IEEE Transactions on Nuclear Science, Vol. NS-26, No. 3, June 1979

SIMULATIONS OF FREE ELECTRON LASER

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Abstract

The generation of coherent electromagnetic radiation by the interaction of a relativistic electron beam with a static helical magnetic field is investigated using one- and two-dimensional relativistic electromagnetic plasma simulation codes. In the one-dimensional simulations, we observed the coupling between the negative energy beam mode and the positive energy electromagnetic wave. Substantial growth rate ($\omega_i \sim 0.1\ \omega_{pe}$) of the unstable electromagnetic wave has been observed and efficiency of radiation production is found to be between 25-30% depending on various parameters. In the two-dimensional simulations, we observed a decrease in the growth rate, but increased efficiency due to the decrease in the phase velocity of the unstable electrostatic wave. In addition, we also observed waves propagating at an angle with respect to the electron beam. Consequently, the beam is bunched in the radial as well as the axial directions. These waves are believed to be generated by another instability which saturates at relatively low level.

Introduction

In recent years, there has been considerable interest in the free electron laser which generates coherent electromagnetic radiation using relativistic electron beams interacting with a static helical magnetic field. Due to the relativistic Doppler effect, the wavelength of the radiation generated by the electron beam is roughly $\lambda_0/2\gamma^2$ where λ_0 is the wavelength of the helical magnetic field and γ is the relativistic factor corresponding to the velocity of the electron beam.[1,2] By varying the beam velocity and/or the wavelength of the helical magnetic field, one can make a laser which is continuously tunable throughout the entire electromagnetic spectrum. Such source of tunable coherent electromagnetic radiation is highly desirable for application to many areas, particularly in plasma heating and diagnostics, laser-pellet fusion, isotope separation, and for diagnostics in atomic, molecular and solid state physics.

Experimental investigations of free electron laser were carried out in various places.[3-5] Experiments at NRL[3] and Columbia University[4] demonstrated the generation of high power submillimeter microwaves, whereas, in Stanford's experiment,[5] a free electron laser oscillator was operated above threshold at a wavelength of 3.4 μm. The theory of free electron laser has been studied extensively in one spatial dimension in the linear regime.[6-8] The nonlinear saturation was found to be due to the trapping of beam by the unstable electrostatic wave.[6] This paper confines itself to the comparison of the one- and two-dimensional simulations.

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Results of One-Dimensional Simulations

We used a one-space and three-velocity electromagnetic relativistic code[6] to study the coupling between the negative energy beam mode and the electromagnetic wave through a static helical magnetic field. In Fig. 1, the unstable electromagnetic and electrostatic spectrum are shown at $T = 40\ \omega_{pe}^{-1}$ for the case with $\lambda = 2.0$ and $\omega_{ce} = 0.7\ \omega_{pe}$, where ω_{ce} is the electron cyclotron frequency and ω_{pe} is the plasma frequency. At this time, the instability has

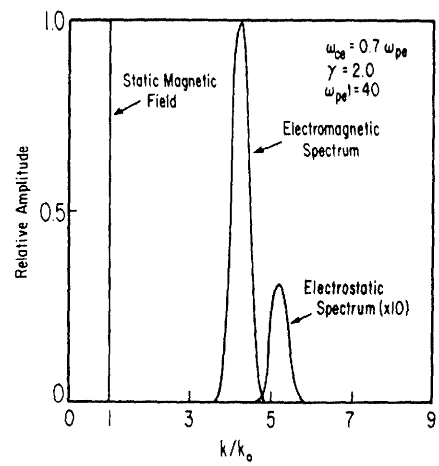

Fig. 1. The unstable electromagnetic and electrostatic spectrum at $T = 40\ \omega_{pe}^{-1}$.

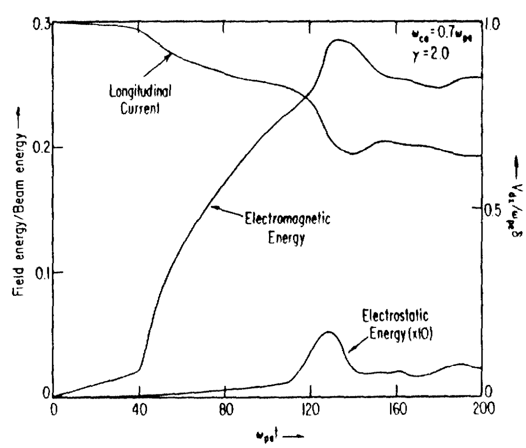

Fig. 2. Time histories of the beam current, the electromagnetic and the electrostatic energy.

started and the waves have grown to considerable amplitude. The wavenumber matching conditions are

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clearly satisfied between the unstable waves and the helical magnetic field. From the time evolution of the most unstable mode, we obtained the maximum growth rate, $\omega_i = 0.105\ \omega_{pe}$. In Fig. 2, we show the time histories of the beam current, the total electromagnetic energy and the electrostatic energy. At the time of saturation, about 28% of the beam energy was converted into electromagnetic energy. Besides the high frequency electromagnetic waves ($\omega \sim 2\gamma^2 k_0 V_0$) which propagate in the beam direction low frequency electromagnetic waves ($\omega \sim \omega_{pe}/\sqrt{\gamma}$) which propagate opposite to the electron beam were also excited. The low frequency waves had smaller growth rates but saturated much later due to their low phase velocities. In this run, the high frequency radiation accounts for 15% of the beam energy. The saturation mechanism was found due to the trapping of the electron beam by the unstable electrostatic waves, and the efficiency of radiation production is given by[6]

$$\eta = \frac{\gamma_0 - \gamma_{ph}}{\gamma_0 - 1} = \frac{\gamma_0^{1/2}(\gamma_0^2 - 1)^{1/2}\,\omega_{pe}}{(\gamma_0 - 1)(k + k_0)c} \qquad (1)$$

Results of Two-Dimensional Simulations

In one-dimensional simulations, all the waves propagate either parallel or antiparallel to the electron beam and, therefore, any instability which generates waves propagating at an angle with respect to the beam will be suppressed. In order to study the two-dimensional effects on the free electron laser, we have used a two-dimensional electromagnetic relativistic code, CCUBE,[10] to simulate the coupling of electromagnetic radiation with the negative energy electrostatic beam mode through the static helical magnetic field in rectangular geometry. Preliminary study from our simulations seemed to indicate that there was another instability which generated waves with large $k_\perp$. This instability developed somewhat earlier than the lasing process and saturated at relatively low level. However, the electron beam was seen bunched up in the perpendicular direction. This induced radial inhomogeniety could have significant effects on the lasing action of the free electron laser.

The configuration space of the electron beam in one of our simulations with $\gamma = 2.0$, $\omega_{ce} = 0.7\ \omega_{pe}$ and $k_0 c = 1.257\ \omega_{pe}$ is shown in Fig. 3 which exhibits

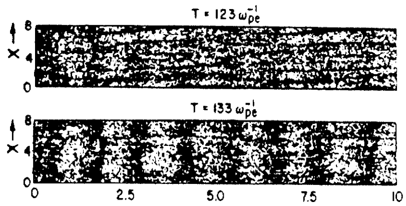

Fig. 3. The configuration space (x,z) of the electron beam at $T = 123\ \omega_{pe}^{-1}$ and at $T = 133\ \omega_{pe}^{-1}$.

bunching in the perpendicular direction at $T = 123\ \omega_{pe}^{-1}$. As the strong coupling between the electromagnetic wave and the slow beam mode develops, the electron beam is strongly bunched in the axial direction at $T = 133\ \omega_{pe}^{-1}$ as shown in Fig. 3.

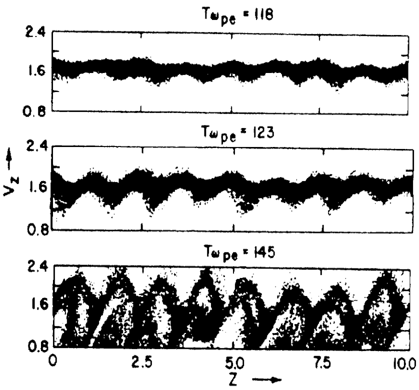

Fig. 4. The phase space (v_z, z) of the electron beam at three stages of the instability.

Figure 4 shows the phase space (v_z vs. z) at three different stages of the instability. The velocity (v_z) of the electron beam was strongly modulated by the unstable electrostatic wave which eventually caused the trapping of the electrons in the electrostatic wells and thus the instability saturated. The trapping process is clearly reflected by the formation of vortices in the phase space as shown in Fig. 4. From the time evolution of the field energy as shown in Fig. 5, we obtained a growth rate ($\omega_i = 0.077\ \omega_{pe}$) which is smaller than the value obtained in the previous one-dimensional simulation.

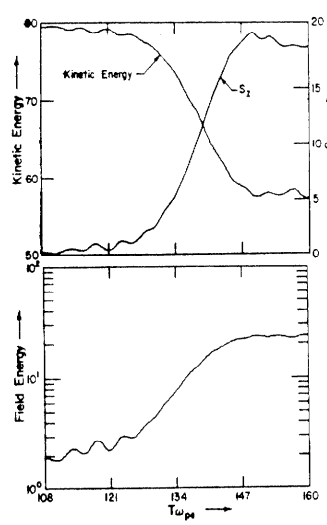

Fig. 5. Time histories of the field energy and the kinetic energy of the electron beam.

At the time of saturation, 28% of the beam energy was converted into radiation. The z-component of the Poynting vector is also shown in Fig. 5 which indicates a rapid increase of energy flow of electromagnetic field in the axial direction during the course of the instability. Since the simulation also showed that the energy flows in the other directions are much smaller in comparison to the axial direction, we believe that the lasing phenomenon is primarily along the direction of electron beam propagation. In Fig. 6, we show the time histories of the electric and the magnetic field at a fixed position as well as their Fourier transforms from which we can conclude that most of the field energy is

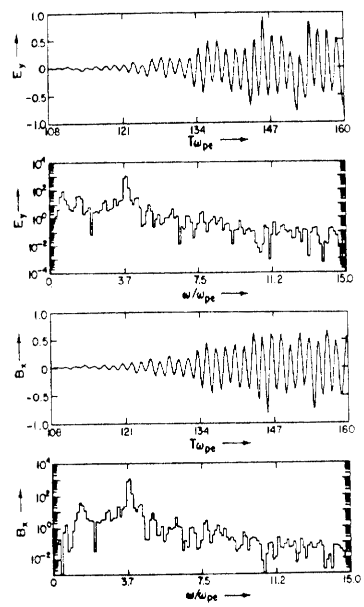

Fig. 6. Time histories of electric and magnetic fields and their Fourier transforms.

concentrated at the unstable high frequency electromagnetic waves. This is in contrast with the one-dimensional simulation in which the field energy was almost equally distributed among the high and low unstable electromagnetic waves. In Fig. 6 the most unstable electromagnetic wave has a frequency at 3.8 ω_{pe} whereas according to the one-dimensional theory,[6] the frequency should be at 4.5 ω_{pe}. This downshift in frequency could be due to the aforenamed instability which had modulated the beam in its perpendicular direction. The decrease in frequency of the unstable waves caused downshift in the phase velocity of the unstable wave and, therefore, the electrons will lose more energy before it can be trapped by the wave. The net result is that the instability could saturate at a higher level than expected due to the decrease in frequency of the unstable waves.

Conclusions

In comparing the one- and two-dimensional simulations, we found that two-dimensional effects could be important in the operation of the free-electron laser. We have observed radial inhomogeniety of the electron beam caused by waves propagating with an angle with respect to the beam. We also observed a decrease in frequency of the unstable electromagnetic waves as well as an increase in the efficiency of radiation production due to the decrease of the phase velocity of the unstable electrostatic waves. However, in order to further understand the underlying physics, a two-dimensional theory of the free electron laser needs to be developed.

Acknowledgments

The one-dimensional simulations were done by Thomas Kwan in cooperation with Dr. J. M. Dawson at the University of California, Los Angeles.

This research was supported by the U.S. Department of Energy.

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