k_e^2} \frac{S+\frac{W_1}{Q_1} S + W_1^2}{(S^2+2\alpha S+W_a^2)(S^2+2\beta S+W_b^2)} \quad (11)$$

where

$W_1$, $W_2$ = primary and secondary tuning frequencies.

$k_e$ = effective coupling coefficient

$$W_{a,b} = \frac{1}{2} \frac{W_1^2 W_2^2}{1-k_e^2} \left[1 \pm \sqrt{1-4(1-k_e^2)\frac{W_1^2 W_2^2}{(W_1^2+W_2^2)^2}}\right] \quad (12)$$

∴ Ideal transform function for energy extraction with no losses is expressed as:

$$\Gamma(s) = \frac{V_2(s)}{V_1(s)} = 1 + I(s)\left[\frac{(S^2+W_a)(S^2+W_b^2)}{k_1 W_2^2 C_2 S^2} - \frac{(S^2+\frac{W_1}{Q_1} S+W_1^2)}{k_1 W_2^2 C_2 S^2}\right] k_2 \quad (13)$$

where

$$k_1 = \frac{k_e G_v V_o}{1-k_e^2}$$

$$k_2 = \frac{W_2^2}{1-k_e^2}$$

The dynamic model for the resonant transformer accelerator is shown in Fig. 2.

This ratio of accelerating voltage transformation is secured below at no losses and with the constraint of any equivalent transformer of odd multi-half wave length.

$$\Gamma = \sqrt{\left(1+\frac{h_1 k_3 - h_2 h_4}{k_5}\right)^2 + \left(\frac{h_1 h_4 + h_2 k_3}{k_5}\right)^2}\ \underline{\left|\ \gamma\right.} \quad (14)$$

with:

$|\Gamma| = 1$ and $\gamma = n\pi$, n is odd.

where

$$k_3 = (\tau^4 - \tau^2(W_a^2 + W_b^2) + W_a^2 W_b^2 + k_2 \tau^2 - k_2 W_1^2)$$

$$k_4 = k_2 \frac{W_1}{Q_1} \tau \quad (15)$$

$$k_5 = -k_1 W_2^2 C_2 \tau^2$$

$$h_1 = \frac{\mathrm{Sin}\,a\tau}{\tau},\quad h_2 = \left(\frac{1}{\tau} - \frac{\cos a\tau}{\tau}\right)$$

a = pulse width of I(t) in seconds

B. To express the optimum condition of tuning: at no-losses

$$Z_2(s) = \frac{1}{I(s)} \frac{k_1 W_2^2 S}{(S^2+W_a^2)(S^2+W_b^2)} + \frac{1}{SC_2} - \frac{S^2+\frac{W_1}{Q_1}S+W_1^2}{SC_2(S^2+W_a^2)(S^2+W_b^2)} \quad (16)$$

From the final value theorm:

$$\lim_{t\to\infty}{}^{-1} Z_2(s) = \frac{1}{C_2}\left[1-\frac{1-k_e^2}{W_2^2}\right] \quad (17)$$

Condition at resonance implies:

$$\frac{1+X^2}{X} = \frac{5}{2}\sqrt{1-k_e^2} \quad (18)$$

where $x = W_1/W$

From 16, 17 and 18, the optimum tuning ratio is secured for x = 1

i.e. $W_1 = W_2$

## Conclusions

A. For the high flux ion collective accelerator:

1. MHD instabilities may occur whenever the ratio of the induced magentic to the axial field exceeds 0.10, or the beam radius exceeds 0.318 of a half axial length.
2. Velocity change across the bean length will diminish since electron temperature will level with the effect of MHD instability

B. For the resonant transformer accelerator:

1. Dynamic model expressing the ratio of transformed accelerating voltage has been established with the constraint of odd multi-half wave length equivalent transformer.
2. Under the condition of resonance and stable steady state response, the optimum ratio of tuning is secured at unity.
3. Stability limit is established in terms of the secondary tuning frequency, coefficient of effective coupling and the secondary capacitance.

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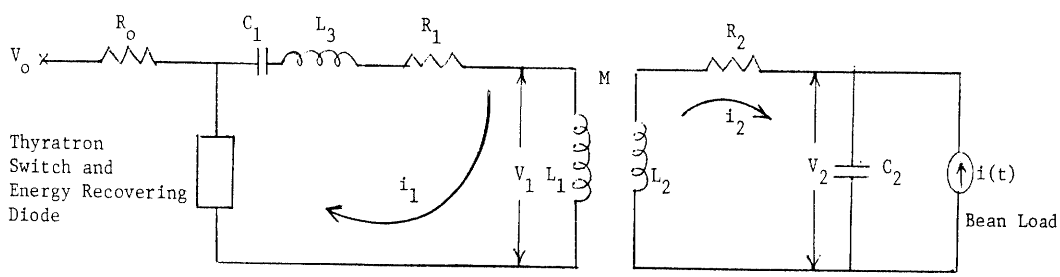


Fig. 1. Equivalent Circuit for Resonant Transformer Accelerator

$$G_1 = I(s) \frac{(S^2+W_a)(S^2+W_b^2)}{k_1 W_2^2 C_2 S_2}$$

$$G_2 = I(s) \frac{(S^2 + \frac{W_1}{Q_1} S + W_1^2)}{k_1 W_2^2 C_2 S_2}$$

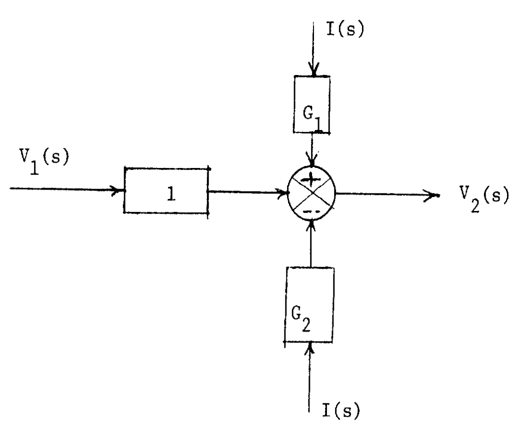


Fig. 2. Dynamic Model for Resonant Transformer Accelerator