

**Abstract**

In 1978 the magnetic field shapes of six ISABELLE prototype dipole magnets were measured in detail. All the harmonic terms that are forbidden by symmetry in dipole magnets are found to have values that are zero to within the allowed tolerances. This result indicates that the random errors in conductor placement are within tolerances and that the assembly techniques are satisfactory in this regard. However, the first allowed error term in a dipole magnet, the sextupole term, is much too large, and varies greatly from magnet to magnet resulting in field errors at the edge of the desired good field region that are  $10^{-3}$  of the central field value, whereas the tolerable field errors are of the order of  $10^{-4}$  of the central field value. This large sextupole term has been traced to assembly errors which yield a coil assembly which instead of being circular in cross section is racetrack shaped, with the vertical axis being 10 to 20 thousandths of an inch greater than the horizontal axis. This problem will be corrected in the future production series.

**Introduction**

Any static magnetic field in the bore of an ISABELLE dipole magnet can be completely described by Equation 1:

$$\vec{B} = \hat{r} \sum_{n=0}^{\infty} B_n r^n \{ a_n \cos(n+1)\theta + b_n \sin(n+1)\theta \} \quad (1)$$

$$+ \hat{\theta} \sum_{n=0}^{\infty} B_n r^n \{ -a_n \sin(n+1)\theta + b_n \cos(n+1)\theta \}.$$

Equation 1 can be derived directly from Maxwell's equations and is actually surprisingly simple since it involves only two sets of coefficients, the  $a_n$  and the  $b_n$ . It is customarily dealt with in a simpler form which considers only the vertical and horizontal fields along the x-axis:

$$B_y = B_0 (1 + b_1 x + b_2 x^2 + b_3 x^3 + \dots + b_n x^n + \dots) \quad (2)$$

$$B_x = B_0 (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n + \dots). \quad (3)$$

In a dipole magnet there should be no horizontal field along the x-axis, and therefore all the  $a_n$  should be zero. The vertical field should be symmetric about  $x = 0$  and therefore all the  $b_n$  for odd  $n$  should be zero. Thus, our discussion of the field shape can be broken up into two classes of terms, those that are allowed and those that are forbidden. The forbidden terms should all be zero within certain tolerances. These tolerances are dictated by the requirements of the storage ring designers on the one hand, and by what it is practical to build on the other hand. Forbidden terms are generated typically by an accumulation of random errors in current block placement, by mis-centering the conductor coil in the iron core, or by building some asymmetry into the coil.

The allowed terms are those that appear in the general expression for the field shape of a dipole magnet:

$$B_y = B_0 (1 + b_2 x^2 + b_4 x^4 + b_6 x^6 + \dots). \quad (4)$$

In the ISABELLE dipole, which has six current blocks, the first 5 terms,  $b_2$  through  $b_{10}$ , have been set to zero in magnets through MK-XIV while in subsequent magnets the design has been modified to give  $b_2$ , the sextupole term, a value desirable for storage ring

operation. However, any systematic errors which occur in producing these magnets will show up as errors in the allowed terms, and errors of this sort are often the most intractable.

**Field Measuring Techniques**

The harmonic expansion of the field shape given in Eqs. (2) and (3) is very useful since the system for measuring the field shape gives results directly in terms of the expansion coefficients. We use an array of harmonic "Morgan" coils; each coil is about one meter long, and all are mounted concentrically on a fiberglass cylinder of 3.5 centimeter radius which spins at 300 rpm in the bore of the magnet. The present system has six coils, the signal from each coil being fed to a digital lock-in amplifier and then to a computer which prints out in a few seconds the  $a_n$  and  $b_n$  for  $n = 0$  to  $n = 5$ . An array of five meter long coils is being designed to measure the field over the full length of a magnet. The measurements reported here are for the coil located at some arbitrary position along the magnet axis, and are not values integrated along the full magnet length. Scanning the coil along the magnet axis gives the results shown in Fig. 1, which shows that the sextupole term,  $b_2$ , varies along the magnet length but within the tolerances expected from a random 0.002" error in conductor block placement as is discussed below. In addition, all the forbidden terms due to the structure of the magnets have been found to be independent of the primary field strength, as shown in Fig. 2 for the quadrupole term.

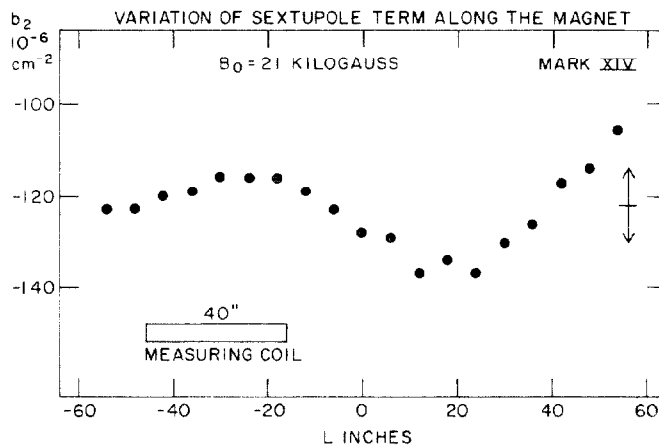


Fig. 1. Variation of Sextupole Term Along the Length of Magnet Mark XIV. This plot shows the variation of  $b_2$  along the length ( $L$ ) of the magnet over a range of  $\pm 60$ " from the magnet center as measured using a 40" long measuring coil. End effects are excluded. The definition of  $b_2$  is  $B = B_0 (1 + b_2 x^2 + \dots)$ . The design value of  $b_2$  is zero with an expected error of  $\pm 8 \times 10^{-6} \text{ cm}^{-2}$ . The variation along the magnet is within the expected tolerance but the average value is far from the design value.

A significant problem in using a harmonic analysis coil, results from the fact that a coil which is off-center in a higher order harmonic field will give a spurious signal. This is a particular problem in measuring the quadrupole ( $b_1, a_1$ ) and octupole ( $b_3, a_3$ ) terms since our magnets have strong sextupole ( $b_2$ ) and decapole ( $b_4$ ) terms. However, the magnets also contain

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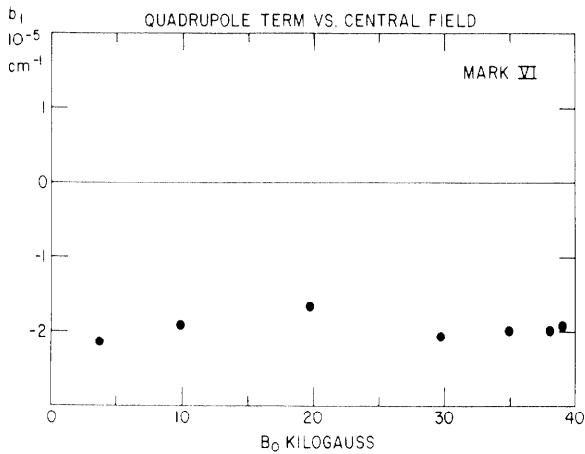


Fig. 2. Quadrupole Terms as a Function of Central Field for Magnet Mark VI. This plot shows that as  $B_0$  varies from 4 to 39 kilogauss, the quadrupole term ( $b_1$ ) is independent of  $B_0$ . The definition of  $b_1$  is  $B = B_0 (1 + b_1 x + \dots)$ . The expected value is  $(0 \pm 8) \times 10^{-5} \text{ cm}^{-1}$ . The measured value is  $(-2) \times 10^{-5} \text{ cm}^{-1}$  with a spread of  $(\pm 0.1) \times 10^{-5} \text{ cm}^{-1}$  which is attributable to the measurement accuracy and not to any changes in the magnet.

sextupole and decapole trim coils, so it is possible to set  $b_2$  and  $b_4$  to zero and, therefore, to get good measurements of  $b_1$ ,  $a_1$ ,  $b_3$ , and  $a_3$ . This process also provides sufficient data for crosschecking all these terms, and requiring that the measured fields be consistent with those produced by the trim windings, and that the off-center distance deduced for the quadrupole coil agree with that deduced for the octupole coil.

#### Required Tolerances

The rms errors in each field harmonic in each magnet which can be tolerated by the ISA storage ring have been calculated<sup>2</sup> and are given in Table I.

Table I. Permitted rms Errors

Term	rms Error	Unit
	$\sigma_n$	
$b_1, a_1$	8	$10^{-5} \text{ cm}^{-1}$
$b_2, a_2$	8	$10^{-6} \text{ cm}^{-2}$
$b_3, a_3$	15	$10^{-7} \text{ cm}^{-3}$
$b_4, a_4$	28	$10^{-8} \text{ cm}^{-4}$
$b_5, a_5$	50	$10^{-9} \text{ cm}^{-5}$

These errors correspond to an rms error of 0.002 inches in placing each of the six conductor blocks in a quadrant. Included in  $b_1$  and  $a_1$  is an additional error of 0.002 inches in centering the conductor coil in the iron core. These tolerances correspond to field errors at the edge of the warm bore tube of about one part in 10,000 for each of the higher harmonics, and several parts in 10,000 for the quadrupole term. As presently specified, the ISABELLE dipoles must meet these rms tolerances. The subject of this paper is a discussion of how close we are to achieving this goal.

#### Forbidden Terms

In the past year we have measured six dipole magnets. The measured forbidden harmonics are shown in

Table II where each term is shown divided by the allowed rms errors from Table I. These results are encouraging in that most of the measurements are within one standard deviation of zero, as they should be. This indicates that we are achieving the required accuracy in our assembly procedures. Figure 3 shows a histogram of these 48 measurements and compares it with the expected normal error curve, which shows good agreement. Examination of Table II shows that seven of the nine measurements of greater than  $2\sigma$  are skew terms in magnets XI and XIV. In both of these magnets we have metallurgical problems which resulted in the upper and lower coil halves being different. Large skew terms are therefore expected although not fully understood in detail. Thus, we attribute these terms to a systematic error and not to random errors in construction.

#### Allowed Terms

The ISABELLE dipole is a cold iron magnet; the iron is close to the coil so that, at a central field of 50,000 gauss, the iron is strongly saturated. The

Table II. Measured Forbidden Harmonics Divided by Permitted rms Errors

Harmonic	Magnet					
	VI	VII	IX	XI	XIV	B0001
$b_1$	-.2	1.2	.6	.1	.2	-.6
$b_3$	.7	-1.5	.2	1.3	-1.3	-.1
$b_5$	.3	1.8	1.2	1.3	.1	.1
$a_1$	-1.6	-1.5	1.5	4.4	1.5	-.6
$a_2$	-.9	.9	-2.5	-2.6	-1.5	.4
$a_3$	0	.8	0	5.5	7.5	2.0
$a_4$	-.5	-.4	-.3	-.2	2.2	-.7
$a_5$	-.4	-1.8	.8	-2.3	-4.0	.1

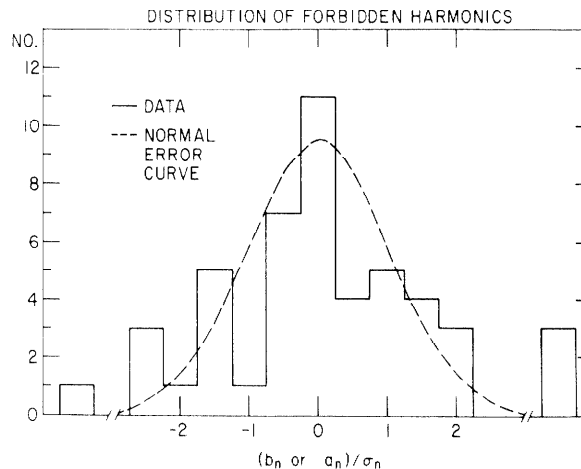


Fig. 3. Distribution of Forbidden Harmonics. This plot is a histogram of the distribution of the measured values divided by the permitted rms errors of Table I for the harmonic terms which are forbidden in a dipole magnet. There are four measurements off-scale but otherwise the results are in good agreement with an error curve normalized to forty-eight measurements.

iron saturation maintains dipole symmetry so that the allowed terms are affected. Figures 4 and 5 show the sextupole ( $b_2$ ) and decapole ( $b_4$ ) terms as a function of  $B_0$ , the central field. At low field, induced currents are permanently circulating in the superconducting filaments giving a large effect which decreases as  $1/B$  and changes sign for decreasing central field - a case not shown. At intermediate fields,  $b_2$  and  $b_4$  are flat with values determined by conductor placement. The  $b_2$  and  $b_4$  values reported here are for this flat region. At high field the iron starts to saturate and  $b_2$  and  $b_4$  change rapidly. At very high fields the iron is fully saturated, the field shape is again determined by the conductor placement, and  $b_2$  and  $b_4$  have the same values they had at intermediate fields. The magnets do not operate at this high a field, but the figures do show the turnovers in  $b_2$  and  $b_4$ . Higher order terms are also affected, but the effects are small and are presently ignored. In order to correct for this saturation each dipole magnet has wound on its bore tube a sextupole and decapole coil each type of which will be powered in series around the ring to compensate for the saturation. It should be noted that the dipole has 800,000 ampere-turns, while the correction coils need only 800 ampere-turns for the sextupole and 90 ampere-turns for the decapole. These and other trim coils are also used for working line corrections.

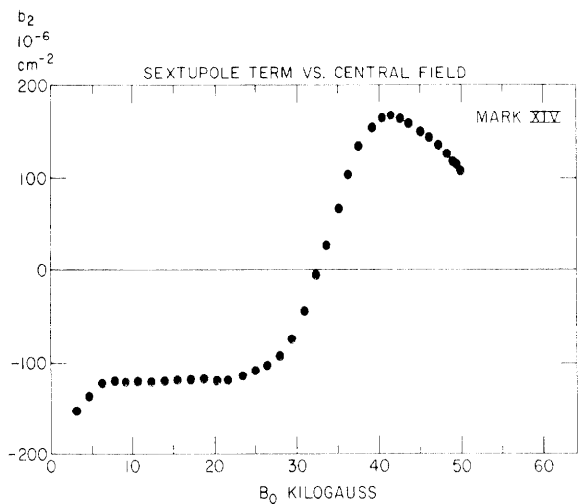


Fig. 4. Sextupole Term as a Function of Central Field for Magnet Mark XIV.

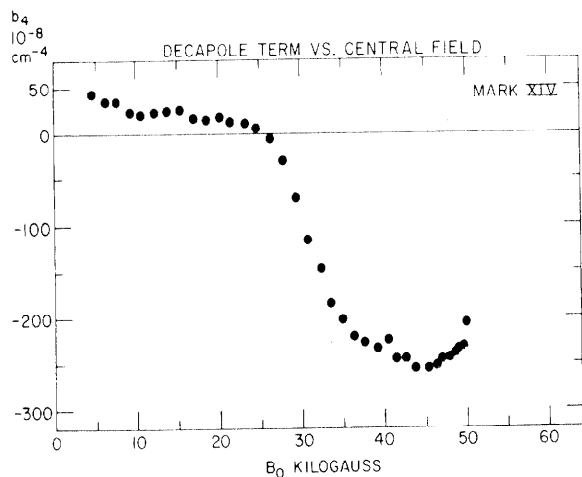


Fig. 5. Decapole Term as a Function of Central Field for Magnet Mark XIV.

Table III lists the measured values of  $b_2$  and  $b_4$  for all the ISABELLE dipoles. It is clear that the results are far from the design values, and that no two of these magnets are similar enough for use in the same accelerator. It should be noted that achieving a good field shape has been a goal secondary to that of achieving the required peak field and the ability to tolerate a quench without coil damage. Figure 6 shows a plot of the data for the sextupole term. The spread in the values of the sextupole terms is serious and corresponds to an error in field strength at the edge of the bore of about one part in 1,000, which is nearly ten times too large.

Table III. Measured Sextupole and Decapole Terms for all ISABELLE Dipoles

Magnet	$b_2$ ( $10^{-6} \text{ cm}^{-2}$ )	$b_4$ ( $10^{-8} \text{ cm}^{-4}$ )
Design	$0 \pm 8$	$0 \pm 28$
I	-40	60
II	-150	140
III	-100	260
IV	-160	270
V	-150	-50
VI	-66	50
VII	-50	140
IX	20	20
XI	-55	27
XIV	-130	20
Design	$-286 \pm 8$	382
B0001	-500	270

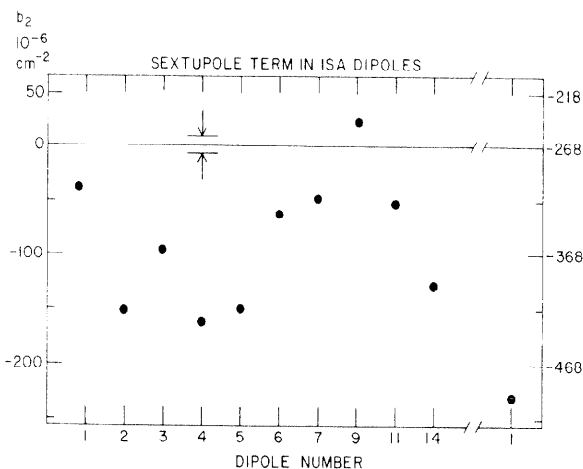


Fig. 6. Sextupole Term in ISABELLE Dipoles. This plot shows the sextupole term ( $b_2$ ) measured for central field values in the range of 10 to 20 kilogauss for Magnets I, II, III, IV, V, VI, VII, IX, XI, and XIV for which the design value is  $(0 \pm 8) \times 10^{-6} \text{ cm}^{-2}$  and for Magnet B0001 for which the design value is  $(-268 \pm 8) \times 10^{-6} \text{ cm}^{-2}$ .

The source of this sextupole field has been found on recent magnets to be faulty assembly procedures resulting in the magnet coil cross section being race-track shaped rather than circular. Figure 7 shows the measured positions of the coil blocks in an assembled coil compared with their design positions as measured

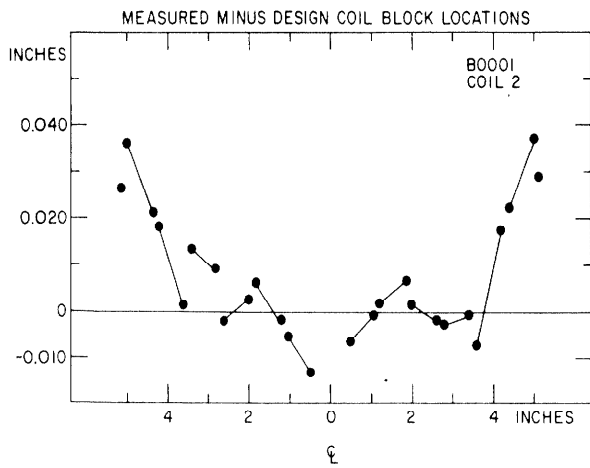


Fig. 7. Measured Minus Design Coil Block Locations. This plot shows the difference between the measured position and the design value of the coil block edges as measured along the outer circumference of the coil from the center of the center post  $\pm 5$  inches to the midplane of the magnet. These results show that the two blocks nearest the midplane are  $10$  to  $20 \times 10^{-3}$  inches too thick.

along the outside circumference of the coil from the center. The two blocks near the midplane are oversized. The field calculated for this shape gives good agreement with the measured value. This assembly problem can be solved and we expect to control the sextupole more tightly. We have yet to demonstrate that we can achieve the tolerances on the allowed terms specified by the theorists, but the good results on the forbidden terms are encouraging with regard to our ultimate assembly accuracy.

#### References

1. G. H. Morgan, Proc. 4th Inter. Conf. on Magnet Technology, Brookhaven, 787 (1972).
2. G. Parzen, Particle Accelerators, 6, 237 (1975).