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DESIGN OF CIRCUITS FOR COUPLING POWER AMPLIFIERS TO RESONATORS

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Abstract

Four resonators and one buncher, each driven by a power amplifier through a coupling circuit, are being planned. The method by which the parameters of various coupling circuits have been calculated is discussed. Expressions for the parameters and inherent resonant frequencies are derived for some well-known coupling circuits.

Introduction

A National Accelerator Centre is at present being established around a 200 MeV separated-sector cyclotron and an 8 MeV solid-pole injector cyclotron. Five rf systems, i.e. two for the separated-sector cyclotron, two for the injector and one for the buncher are being designed and built.

Several possible circuits for coupling the power amplifiers to the resonators have been analysed. While the circuit parameters and inherent resonant frequencies can be obtained by graphical and iterative methods, the present paper shows that relatively simple analytical expressions can be obtained for these parameters and frequencies for several well-known coupling circuits.

Method of Calculation

The method is based on the use of a parallel-resonant circuit, shown in figures 1a and 1b, as an equivalent circuit for a resonator. The circuit with the parallel resistor is used for analysing capacitive-coupling circuits and the circuit in figure 1b for inductive-coupling circuits. Terminals A and B represent the accelerating electrode and vacuum chamber lining respectively. Parameters L, C, R and R_s are calculated from the resonator characteristics which may be obtained either by measurement or by calculation. R_s is dependent on the position of the coupling loop in the resonator.

Although the parameters of the equivalent circuit are calculated at a particular frequency, they may be applied over a frequency range of several bandwidths centred at the resonant frequency of the resonator. This is of importance since the resonant frequency of the resonator differs in general from the operating frequency.

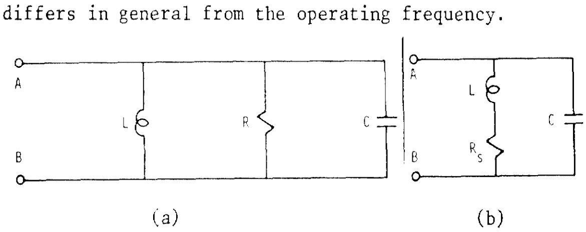

Fig. 1. Equivalent circuits for a resonator with (a) a parallel resistor and (b) a series resistor.

The following equations show the relation between the parameters of the equivalent circuit and the resonator characteristics:

$$R = \frac{V^2}{P},\ L = \frac{R}{2\pi FQ},\ C = \frac{1}{4\pi^2 LF^2} \text{ and } R_s = \frac{P}{I^2}$$

P is the power dissipated in the resonator at an rms dee voltage V and an rms current I in the resonator opposite the coupling loop at a frequency F. Q is the Q-value of the resonator at frequency F.

Once the coupling circuit to be analysed and the resonant frequency of the equivalent circuit are specified, the operating frequency can no longer be specified independently. The amount by which the operating frequency differs from the resonant frequency is however not yet known. This problem is solved by specifying the operating frequency equal to the resonant frequency of the equivalent circuit and by modifying either L or C (of the equivalent circuit) slightly to obtain a new resonant frequency. By doing this it is implicitly assumed that R and R_s (and therefore also the current distribution in the resonator) vary slowly with frequency in the vicinity of the resonant frequency. This assumption was checked and found valid over a frequency range of several bandwidths centred around the resonant frequency. It is further known that the difference between the resonant frequency and operating frequency is very small (typically 1 kHz to 100 kHz) for resonators with high Q-values (5 000 to 20 000). The procedure is further illustrated in the next few sections in which several coupling circuits are analysed.

Direct Coupling

Figure 2 shows a circuit which represents direct coupling, i.e. the power amplifier is coupled to the inner conductor of the resonator without any components between. The input impedance can be varied by adjusting the ratio between L_1 and L_2 and by varying C slightly to obtain a new resonant frequency. C and $k = L1/(L_1+L_2) = L_1/L$ are therefore considered as unknown variables.

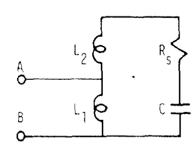

Fig. 2. Direct coupling to a resonator.

The required input impedance at the operating frequency $F = W/2\pi$ is specified as $Z_i = R_i$:

$$R_i = \frac{L_1 jW(L_2 jW + R_s + \frac{1}{CjW})}{LjW + R_s + \frac{1}{CjW}}. \quad (1)$$

The new capacitance value follows from the imaginary part of this equation:

$$C = \frac{R_i}{LW^2(R_i - kR_s)}. \quad (2)$$

By substituting (2) in the real part of (1) an expression for k is obtained:

$$k = \frac{R_i}{LW}\sqrt{\frac{R_s}{R_i - R_s}}. \quad (3)$$

By eliminating R_i between the real and imaginary parts of (1) the following equation for the inherent resonant frequencies are obtained:

$$W^4(L^2C_1^2\ (1-k) + W^2(C_1^2R_s^2 - LC_1(1-k)-LC_1) + 1 = 0, \quad (4)$$

which indicates two positive resonant frequencies. It is to be noted that C_1 and k appearing in (4) are the values as calculated from (2) and (3) respectively.

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The new resonant frequency F_1 can be calculated from $F_1 = 1/2\pi\sqrt{LC_1}$ and the difference ΔF between the operating frequency and the new resonant frequency by $\Delta F = F - F_1 = (1/2\pi)(1/\sqrt{LC} - 1/\sqrt{LC_1})$.

The same procedure is applied in the next few sections in which more practical circuits than the direc-coupling circuits are considered.

Capacitive Coupling

Figure 3 shows a capacitive coupling circuit consisting of only a series capacitor. The series capacitor S and the inductance L of the equivalent circuit are the unknown variables.

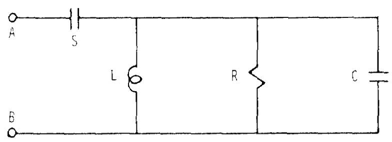

Fig. 3. Capacitive coupling with a series capacitor S.

The required input impedance is specified as $Z_i = R_i + j0$ at a frequency F equal to the resonant frequency of the equivalent circuit:

$$R_i = \frac{RLjW}{R(1-W^2LC) + LjW} + \frac{1}{SjW}. \quad (5)$$

The following equation is obtained for the new L-value:

$$L^2(R_iR^2W^4C^2 + R_iW^2 - RW^2) - L(2R_iR^2W^2C) + R_iR^2 = 0. \quad (6)$$

The new resonant frequency F_1 can be calculated from $F_1 = 1/2\pi\sqrt{L_1C}$, where L_1 is the L-value as calculated from equation (6). The difference ΔF between the operating and new resonant frequency F_1 is given by:

$$\Delta F = F - F_1 = (1/2\pi)(1/\sqrt{LC} - 1/\sqrt{L_1C}). \quad (7)$$

The following expression is obtained for S:

$$S = \frac{R^2(1 - W^2L_1C) + L_1^2W^2}{W^2L_1R^2(1 - W^2L_1C)}. \quad (8)$$

The inherent resonant frequencies are determined by:

$$W^4(R^2L_1^2C^2 + R^2L_1^2SC) + W^2(L_1^2 - R^2L_1S - 2L_1CR^2) + R^2 = 0. \quad (9)$$

For variable frequency operation the series capacitor should be variable. In practice it is difficult to cover a wide frequency range (more than three to one) with only an adjustable series capacitor. The frequency range can be extended by using a second adjustable capacitor between terminals A and B. This circuit is analysed in the next section.

Capacitive Coupling Including a Parallel Capacitor

Figure 4 shows the circuit under consideration. Either P or S can be specified.

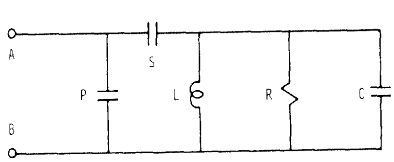

Fig.4. Capacitive coupling including a parallel capacitor P.

If S is specified, P and L can be calculated for a given input impedance R_i:

$$L^2(W^2R_i + 2R_iR^2W^4CS + R^2R_i\,S^2W^4 - R_i^2RS^2W^4 + R^2R_iC^2W^4)$$

$$+ L(-2R^2R_iCW^2 - 2R^2R_iSW^2) + R^2R_i = 0 \quad (10)$$

$$\text{and} \quad P = \frac{SRL_1W^2 - R_iSL_1W^2 - R + RL_1CW^2}{R_iL_1W^2}, \quad (11)$$

where L_1 is obtained by solving for L in equation (1). Using the same symbols as before, $F_1 = 1/2\pi\sqrt{L_1C}$ and $\Delta F = 1/2\pi(1/\sqrt{LC} - 1/\sqrt{L_1C})$.

The resonant frequencies are obtained by solving:

$$W^4(PR^2L_1^2C^2 + PR^2L_1^2CS + PR^2SL_1^2C + PS^2R^2L_1^2 + R^2L_1^2C^2S + R^2S^2L_1^2C)$$

$$+ W^2(PL_1^2 + SL_1^2 - 2PR^2L_1C - 2PR^2SL_1 - 2R^2SLC - R^2S^2L_1)$$

$$+ R^2(P + S) = 0. \quad (12)$$

Analysis of the transient behaviour of this circuit showed that since P is in general much larger than S, it has the advantage that transient voltages coming from the resonator, due to sparking, are strongly attenuated.

This coupling circuit has been selected as the most suitable for the resonators of the 200 MeV separated-sector cyclotron.

Inductive Coupling Including a Series Capacitor

The same procedure as before is followed for the inductively coupled circuit shown in figure 5. A series capacitor is included because it would otherwise be necessary to adjust the mutual inductance for variable-frequency operation.

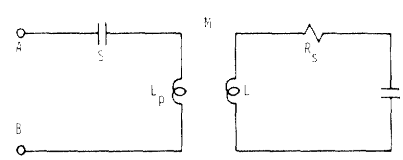

Fig. 5. Inductive coupling including a series capacitor S.

The series reactance in the secondary part of the circuit is given by $X_s = LW - \frac{1}{CW}$. The required input impedance R_i is given by:

$$R_i = \frac{M^2W^2R_s}{X_s^2+R_s^2} + j\left(L_pW - \frac{1}{SW} - \frac{M^2W^2X_s}{X_s^2+R_s^2}\right). \quad (13)$$

X_s and S are chosen as variables to be solved for. The following expression is obtained from equation (13):

$$X_s = \pm\sqrt{\frac{M^2W^2R_s - R_iR_s^2}{R_i}} \quad (14)$$

$$\text{and} \quad S = \frac{X_s^2 + R_s^2}{L_pW^2X_s^2 + L_pW^2R_s^2 - M^2W^3X_s},$$

$$= \frac{1}{W(L_pW \mp \sqrt{(R_iM^2W^2 - R_sR_i^2)/R_s})}. \quad (15)$$

By using X_s as calculated from (15) a new L-value can be calculated from $L_1 = X_s/W + (1/CW^2)$. The new resonant frequency F_1 and ΔF can be calculated as before.

The resonant frequencies of the coupling circuit and resonator is given by:

$$W^6(M^2L_1SC^2 - L_pL_1^2SC^2)$$

$$+W^4(L_1^2C^2 + 2L_1L_pCS - R_s^2L_pC^2S - M^2CS)$$
$$+W^2(R_s^2C^2 - 2L_1C - L_pS)$$
$$+ 1 = 0 \qquad (16)$$

The mutual inductance M and the self-inductance L_p for a rectangular loop, in vacuum, as shown in figure 6 is given by:

$$M = 2 \times 10^{-7} g \ln\left(\frac{b}{b-d}\right) \qquad (17)$$

and
$$L = \frac{\mu_0}{2}\left\{\left[\sqrt{4d^2+g^2} - g - 2d\ln\left(\frac{2d+\sqrt{4d^2+g^2}}{g}\right) - \sqrt{4d^2+c^2} + c + 2d\ln\left(\frac{2d+\sqrt{4d^2+c^2}}{c}\right)\right] + \left[\sqrt{g^2+4d^2} - 2d - g\ln\left(\frac{g+\sqrt{g^2+4d^2}}{2d}\right) - \sqrt{g^2+c^2} + c + g\ln\left(\frac{g+\sqrt{g^2+c^2}}{c}\right)\right]\right\}, \qquad (18)$$

for a conductor diameter of 2a.

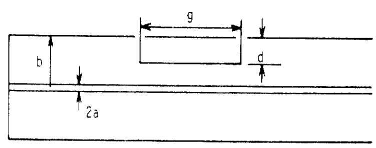

Fig. 6. Resonator with rectangular coupling

Inductive Coupling Including a Parallel Capacitor

Figure 7 shows an inductive coupling circuit consisting of a capacitor in parallel with the loop.

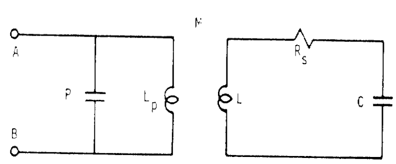

Fig. 7. Inductive coupling including a parallel capacitor P.

For a given input impedance R_i, P and X_s can be determined:

$$X_s^4(L_p^2) + X_s^3(-2M^2L_pW) + X_s^2(2L_p^2R_s^2 - R_iR_sM^2+W^2M^4) + X_s(-M^2W\,L_pR_s^2 - M^2L_pR_s^2W) + (M^4R_s^2W^2 - R_s^3R_iM^2 + L_p^2R_s^4) = 0 \qquad (19)$$

A new L-value, L_1, can be calculated as in the previous section. The parallel capacitance is given by:

$$P = \frac{L_pR_s^2 + L_pX_s^2 - M^2WX_s}{R_iR_sM^2W^2}. \qquad (20)$$

The resonance frequencies are given by:

$$W^6(M^2-L_pL_1)^2 + W^4(M^2L_1C - L_pL_1^2C + L_p^2R_s^2CP - 2L_p^2L_1P + 2M^2L_pP)C + W^2(2L_pL_1C - L_pR_s^2C^2 - M^2C + L_p^2P) - L_p = 0 \qquad (21)$$

Inductive Coupling Including Series and Parallel Capacitors

In cases where a wide frequency range has to be covered with the matching circuit it may be necessary to include two variable capacitors, one in series and one in parallel, in the primary circuit as shown in figure 8.

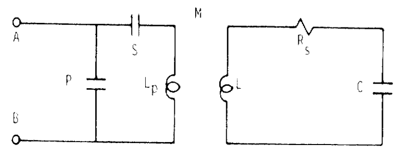

Fig. 8. Inductive coupling including both series and parallel capacitors S and P.

For a given series capacitance S and impedance R_i between terminals A and B the parallel capacitance P and the reactance in the secondary circuit can be calculated:

$$AX_s^4 + BX_s^3 + DX_s^2 + EX_s + F = 0, \qquad (22)$$

where
$$A = 2W^2L_pS - 1 - W^2L_p^2S^2,$$
$$B = 2W^5L_pS^2M^2 - 2W^3SM^2,$$
$$D = R_iW^4R_sS^2M - 2R_s^2 - 2W^4L_p^2S^2R_s^2 + 4W^2L_p^2SR_s^2 - W^6S^2M^4,$$
$$E = 2W^5L_pS^2M^2R_s^2 - 2W^3SM^2R_s^2,$$
$$F = R_iW^4R_s^3S^2M^2 - R_s^4 - W^4L_p^2S^2R_s^4 + 2W^2L_pSR_s^4 - W^6R_s^2S^2M^4 \text{ and}$$
$$P = \frac{(W^2L_pS-1)(X_s^2 + R_s^2) - W^3M^2X_sS}{R_iR_sW^4S\,M^2}. \qquad (23)$$

The new resonant frequency for the resonator and the frequency difference ΔF can be calculated as before.

The inherent resonance frequencies are given by:

$$W^8(2M^2L_pL_1 - L_p^2L_1^2 - M^4)\,PS^2C^2$$
$$+W^6(L_pL_1^2SC - M^2L_1SC - L_p^2R_s^2PSC + 2L_p^2L_1PS + 2L_pL_1^2PC - 2M^2L_pPS - 2M^2L_1PC)SC$$
$$+W^4(L_pR_s^2S^2C^2 - 2L_pL_1S^2C - L_1^2C^2S + M^2S^2C - L_p^2PS^2 + 2L_pPR_s^2SC^2 - 4L_pL_1PSC - L_1^2PC^2 + 2M^2PSC)$$
$$+W^2(L_pS^2 - R_s^2SC^2 + 2L_1SC + 2L_pPS - R^2PC^2 + 2L_1PC)$$
$$-(P + S) = 0. \qquad (24)$$

This type of matching circuit has been selected as the most suitable for the resonators of the injector cyclotron.

Acknowledgements

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