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THE EFFECT OF LARGE DELAYS ON BEAM RF PHASE LOCK LOOPS

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Introduction

The frequency of the rf accelerating voltage in many synchrotrons is generated by a voltage controlled oscillator (VCO) which is driven by a phase comparison between the phase of the bunched beam and the phase of the rf accelerating cavity gap voltage.[1] Frequently several rf accelerating cavities located at different parts of the ring must be used. In an accelerating regime with particle speeds significantly smaller than c it is normally necessary to sweep the rf frequency. In such a situation it is necessary that the cable delay from the VCO to all accelerating cavities be equal to that of the most distant cavity. Additional cable delay must be inserted to return a gap voltage phase signal to the phase comparitor. As accelerators grow larger the total cable delay which must be introduced into the VCO phase lock loop also grows larger. These large delays affect the stability and response characteristics of the phase lock loop and dictate, to some extent, the nature of other compensating networks which may be introduced. The stability of such systems has been investigated by Gumowski[2] and others[3] for both the linearized small signal case and for large signal cases. It is the purpose of this note to present a graphical way of examining these effects which may assist in understanding the requirements for loop compensating networks.

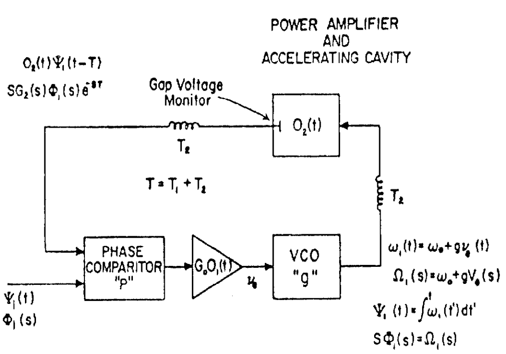

Fig. 1. Block Diagram Showing the Essential Features of a Beam-RF Phase Lock Loop.

Figure 1 is a block diagram representing an accelerator phase lock loop. The input signal $\phi_i(t)$ would be a signal representing the beam phase, delayed by a time T_2 to make it compatible at all frequencies with the accelerating cavity gap voltage. Assume that the phase comparitor creates an output voltage which is related linearly to the phase difference $\phi_i(t) - \phi_o(t-T)O_2(t)$ through the factor p (which has dimensions of volts and magnitude about 1 volt per radian). $O_1(t)$ is an operator function of time which represents a linear passive network and an amplifier with real gain G. The VCO generates an output angular frequency

$$\omega_1(t) = \omega_0 + gv_e$$

where ω_0 is the rest (or zero input voltage) frequency, v_e is the error voltage delivered to the VCO input, and g is the response characteristic of the VCO, having dimensions of (volt-seconds)$^{-1}$ and magnitude typically of order $2\pi \times 10^6$ radians per volt-second. $O_2(t)$ is an operator function of time describing the response of the rf cavity to small amplitude frequency modulated signals. To first order the cavity can be represented by a phase lag network with corner frequency determined by the cavity time constant $2Q/\omega$. The LaPlace transform of an operator function of time operating on some function of time is expressed:

$$\mathcal{L}\, O_1(t)\phi(t) \rightarrow G_1(s)\Phi(s).$$

The transformed transfer function of the phase-lock-loop is:

$$\Phi_o(s) = \frac{gPG_0G_1(s)\Phi_i(s)}{s + gPG_0G_1(s)G_2(s)e^{-sT}} = \frac{gPG_0G_1(s)}{D(s)}\,\Phi_i(s). \qquad (1)$$

Let $gpG_0G_1(s)G_2(s) = AG(s)$ with dimensions of (time)$^{-1}$. The small signal transient response of the phase-lock loop can be expressed in terms of the roots of the characteristic equation D(s)[1,3]. In general negative real parts of the roots result in damped solutions while positive real parts result in anti-damped (or exponentially growing) solutions. Imaginary parts of the roots contribute oscillatory components which decay or grow depending on the sign of the real parts. It is clear that only solutions with negative real parts are allowed for stability. It is very probable that imaginary components to the roots, which allow damped oscillatory response to noise trasients, may contribute to phase space dilution of bunched beams resulting from bucket shaking at frequencies near the synchrotron frequency. This is especially true in proton machines with long acceleration times, or those which store bunched beams. It may, in some cases, be possible to increase the frequency of the damped oscillatory response to such an extent that no adverse effect occurs, but when appreciable delay is introduced, the required increase in frequency will almost certainly generate an anti-damped solution. So, while small signal stability is assured by admitting only those feedback configurations which result in roots with negative real parts, an added requirement, that the imaginary parts of the roots be zero, may be imposed.

Single Phase Lag Network

Consider first the very simple case in which the amplifier response is independent of frequency, i.e. $G_1(s) = 1$. If the cavity transfer function is represented by a single lag network,

$$G_2(s) = \frac{\alpha}{s+\alpha}, \quad \alpha = \omega/2Q,$$

then the characteristic equation becomes

$$D(s) = s(s+\alpha) + \alpha A e^{-sT}. \qquad (2)$$

For the limiting case of T=0, a root locus plot as a function of gain A is shown in Figure 2a. For A=0 a pair of real roots exist at s=0 and s=-α. As the gain is increased, these roots move together along

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negative real axis, and for gain $A = \alpha/4$ the roots meet at $s = -\alpha/2$. For higher gains a conjugate pair of complex roots with constant negative real part appears, with the complex part increasing without bound as the gain is increased.

Since we are concerned only with negative real roots, it is useful to equate s in equation 2 with a negative real number $-\delta$.

$$s = \sigma + j\omega \rightarrow -\delta. \tag{3}$$

Then equation 2 can be rewritten, expressing the gain A as a function of δ

$$A(\delta) = \frac{\delta(\alpha - \delta)e^{-\delta T}}{\alpha} \tag{4}$$

This expression is shown in Figure 2b for the case $T = 0$ (solid curve). The behavior of the negative real roots can now be examined by observing the points at which constant values of A intercept the curve. For $A = 0$ the intercepts are at $\delta = 0;\alpha$. As A is increased the roots move toward each other and coalesce at the maximum point on the curve where $A = \alpha/4$. For values of A larger than that no real solutions exist.

The delay time T can be expressed in terms of the inverse of the corner frequency α. The dashed curve in Figure 2b is $A(\delta)$ plotted with the delay time $T = 0.5\alpha^{-1}$. The effect of introducing the delay is immediately obvious. The two roots start again at 0; $-\alpha$ but the decaying exponential function forces the magnitude of the entire curve down so that, for the chosen value of T, the maximum allowable gain is reduced to about 0.2α.

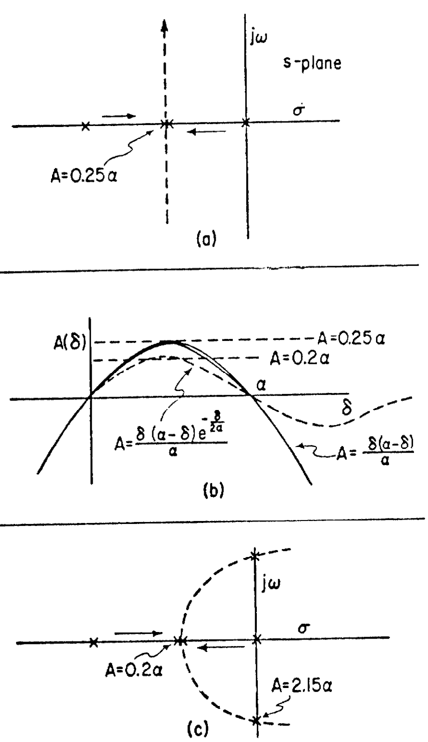

Fig. 2a. Root locus plot of phase-lock look with single lag network. b. Loop gain plotted as a function of the negative real part of the transform variable s. Solid curve-no delay. Dashed curve-moderate delay. c. Root locus plot with single lag network and moderate delay.

Another limitation resulting from the delay can be seen by examining Equation 2 for purely imaginary solutions, i.e. let $s = j\omega$. This results in the pair of equations

$$\begin{aligned} A\alpha \cos \omega T &= \omega^2 \\ A \sin \omega T &= \omega. \end{aligned} \tag{5}$$

For the same delay, $T = 0.5\alpha^{-1}$, as A is increased a solution occurs when $A \approx 2.15\alpha$ and $\omega = 1.3\alpha$. For gains larger than this the complex poles move into the right half-plane and oscillations with exponentially growing amplitudes will occur. A root locus plot for this situation is shown in Figure 2c.

For the simple phase lag situation the gain of the loop is limited to magnitude less than 0.25α. For an rf system operating near 40 MHz with Q =1000, $\alpha = 4\pi x10^4$. A natural "gain" factor for the PLL is the product of the VCO and the phase comparitor "gain" functions, which might well be $2\pi x10^6$. So the gain limitations imposed by loop stability require attenuation of the signal levels by a factor near 10^3 with attendant degeneration of performance of the feedback loop.

Phase Retard Compensation

The PLL transient performance can sometimes be improved by introduction of an active network such that the product $O_1(t)O_2(t)$ is a phase-retard (lag-lead) network with the transformed transfer function

$$G(s) = \frac{s + \alpha}{ks + \alpha} \tag{6}$$

where k is a factor usually in the range from 10 to 100. If s is again limited to values $-\delta$ the characteristic equation becomes

$$-\delta(\alpha = k\delta) + A(\alpha - \delta)e^{-\delta T} = 0 \tag{7}$$

or, again expressing the gain as a function of δ

$$A(\delta) = \frac{\delta(\alpha - k\delta)e^{-\delta T}}{(\alpha - \delta)} \tag{8}$$

$A(\delta)$ is shown in Figure 3a for the zero delay case.

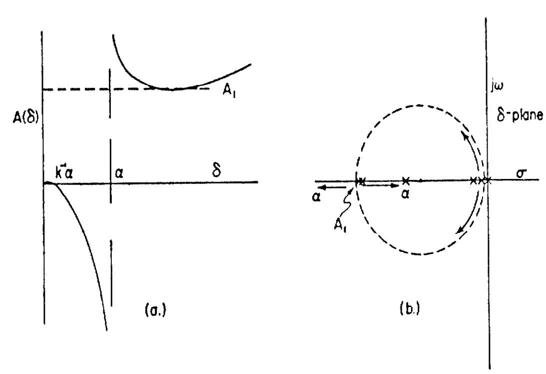

Fig. 3a. $A(\delta)$ for PLL with phase retard compensation and no delay. b. Corresponding root-locus plot.

For zero gain real roots exist at $\delta = 0$; αk^{-1}. At a very small gain the real roots coalesce near the origin and there follows an interval of gain for which no real roots exist. For much larger values of gain a pair of real roots appears near 2α and as the gain is increased toward infinity one root moves to α while the other moves toward infinity. For $k = 10$ the value of gain at which a pair of real roots reappears is about $A = 39\alpha$. The well known root locus plot for this system is shown for reference in Figure 3b.[4]

The effect of finite delay can now be seen graphically by examining $A(\delta)$ for several values of the delay time T. Clearly the effect of the decaying exponential factor is to decrease $A(\delta)$ for large δ. In Figure 4a the function $A(\delta)$ is shown for a "small" delay T. For zero gain an additional root appears at infinity. As the gain is increased this root moves to smaller values of δ. $A(\delta)$ still has a minimum near 2α where two real roots appear. As the gain is increased beyond this point one root moves to α, the other coalesces with the root moving from infinity when A reaches the maximum of $A(\delta)$ at some large value of δ. Beyond this point these two roots become complex and eventually move into the right half-plane. There now exists a limited range of higher gain in which real roots can exist and the detailed nature of this range depends on the relationship which exists between k, α, and T. A root locus plot for this case is shown in Figure 4b.

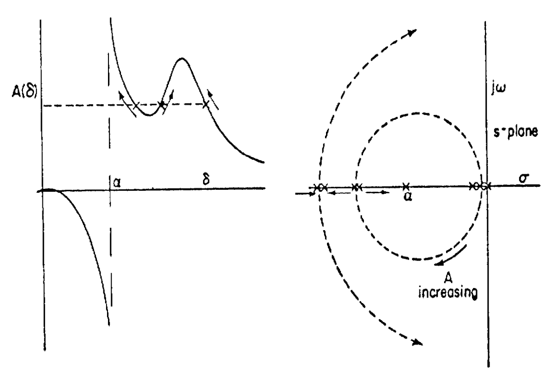

Fig. 4a. $A(\delta)$ for phase retard compensation and small delay. b. Corresponding root-locus plot.

As the delay is increased without changing α the minimum-maximum interval in $A(\delta)$ coalesces into an inflection point. The root-locus plot for that case just touches the real axis at a single point (after the low gain region of real roots). For larger delays the one real root moves from infinity to α while the pair of complex roots dip toward the real axis, then return back into the positive half-plane for very large gains.

By examining the derivative of Equation 8 with respect to δ for large k it is easy to show that the requirement for some region of high gain real roots is αT be less than 0.17. As the delay is increased the upper corner frequency α must be decreased resulting in a decrease in the allowable loop gain. For αT products which allow a range of real roots it is a simple matter to calculate the range of allowable gains. If, for example, $T = 10^{-6}$s, $\alpha = 2\pi x10^4$ (corner frequency 10 KHz), and k = 10, the allowable gain ranges from 35α to 65α. A gain of 50α is $\pi x10^6$ s^{-1}, which is still relatively small compared to the "gain" available from the VCO described earlier.

The phase retard network described, standing alone, is somewhat unrealistic because, in the presence of the rf cavity corner frequency, it implies an amplifier with gain increasing with frequency without bound. An additional lag network with a corner frequency well above α can be added to the formalism easily. It is apparent that the effect of such an additional lag network will be to exacerbate the effect of the delay by further decreasing the function $A(\delta)$ at large δ, making it more difficult to realize a region of real roots at large gain.

Conclusion

The large delays introduced into beam control feedback systems by the requirement that widely spaced components be contained within the loop can result in deterioration of feedback loop performance. If the loop is to have non-oscillatory damped response to noise transients, the delay introduces stringent requirements on the characteristics of other networks within the loop and on the loop gain. These requirements can be extracted rather easily for various internal network configurations by expressing the "gain" A as a function of the negative real part of the transform variable.

References

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