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FREE ELECTRON LASER AMPLIFIER OPERATION IN STORAGE RING

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ABSTRACT

The steady state features of the free electron laser amplifier operating in a storage ring is presented. The electron distribution relative to the longitudinal motion and the amplifier output are evaluated as functions of the storage ring parameters and the laser power density. It is shown that the energy transferred from the electrons to the laser beam is proportional to the energy emitted via synchrotron radiation in the whole machine.

1. INTRODUCTION

In this paper we investigate, following the Refs. [1] and [2], the main features of a free electron laser (FEL) amplifier operating in a storage ring (SR). For an overview of the experimental and theoretical status of the FEL devices, see Refs. [1-5].

The machine layout is sketched in Fig. 1.

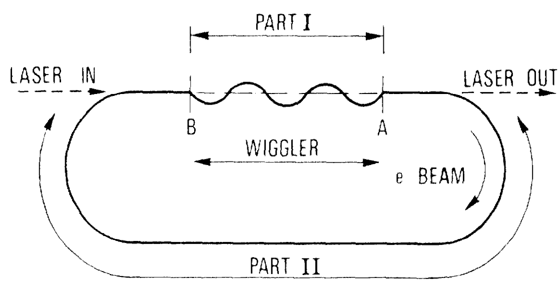

Fig. 1 - Machine layout

Part I is an achromatic special insertion (i.e. with zero off-energy closed orbit), where it is located the FEL region. Part II is a conventional SR. From A to B (part II) the electrons follow the usual synchrotron and betatron equations. From B to A (Part I) they experiment the FEL interaction, which, for the achromaticity of the insertion would affect the longitudinal motion only [6] (apart a betatron tune shift due to the focusing properties of the wiggler magnetic field). As a consequence we can neglect the transverse motion and only deal with the longitudinal one.

The operating conditions are,

i) small signal FEL regime [2],
ii) Inhomogeneous frequency broadening due to the transverse emittance negligible with respect to the homogeneous one [7],
iii) laser cross section greater than the electron beam (e-beam) one,
iv) wiggler magnetic field vertically polarized (in order to have larger radial admittance),
v) small amplitude synchrotron motion.

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2. FOKKER-PLANCK EQUATION FOR THE ELECTRON DISTRIBUTION

The linearized recurrence electron motion equations (turn by turn) are the usual synchrotron equations perturbed by the FEL interaction. Namely we have,

$$\begin{cases} \bar{z} = z - (cT \cdot \alpha_c)\,\bar{\varepsilon} \\ \bar{\varepsilon} = \varepsilon + (\omega_s^2 T/c\alpha_c)\, z - 2(T/\tau)\,\varepsilon + (u/E_0) + \delta\varepsilon, \end{cases} \quad (1)$$

where (z,ε) and $(\bar{z},\bar{\varepsilon})$ are the synchrotron coordinates before and after one machine turn,

z = longitudinal displacement from the synchronous particle,
ε = $(E-E_0)/E_0$ = relative shift from the synchronous particle energy (E_0),
T = revolution period, c = light velocity,
α_c = momentum compaction,
ω_s = synchrotron frequency,
u = synchrotron radiation quantum noise,
$\delta\varepsilon$ = relative energy change due to the FEL interaction,
U_0 = synchrotron energy radiated in one turn

$$\tau \cong T\,\frac{E_0}{U_0} = \text{longitudinal damping time.} \quad (2)$$

The energy change $\delta\varepsilon$ is a periodic function of the phase difference between the laser and the wiggler fields at the time entrance of the electron in the FEL. It can be shown that, if the laser power density is sufficently high, this phase behaves like a random variable and $\delta\varepsilon$ may be regarded as an additional noise perturbing the synchrotron motion [8]. In this connection the electron distribution function $f(z,\varepsilon|t)$ describing the longitudinal motion can be derived from a Fokker-Planck equation (FPE).

In our case the time independent FPE ($\partial f/\partial t = 0$), reads (see Ref. [8], Sect. 4),

$$\omega_s\left[\left(\frac{c\alpha_c}{\omega_s}\right)\varepsilon\,\frac{\partial f}{\partial z} - \left(\frac{\omega_s}{c\alpha_c}\right) z\,\frac{\partial f}{\partial \varepsilon}\right] + \\ + \frac{1}{T}\frac{\partial}{\partial\varepsilon}\left(2\,\frac{T}{\tau}\,\varepsilon f + \left(2\sigma_\varepsilon^2\,\frac{T}{\tau} + \frac{\langle\delta\varepsilon^2\rangle}{2}\right)\frac{\partial f}{\partial\varepsilon}\right) = 0, \quad (3)$$

where we have defined (< > = average value),

$$\sigma_\varepsilon = \left(\frac{\langle u^2\rangle\tau}{4E_0^2\,T}\right)^{\frac{1}{2}} = \text{r.m.s. energy spread for zero laser field } (\langle\delta\varepsilon^2\rangle = 0) \quad (4)$$

and we have utilized the relationships [6]

$$\langle\delta\varepsilon\rangle = (1/2)\,(\partial/\partial\varepsilon)\,\langle\delta\varepsilon^2\rangle, \quad \langle u\rangle = 0. \quad (5)$$

The diffusion term due to the FEL interaction reads (see Ref. [6], Sect. 3.5),

$$\langle\delta\varepsilon^2\rangle = \frac{T}{\tau}\left(\frac{\Delta\omega}{\omega}\right)_0 W\, G(2(\varepsilon+\varepsilon_0)/(\Delta\omega/\omega)_0), \quad (6)$$

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where we have defined,

$$W = \frac{\tau}{T} \cdot \frac{2\pi r_0}{m_0 c^3} \left(\frac{\Delta\omega}{\omega}\right)_0^{-4} \frac{\lambda^2 k^2}{(1+k^2)^2} \frac{dP_L}{dS}\ , \qquad (7)$$

$(\Delta\omega/\omega)_0$ = $\lambda_q/2L_w$ = homogeneous linewidth
λ_q, λ = laser and wiggler wavelength
L_w = wiggler length
k^2 = $(1/2)\ (r_0/m_0c^2)\ (B_0\ \lambda_q/2\pi)^2$
r_0 = classical electron radius
m_0c^2 = electron rest energy
B_0 = wiggler peak magnetic field
dP_L/dS = laser power density
ε_0 = $(E_0 - E^\star)/E_0$
$E^\star$ = $m_0c^2(\lambda_q(1+k^2)/2\lambda)^{\frac{1}{2}}$ = resonant energy [2]

$$G(x) = (\sin(\pi x/2)/(\pi x/2))^2 \qquad (8)$$

For $\tau \gg (2\pi/\omega_s)$ the integration of the FPE (3) is straightforward. In this connection $f(z,\varepsilon)$ is simply a function of the synchrotron invariant, and reads [8],

$$f(z,\varepsilon) = h(H) = h_0 \exp\left\{-\int_0^H dH'/R(H')\right\}, \qquad (9)$$

where h_0 is a normalization factor, H is the synchrotron invariant, defined by the equation

$$H = 1/2\ (\varepsilon^2 + (\omega_s/c\alpha_c)^2 z^2)/(1/2(\Delta\omega/\omega)_0)^2, \qquad (10)$$

and we have defined

$$R(H) = \tilde{\sigma}_\varepsilon^2 + W\ \frac{2}{\pi} \int_0^\pi \cos^2\theta\ G(\sqrt{2H}\ \cos\theta + \tilde{\varepsilon}_0)d\theta, \qquad (11)$$

$$\tilde{\sigma}_\varepsilon = 2(\Delta\omega/\omega)_0^{-1}\ \sigma_\varepsilon, \qquad \tilde{\varepsilon}_0 = 2(\Delta\omega/\omega)_0^{-1}\ \varepsilon_0. \qquad (12)$$

3. E-BEAM PARAMETERS

a) Energy spread and bunch length

The average invariant <H> is given by

$$\langle H\rangle = \sigma^2 = 2\pi \int_0^\infty Hh(H)\ dH. \qquad (13)$$

In Figure 2 it is plotted, as example, σ vs W in some cases. We see that (for $\tilde{\sigma}_\varepsilon \neq 0$), σ starts from the umperturbed value $\tilde{\sigma}_\varepsilon$ (W = 0) and, as W increases, approaches asymptotically the $\tilde{\sigma}_\varepsilon = 0$ case. For large W we have

$$\sigma \propto W^{\frac{1}{4}}\ (W \gg 1).$$

The energy spread and the bunch length are proportional to σ, namely

$$\langle\varepsilon^2\rangle^{\frac{1}{2}} = 1/2\ (\Delta\omega/\omega)_0\ \sigma,$$
$$\langle z^2\rangle^{\frac{1}{2}} = 1/2\ (\Delta\omega/\omega)_0\ (c\alpha_c/\omega_s)\ \sigma. \qquad (14)$$

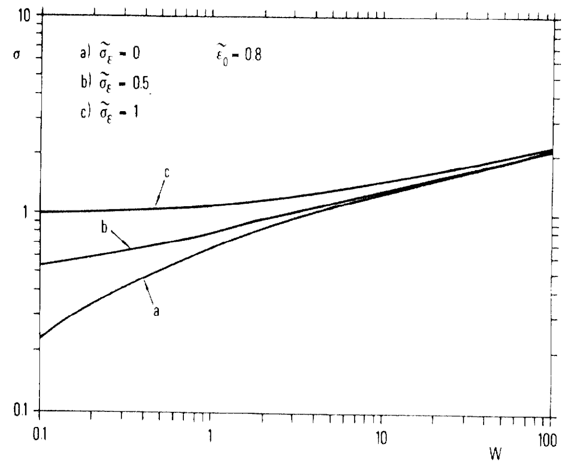

Fig. 2 - σ vs W

b) Longitudinal quantum lifetime

The energy spread blowing up reduces the longitudinal quantum lifetime τ_q, which reads (see Ref. [9], Sect. 5.8),

$$\tau_q = \tau\cdot 2\pi/(\tilde{\varepsilon}_M^2\ h(\tilde{\varepsilon}_M^2/2)),\ \tilde{\varepsilon}_M = 2(\Delta\omega/\omega)_0^{-1}\ \varepsilon_M, \qquad (15)$$

where ε_M is the SR energy acceptance. In Figure 3 it is plotted, as example, $\tilde{\varepsilon}_M$ vs W for

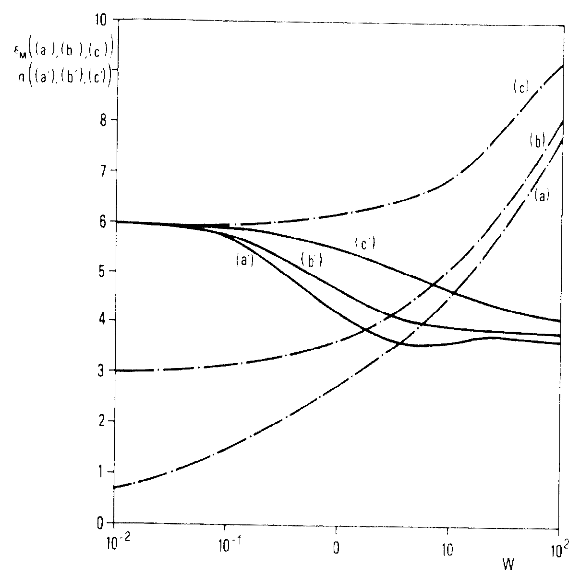

Fig. 3 - $\tilde{\varepsilon}_M$ ((a),(b),(c)) and n ((a'), (b'), (c')) for $\tilde{\varepsilon}_0$ = .8, $\tau_q/\tau = 1.8 \times 10^6$ (a),(a') $\tilde{\sigma}_\varepsilon$ = 0.1; (b),(b') $\tilde{\sigma}_\varepsilon$ = 0.5; (c),(c') $\tilde{\sigma}_\varepsilon$ = 1.0

$\tau_q/\tau = 1.8 \times 10^6$ (which, for $\tau \sim 10^{-2}$ s, gives $\tau_q \sim 5$h) and the ratio $n = \tilde{\varepsilon}_M/\sigma$. For W = 0 we have n = 6 (the well know "rule of thumbs" for gaussian beams), while for W > 0 ⇒ n < 6. Indeed the FEL interaction mainly affects the central part of the distribution.

4. FEL AMPLIFIER PARAMETERS

The energy variation of the laser beam is given by the folding of the small signal gain (SSG) for monoenergetic beam [1,2] with the electron distribution (9). The power profile (along the z-direction) is given by

$$p(\tilde{z}) = \int_{-\infty}^{+\infty} d\tilde{\varepsilon}\; h((\tilde{z}^2 + \tilde{\varepsilon}^2)/2)\; S(\tilde{\varepsilon} + \tilde{\varepsilon}_0), \qquad (16)$$

where we have put

$$\tilde{z} = 2(\Delta\omega/\omega)_0^{-1}\,(\omega_s/c\alpha_c)z,\quad \tilde{\varepsilon} = 2(\Delta\omega/\omega)_0^{-1}\,\varepsilon, \qquad (17)$$

$$S(x) = -\,dG(x)/dx = \text{SSG profile}. \qquad (18)$$

In Figure 4 it is plotted $p(\tilde{z})$ in some

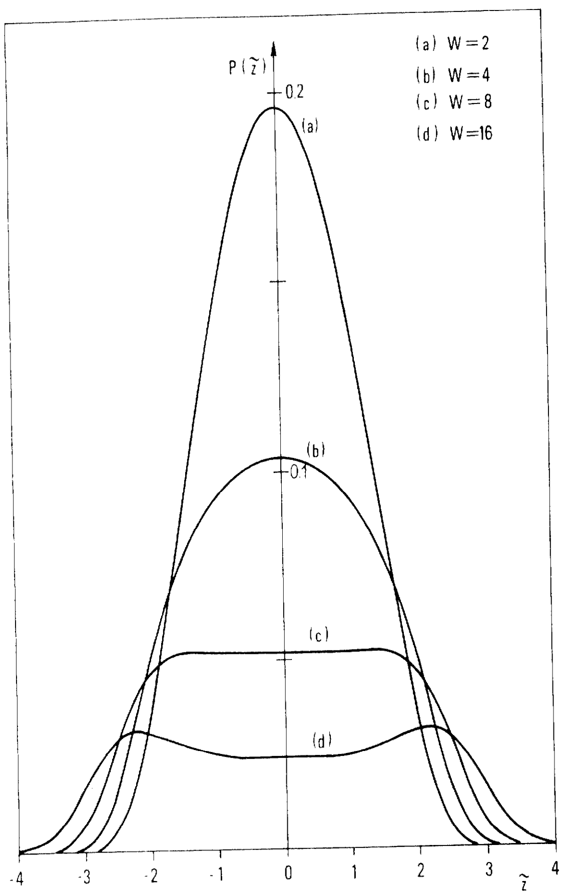

Fig. 4 - Output power profile $p(\tilde{z})$ for $\tilde{\sigma}_\varepsilon = 0.5$ and $\tilde{\varepsilon}_0 = 0.8$

cases. As W increases, the broadening of the e-beam energy distribution lowers the gain. Above a W threshold (in the case of Fig. 4 for $W \sim 8$), the power profile has a rippled structure with two peaks. As a consequence, there is the appearance of sidebands in the FEL output frequency spectrum.

The total energy gained by the laser beam is given by (see eqs (2), (5), (6), (7))

$$\Delta E = (\Delta\omega/\omega)_0\, U_T \cdot \chi(W, \tilde{\sigma}_\varepsilon, \tilde{\varepsilon}_0), \qquad (19)$$

where U_T is the total synchrotron energy radiated per turn by the whole beam, and χ is defined by

$$\chi = W \int d\tilde{\varepsilon}\, d\tilde{z}\; h((\tilde{z}^2 + \tilde{\varepsilon}^2)/2)\, S(\tilde{\varepsilon} + \tilde{\varepsilon}_0). \qquad (20)$$

The most interesting result in eq. (19) is that the energy radiated into the laser mode is proportional to the emitted synchrotron energy. As example, in Fig.5 it is plot-

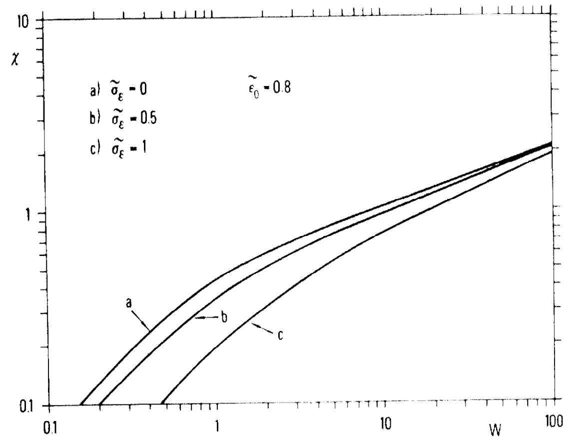

Fig. 5 - χ vs W

ted (in some cases) χ vs W. For $\tilde{\sigma}_\varepsilon = 0$ (negligible synchrotron radiation noise), we distinguish two limit regimes

i) $W \ll 1$, χ follows the SSG for monoenergetic beam

$$\chi\,(W \ll 1) = S(\tilde{\varepsilon}_0)\cdot W. \qquad (21)$$

ii) $W \gg 1$, the large e-beam energy spread lowers the gain, and χ depends weakly on W, namely

$$\chi\,(W \gg 1)\ \alpha\ W^{\frac{1}{3}}. \qquad (22)$$

The behaviour of χ for $\tilde{\sigma}_\varepsilon \neq 0$ is similar to the $\tilde{\sigma}_\varepsilon = 0$ case. We have merely a lowering of χ.

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