

THE FREE ELECTRON LASER AND ITS POSSIBLE DEVELOPMENTS\*  
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I. Introduction

The experiments<sup>1,2</sup> made at Stanford University starting from 1975 have shown that it is possible to build and operate a free electron laser. The theoretical works preceding and following these experiments have also given us a good understanding of the basic principles underlying a free electron laser and of the main characteristics of this device.

There is now much interest in the free electron laser as a powerful source of continuously tunable, coherent, electromagnetic radiation in a wavelength region extending from about 0.1 to 10  $\mu\text{m}$ , and possibly also beyond these limits. The advantages and disadvantages of using either a linear accelerator or an electron storage ring as the electron source are being explored. Many new ideas are being introduced to increase the output laser power, the efficiency of the system, and to extend the range of tunability in the 0.1 mm or in the soft x-ray region.

In this paper I will first review the basic characteristics of a free electron laser in a storage ring. In the end I will try to describe some of the more recent works and ideas and how they might improve the performance of the laser.

In discussing the free electron laser-electron storage ring system, we will see that, on the basis of our understanding of this system and of the results of the Stanford experiments, and without taking into account the possible improvements which have already been proposed, it is possible to build such a system capable of providing laser light in the region 0.1 to 10  $\mu\text{m}$  with an average power of the order of hundred to a thousand watt and a line-width,  $\Delta\lambda/\lambda$ , between  $10^{-7}$  and  $10^{-4}$ . This system would be a unique source of electro-magnetic radiation, providing both synchrotron radiation and laser light, of great usefulness for research in solid-state physics, photochemistry, surface physics, and biology.

If, using the proposed improvements, it would be possible to raise the output laser power to the ten kWatt level and the efficiency to several percent, such a system might be of interest also for industrial applications.

As a final remark, I want to point out that a free electron laser-electron storage ring system might also be used to provide laser light in the soft x-ray region, providing us again with a unique research tool.

II. Principles of Operation of a Free Electron Laser

The basic scheme of a free electron laser is shown in Fig. 1. A plane electromagnetic wave and a bunch of

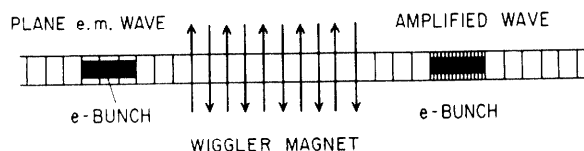


Fig. 1. Schematic representation of a free electron laser.

relativistic electrons are traversing a wiggler magnet. At the exit of the wiggler magnet, one can obtain a plane wave of higher intensity, while the average electron energy is decreased.

A wiggler magnet produces a field with alternating polarity along its axis. It can be either transverse or helical. For a transverse wiggler, the field can be represented by

$$\vec{B}_w = B_w \hat{x} \cos 2\pi \frac{z}{\lambda_w} \quad (2.1)$$

where  $z$  is the direction along the magnet axis and  $\lambda_w$  is the field period. The two axis  $x$  and  $y$  are orthogonal to  $z$  and  $\hat{x}$ ,  $\hat{y}$  are unit vectors in their directions. For a helical wiggler one has

$$\vec{B}_w = B_w \left\{ \hat{x} \cos 2\pi \frac{z}{\lambda_w} + \hat{y} \sin 2\pi \frac{z}{\lambda_w} \right\} \quad (2.2)$$

In the following we will consider the case of a helical wiggler. However, all the results can be applied, with only minor modifications, to the case of a transverse wiggler magnet.

A simplified description of how energy is transferred from the electrons to the radiation field can be given in two ways.<sup>3</sup> In a quantum mechanical description it is easier to consider the interaction between the electron and the radiation field in the frame of reference moving with the electron velocity parallel to the magnet axis. In this frame the wiggler magnetic field can be approximated by a plane electromagnetic wave. Electrons can backscatter the photons of this wave and the scattering can be enhanced (stimulated radiation) by the incoming plane wave.<sup>4,5</sup>

In a classical description the wiggler magnetic field determines the electron trajectory so that work can be done by the plane wave on the electrons. In other terms, the electron trajectory makes a nonzero angle with the direction of propagation of the incoming plane wave and the term  $\vec{E} \cdot \vec{v}$  has a nonzero average value.<sup>6</sup>

The initial works<sup>4,5</sup> on the subject used a quantum mechanical picture describing the interaction as stimulated radiation in the wiggler magnetic field or stimulated bremsstrahlung. However, when the radiation wavelength,  $\lambda$ , is larger than the electron Compton wavelength,  $\lambda_c$

$$\lambda > \lambda_c$$

and the number of photons in a volume  $\lambda^2 \lambda_w$ ,  $\lambda_w$  being the wiggler period, is much larger than one, the classical description is appropriate.<sup>7</sup> The second condition means that the fluctuations in the number of photons emitted or absorbed is small and can be neglected. In the following, we will assume that these conditions are satisfied and use a classical description.

The classical theories of the free electron laser can be divided into two groups. In one we find the calculations of stimulated radiation from a single electron.<sup>6,8</sup> In the second group are the calculations based on the Boltzmann-Maxwell equations or analogous techniques.<sup>9</sup> In the next section we will use the single electron approach to obtain the main characteristics of a free electron laser.

III. Single Particle Free Electron Laser Theory

In this section we will derive the equations describing the free electron laser following the approach

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developed by Colson.<sup>6</sup> We consider one electron moving in the wiggler magnetic field given by (2.2). We assume the wiggler to have  $N$  periods so that its total length is  $N\lambda_w$ . We also assume that the plane electromagnetic wave propagating along the wiggler axis (Fig. 1) is circularly polarized and its field is given by

$$\underline{E} = E_0 \{ \hat{x} \sin(\frac{2\pi z}{\lambda} - \omega t + \phi_0) + \hat{y} \cos(\frac{2\pi z}{\lambda} - \omega t + \phi_0) \} \quad (3.1)$$

$$\underline{B} = \hat{z} \times \underline{E} \quad (3.2)$$

The electron is supposed to be relativistic and with a small transverse velocity. If  $\underline{\beta}$  is the electron velocity, in light velocity units, we assume

$$\underline{\beta} = \beta_{\parallel} \hat{z} + \underline{\beta}_{\perp} \quad (3.3)$$

with

$$\beta_{\parallel} \approx 1, \beta_{\perp} \ll 1 \quad (3.4)$$

To order  $1/\gamma^2$  ( $\gamma$  being the electron energy divided by its rest mass,  $m_0 c^2$ ) the equations of motion of the electron can be written as

$$\dot{\underline{\beta}} = \frac{e}{m_0 c \gamma} \underline{\beta} \times \underline{B}_w \quad (3.5)$$

$$\dot{\gamma} = \frac{e}{m_0 c} \underline{E} \cdot \underline{\beta} \quad (3.6)$$

showing that the wiggler magnetic field determines the trajectory and the incoming wave the energy exchange.

Solving eq. (3.5), one finds that to order  $\beta_{\perp}/\gamma$ , the electron describes a helical trajectory<sup>10,11</sup> of pitch

$$\delta = \beta_{\perp} \quad (3.7)$$

and radius

$$\rho = \frac{\beta_{\perp} \lambda_w}{2\pi} \quad (3.8)$$

The electron energy remains constant along the trajectory. The perpendicular velocity is

$$\underline{\beta}_{\perp} = \frac{K}{\gamma} \{ \hat{x} \cos(2\pi z/\lambda_w) + \hat{y} \sin(2\pi z/\lambda_w) \} \quad (3.9)$$

with the wiggler parameter,  $K$ , given by

$$K = \frac{e B_w \lambda_w}{2\pi m_0 c^2} \quad (3.10)$$

Using (3.9), (3.1), the equation (3.6) describing the energy exchange can be written as

$$\dot{\gamma} = - \frac{e E_0 \beta_{\perp}}{m_0 c} \sin \phi \quad (3.11)$$

where the phase  $\phi$  is

$$\phi = (\frac{2\pi}{\lambda_w} + \frac{2\pi}{\lambda}) z - \omega t + \phi_0 \quad (3.12)$$

Since

$$\dot{z} = \beta_{\parallel} c \quad (3.13)$$

and

$$\beta_{\parallel} = \{ 1 - \frac{1+K^2}{2\gamma^2} \} \approx 1 - \frac{1+K^2}{2\gamma^2} \quad (3.14)$$

one can obtain from (3.12) that

$$\phi \approx \frac{2\pi c}{\lambda_w} \{ 1 - \frac{\lambda_w}{\lambda} \frac{1+K^2}{2\gamma^2} \} \quad (3.15)$$

We can define a resonant electron energy,  $\gamma_r^2$ , for which  $\phi = 0$ . This energy is given by

$$\gamma_r^2 = \frac{\lambda_w}{2\lambda} (1+K^2) \quad (3.16)$$

It is worth noting that electrons of energy  $\lambda_r$  moving in a wiggler of period  $\lambda_w$  and constant  $K$ , emit spontaneous radiation at the wavelength<sup>11</sup>

$$\lambda = \frac{\lambda_w}{2\gamma_r} (1+K^2) \quad (3.17)$$

Using (3.16) we can write the electron equation of motion as

$$\dot{\gamma} = - \frac{e E_0 \beta_{\perp}}{m_0 c} \sin \phi \quad (3.18)$$

$$\dot{\phi} = \frac{2\pi c}{\lambda_w} \{ 1 - \frac{\gamma_r^2}{\gamma^2} \} \quad (3.19)$$

These equations can be derived from a Hamiltonian

$$H = \frac{2\pi c}{\lambda_w} \{ 1 + \frac{\gamma_r^2}{\gamma^2} \} \gamma - \frac{e E_0 \beta_{\perp}}{m_0 c} \cos \phi \quad (3.20)$$

#### IV. The Small Signal Regime

If we assume that the electron energy change in traversing the wiggler is small and that the initial electron energy is near to  $\gamma_r$ , we can simplify equations (3.18), (3.19), (3.20), defining

$$\eta = \frac{\gamma - \gamma_r}{\gamma_r} \quad (4.1)$$

and assuming

$$\eta \ll 1 \quad (4.2)$$

we have

$$\dot{\eta} = - \frac{e E_0 \beta_{\perp}}{m_0 c \gamma_r} \sin \phi \quad (4.3)$$

$$\dot{\phi} = \frac{4\pi c}{\lambda_w} \eta \quad (4.4)$$

$$H = \frac{2\pi c}{\lambda_w} \eta^2 - \frac{e E_0 \beta_{\perp}}{m_0 c \gamma_r} \cos \phi \quad (4.5)$$

A phase-space representation of the Hamiltonian is given in Fig. 2. We can see that the Hamiltonian is the same as that describing the synchrotron oscillation motion in electron or proton synchrotrons. The small amplitude oscillation frequency  $\Omega$  is given by

$$\Omega^2 = \frac{4\pi e E_0 \beta_{\perp}}{m_0 \gamma_r \lambda_w} \quad (4.6)$$

The separatrix is defined by the condition

$$\dot{\phi}_0 = \pm 2\Omega \quad (4.7)$$

or, equivalently,

$$\eta_0 = \pm \frac{e E_0 \beta_{\perp} \lambda_w}{\pi m_0 c^2} \quad (4.7a)$$

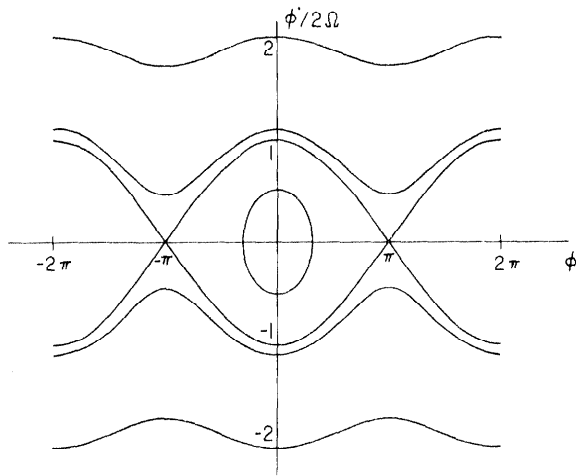


Fig. 2. Phase space electron trajectories.

The electron energy transfer to the radiation field depends on what trajectory the particle is following in phase-space. It is important to notice that a change of  $2\pi$  in phase-space corresponds to a change equal to the radiation wavelength,  $\lambda$ , in the longitudinal electron position. Hence, in the most usual experimental conditions, the initial electron beam distribution in phase is a uniform distribution. The initial energy distribution or equivalently the  $\phi$  distribution, can be very narrow. In this case, the initial electron distribution function can be represented in the  $\frac{1}{2} \phi$  plane by a narrow strip parallel to the  $\phi$ -axis.

We are interested in calculating the energy transfer to the radiation field, i.e. the change in electron energy averaged over the initial electron distribution. We assume that all electrons in the beam have the same initial energy and are uniformly distributed in phase, and discuss in some detail the free electron laser characteristics for this case. For this distribution the average energy change is zero if  $\phi_0 = 0$ . To describe the Stanford experiments, we can assume  $\phi_0 > \Omega$ . We will call the "small signal regime" the case of  $\phi_0 > \Omega$  and uniform phase distribution. At the end of this section we will also discuss briefly the laser operation under other possible conditions.

In the small signal regime we can calculate the electron energy change using an expansion in the small parameter  $(\Omega/\omega_0)^2$ , where  $\omega_0 = 2\pi c/\lambda_w$ . This is given by

$$\left(\frac{\Omega}{\omega_0}\right)^2 = \frac{e E_0 \beta_{\perp} \lambda_w}{\pi m_0 c^2 \gamma_r} \quad (4.8)$$

This quantity is proportional to the maximum electron energy change in one wiggler period,  $e E_0 \beta_{\perp} \lambda_w$ , divided by the resonant electron energy. The electron energy change can be written, to first order in  $(\Omega/\omega_0)^2$  as<sup>6</sup>

$$\left(\frac{\Delta\gamma}{\gamma}\right)_{\text{MAX}} = \pi \left(\frac{\Omega}{\omega_0}\right)^2 N g(x, \phi_0) \quad (4.9)$$

where

$$g(x, \phi_0) = \frac{\cos(x + \phi_0) - \cos \phi_0}{x} \quad (4.10)$$

$$x = 4\pi N \frac{\gamma_0 - \gamma_r}{\gamma_r} \quad (4.11)$$

To evaluate the increase in the electromagnetic field intensity we need to know the average value of the energy change,  $\langle (\Delta\gamma/\gamma) \rangle$ , averaged with respect to  $\phi_0$  between 0 and  $2\pi$ . This quantity is given to the lowest order in  $(\Omega/\omega_0)^2$  by

$$\langle (\Delta\gamma/\gamma) \rangle = 4\pi^3 \left(\frac{\Omega}{\omega_0}\right)^4 N^3 f(x) \quad (4.12)$$

where

$$f(x) = \frac{1}{3} \left[ \cos x - 1 + \frac{1}{2} x \sin x \right] \quad (4.13)$$

and  $x$  is again given by (4.11). The function  $f(x)$  is given in Fig. 3. One can see that the electron lose energy if  $x > 0$  and gain energy if  $x < 0$ . The maximum energy loss is obtained for  $x = 2.5$ , or

$$\frac{\gamma_0 - \gamma_r}{\gamma_r} \approx \frac{0.2}{N} \quad (4.14)$$

For the same value of  $x$ , one obtains from (4.9), (4.10)

$$\left(\frac{\Delta\gamma}{\gamma}\right)_{\text{MAX}} \approx 2.4 \left(\frac{\Omega}{\omega_0}\right)^2 N. \quad (4.15)$$

For a nonmonochromatic electron beam the average energy change is obtained by folding (4.12) with the electron energy distribution function. It is clear from Fig. 3 that if the electron energy distribution function

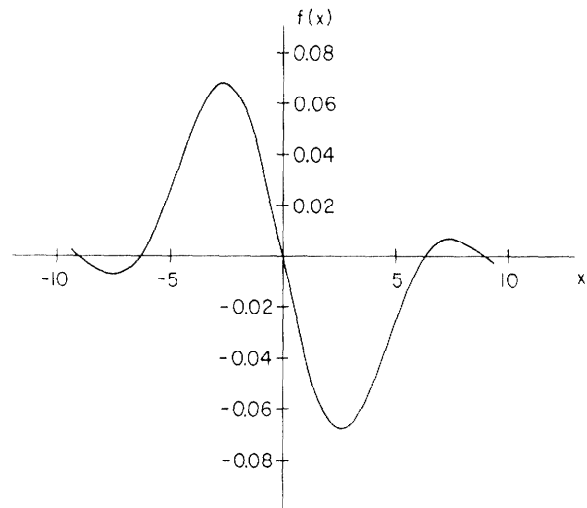


Fig. 3. The gain function  $f(x)$ .

has a width of the order or larger than  $1/2N$ , the average energy change is strongly reduced and becomes nearly zero. This also means that when the maximum energy change for a single electron, as given by (4.15), becomes of the order of  $1/2N$ , the expression (4.12) for the average energy change does not hold any more. This condition  $(\Delta\gamma/\gamma)_{\text{MAX}} \approx 1/2N$  gives, using (4.15), a maximum or saturation value for  $\Omega/\omega_0$ ,

$$\left(\frac{\Omega}{\omega_0}\right)_{\text{sat}} \approx \frac{1}{2N} \quad (4.16)$$

or, using (4.8), a saturation value for  $E_0$ . Notice that the saturation value (4.16) for  $(\Omega/\omega_0)$  gives a value of this quantity which is much less than one for all cases of interest, in which  $N$  is of the order of 10 or larger.

The interaction of the electrons with the radiation field produces not only a change in the electron energy, but also a change in the phase distribution function.<sup>6,9</sup> This change is such that the phase distribution function at the wiggler exit is modulated on the scale of the radiation wavelength.

The interaction between the electron beam and the radiation field, as described by the Hamiltonian (4.5), is completely analogous to the interaction of an electron or proton beam with the radio frequency system of a synchrotron or storage rings, the main difference coming from the much higher field frequency in the laser case. Hence it is possible to apply to the free electron laser most of the concepts and techniques developed for accelerators.<sup>12</sup> One can think of using phase displacement acceleration of the electron beam or of capturing electrons in "radiation buckets" and then decelerating. Several ways of operating a free electron laser in a regime different from the small signal regime are now being studied, with the aim of improving the performance of the system. We will briefly discuss some of this proposal in Section VII.

### V. The Free Electron Laser Gain

From the expression (4.12) for the average electron loss, we can obtain an expression for the gain,  $G_0$ , in the electromagnetic field intensity,  $I$ , during one traversal of the wiggler. Defining

$$G_0 = \Delta I / I \quad (5.1)$$

we obtain

$$G_0 = -32/2 \lambda^{3/2} \lambda_w^{1/2} \frac{K^2}{(1+K)^2} \frac{i_p}{\Sigma i_A} N^3 f(x) \quad (5.2)$$

Where  $i_p$  is the peak electron current,  $i_A = ec/r_0 = 1.7 \times 10^4 A$ , and  $\Sigma$  is the transverse cross section of the electromagnetic wave. To derive (5.2) we have assumed that the electron beam is bunched, with a bunch length  $\ell_e$  and transverse cross section  $\Sigma_e$  equal to  $\Sigma$ . It is easy to generalize (5.2) to the case  $\Sigma_e \neq \Sigma$ .

From the behavior of the function  $f(x)$ , with  $x = 4\pi N (\gamma - \gamma_r) / \gamma_r$ , as illustrated in Fig. 3 and from the discussion of Section IV we can obtain the following results:

1. The electrons lose energy if  $\gamma > \gamma_r$ , gain energy if  $\gamma < \gamma_r$ ; no energy change occurs if  $\gamma = \gamma_r$ ;
2. The maximum gain is obtained for  $(\gamma - \gamma_r) / \gamma_r \approx 0.2$ ;
3. The maximum energy transfer from the electron to the electromagnetic field is

$$(\Delta\gamma)_{MAX} \approx \gamma_r \frac{1}{2N} \quad (5.3)$$

which gives for the system an efficiency

$$\eta \leq \frac{1}{2N} \quad (5.4)$$

4. for a nonmonochromatic electron beam the gain is obtained by folding (5.2) with the electron energy distribution function; if the width of this distribution,  $\sigma_e$  is large compared to  $1/2N$ , the effective gain is strongly reduced and can become nearly zero.

To estimate the order of magnitude of  $G_0$  we assume  $f(x) \approx -7 \times 10^{-2}$ ,  $K \approx 1$  and that  $\Sigma_p$  is determined by diffraction limit of  $\Sigma_p \approx \lambda(N\lambda_w)$ . With these assumptions (5.2) becomes

$$G_0 \approx \pi^2 \left(\frac{\lambda}{\lambda_w}\right)^{3/2} \frac{i_p}{i_A} N^2 \quad (5.5)$$

For  $\lambda = 1\mu m$ ,  $\lambda_w = 5$  cm,  $N = 100$ , we obtain

$$G_0 \approx 2.5 \times 10^{-2} i_p, \text{ with } i_p \text{ in Ampere.}$$

This result shows that to obtain a peak current of the order of several percent, the electron beam peak current must be of the order of several ampere. We can now summarize the characteristics that the electron beam must have in order to obtain a good amplification of the electromagnetic field:

1. the energy spread must be small,  $\sigma_e < \frac{1}{2N}$
2. the transverse cross section must be smaller or equal to that of the electromagnetic wave or  $\Sigma_e \leq \Sigma$ .
3. the angular divergence must be such that  $\langle \theta^2 \rangle < \frac{1}{N^2}$
4. the peak current must be of the order of several ampere or larger.

### VI. A Free Electron Laser Oscillator

To obtain a free electron laser oscillator we modify the system of Fig. 1 by adding an optical cavity, i.e. two mirrors. One of these mirrors is assumed, for simplicity, to be a perfect reflector, while the other is assumed to transmit a fraction  $\alpha$  of the incident light (Fig. 4). We neglect the other possible losses of the light in the cavity.

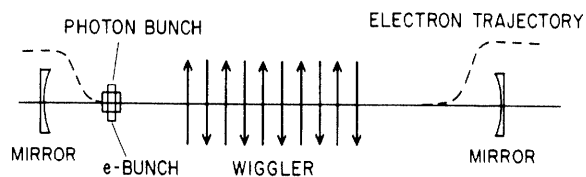


Fig. 4. A free electron laser oscillator.

This system can be a laser oscillator if the gain is larger than the loss, or

$$G \geq \alpha \quad (6.1)$$

The case  $G = \alpha$  describes the steady state operation of the system.

Condition (6.1) must be satisfied for the effective gain, corresponding to a beam with its own energy spread. The output laser power in the steady state operation,  $G = \alpha$  is given by

$$P_L = \eta (E i_{av}) \quad (6.2)$$

where the efficiency,  $\eta$ , has a maximum value of the order of  $1/(2N)$ ,  $E = m_0 c^2 \gamma$  is the electron energy and  $i_{av}$  is the average electron current. Equation (6.2) shows us that a fraction  $\eta$  of the electron beam power is transformed into laser beam power.

The space-time structure of the laser beam reflects the electron beam structure. We assume that the electron beam is bunched, with a bunch length  $\ell_e$  and a bunch separation  $L_e$ . We also assume, for simplicity, that  $L_e \gg \ell_e$ . Since the gain is different from zero only where the electron density is nonzero, one obtains in the laser cavity a radiation pulse of length  $\ell_p \approx \ell_e$ . The length of the radiation pulse determines the line-width,  $\delta_\omega$ , of the laser radiation

$$\delta_\omega = \frac{2\pi c}{\ell_e} \quad (6.3)$$

or

$$\frac{\Delta \omega}{\omega} = \frac{\lambda}{L_e} \quad (6.4)$$

The distance between successive radiation pulses in the output laser beam is determined by the length of the optical cavity,  $L_{op}$ , and is equal to  $2L_{op}/c$ .

To keep the system running, the electron bunch and the radiation pulse must be synchronized. The distance,  $L_e$ , between bunches must be a multiple of  $2L_{op}$ . In the simplest case this synchronization condition can be written as

$$L_e = 2L_{op} \quad (6.5)$$

Equations (6.2), (6.4), (6.5) describe the characteristics of the output laser beam. The gain of the system, as well as the line-width, the output power and the space-time structure of the laser beam are determined by the electron beam characteristics. For radiation at wavelength in the  $10 \mu\text{m}$  region or shorter, one needs high energy electrons, with  $\gamma \approx 100$  or higher. There are two possible sources of high energy, high intensity electron beams: A linear accelerator and a storage ring. The superconducting linear accelerator of Stanford University has been used in two experiments already mentioned.<sup>1,2</sup> Specially designed linear accelerator might be built in the future to operate free electron laser. A linear accelerator with a peak current of the order of 10A and an average current of 0.1 A at 100 MeV operating a free electron laser with an efficiency of 1% could provide a laser beam with 100 KW average power and wavelength of the order of 0.1 to  $10 \mu\text{m}$ .

Electron storage rings can also provide beams with high peak current,  $i_p \gg 100\text{A}$ , high average current,  $i_{av} \approx 1\text{A}$ , and good energy spread,  $\sigma_E \leq 0.1\%$ , and emittance. The wiggler magnet can be inserted in a straight section without substantial alteration to the single particle dynamics. However, in this case some of the equations describing the laser must be modified. In fact, in the linear accelerator case each electron bunch traverses the wiggler only once and the electron distribution function at the wiggler entrance is determined only by the accelerator characteristics. Instead in the storage ring case the same electron bunch is repetitively traversing the wiggler and its characteristics are determined also by the interaction with the radiation field.

As we discussed in Section III, the electron bunch at the wiggler exit has an energy spread larger than that at the wiggler entrance. Hence we can expect that the electron bunch-radiation field interaction will increase the electron beam energy spread thus reducing the effective gain. This problem has been studied by Renieri<sup>13</sup> who has shown that the interaction is such that the energy spread would continue to grow and the effective gain would become zero. The only mechanism opposing this process is the synchrotron radiation damping which tends to decrease the energy spread. An equilibrium between these two processes can be reached such that the laser output power is determined by the strength of the radiation damping process, which is proportional to the total amount of synchrotron radiation energy,  $U_o$ , lost by the electrons in one revolution. Hence for a storage ring case equation (6.2) must be substituted by

$$P_L = \eta(U_o i_{av}) \quad (6.6)$$

The effective gain is also reduced so that to obtain a value of  $\eta$  near to  $1/2N$  one needs a gain calculated for the unperturbed beam larger than the losses by a factor of 5-10.<sup>13</sup>

The synchrotron radiation energy loss per revolution is a strong function of electron energy. For a bending radius,  $\rho$ , in the ring one has

$$U_o = \frac{4}{3} \pi \frac{r_o}{\rho} \gamma^4 m_o c^2 \quad (6.7)$$

so that to increase  $P_L$  is very convenient to increase  $\gamma$ .

In Figures. 5, 6 we give, as an example, the wavelength, gain per unit current, magnetic field in the wiggler, for a free electron laser operating in a storage ring. The parameters used in this calculation are given in Table 1. We also assume that the beam energy spread and emittance satisfy all requirements of Section V when there is no interaction with the laser field.

Table 1.		
Wiggler period	$\lambda_w = 5 \text{ cm}$	or $\lambda_w = 10 \text{ cm}$
Number of periods	N=120	N=60
Number of electron bunches	B=3	B=3
Bunch length	$l_e = 5 \text{ cm}$	$l_e = 5 \text{ cm}$
Electron energy	E=700 MeV	E=500 MeV
Average electron current	$i_{av} = 1\text{A}$	$i_{av} = 1\text{A}$
Bending radius	$\rho = 1.5 \text{ m}$	$\rho = 1.5 \text{ m}$
Synchrotron radiation energy loss per revolution	$U_o = 1.6 \times 10^4 \text{ eV}$	$U_o = 3 \times 10^3 \text{ eV}$

In the case of Fig. 5, electron energy  $\gamma = 1000$  or 500 MeV, the line-width changes between  $1.2 \times 10^{-6}$  at  $\lambda = 0.06 \mu\text{m}$  and  $1.4 \times 10^{-4}$  at  $\lambda = 7.2 \mu\text{m}$ . The laser output power, as given by (6.6), is  $P_L = 35 i_{av}$  in ampere. In the case of Fig. 6, electron energy 1400 or 700 MeV, the line width changes between  $3.2 \times 10^{-7}$  at  $\lambda = 0.016 \mu\text{m}$  and  $9.4 \times 10^{-6}$  at  $\lambda = 0.47 \mu\text{m}$ . The laser output power is  $P_L = 67 i_{av}$  EW.

To increase the laser output power we can increase the strength of the synchrotron radiation damping by adding an auxiliary wiggler radiator. In this way we might bring the laser power to the KWatt level, for an average current of the order of a few ampere.

## VII. Conclusions

The main characteristics of the free electron laser oscillator operating on the Stanford Superconducting Linac are in good agreement with the theoretical calculations, although there are still some uncertainties on how to explain some details of the radiation pulse structure. We can say that we see no reason why the prediction on the free electron laser performance based on the small signal gain regime should differ from the real performance.

The operation of a free electron laser in a storage ring, in the same small signal regime, should also follow the theoretical predictions based on the small signal regime model and the results of Renieri's work as given in formula (6.6).

However it is important to notice that there are other possibilities to operate a free electron laser device. The results of Section V and VI are based on two assumptions:

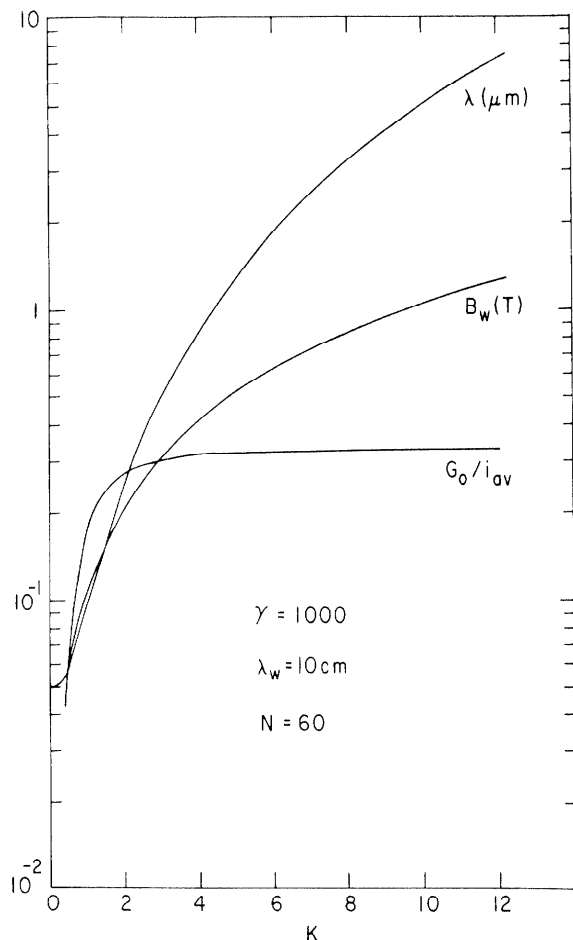


Fig. 5. Wavelength, gain, and wiggler magnetic field versus the wiggler parameter  $K$  for a free electron laser operating in a storage ring.

1. The wiggler has a constant period, the same magnetic field in all periods, and is uniform in a plane perpendicular to its axis;
  2. the system is operated in the small signal regime, starting from a particle distribution function uniform in phase and very narrow in energy.
- The removal of any of these assumptions can open up new possibilities to obtain a higher laser power or a higher efficiency.

The use of a wiggler with a transverse magnetic field gradient has been studied by Smith et al.<sup>14</sup> Their idea is that electrons having different energies will enter the wiggler at different distances from the axis and see a different magnetic field. In this way the effect of the energy spread can be reduced and the efficiency of the system can be increased. One such wiggler operating in a storage ring should allow to obtain a laser power nearly equal to the synchrotron radiation power, thus increasing the efficiency by nearly two orders of magnitude.<sup>15</sup>

Longitudinally variable wiggler fields, in which either the period or the field intensity or both can vary along the electron trajectory, have also been studied by several authors. Two possible schemes have been proposed by Hopf et al.<sup>16</sup> and by Morton,<sup>12</sup> to extract energy from the electron producing a nearly zero energy spread. These systems might increase by a large factor the laser power obtainable from a free electron laser operating in a storage ring, where the main limitations come from the electron beam energy spread produced by the interaction with the laser fields, while one needs to extract only a small amount of energy in a single passage.

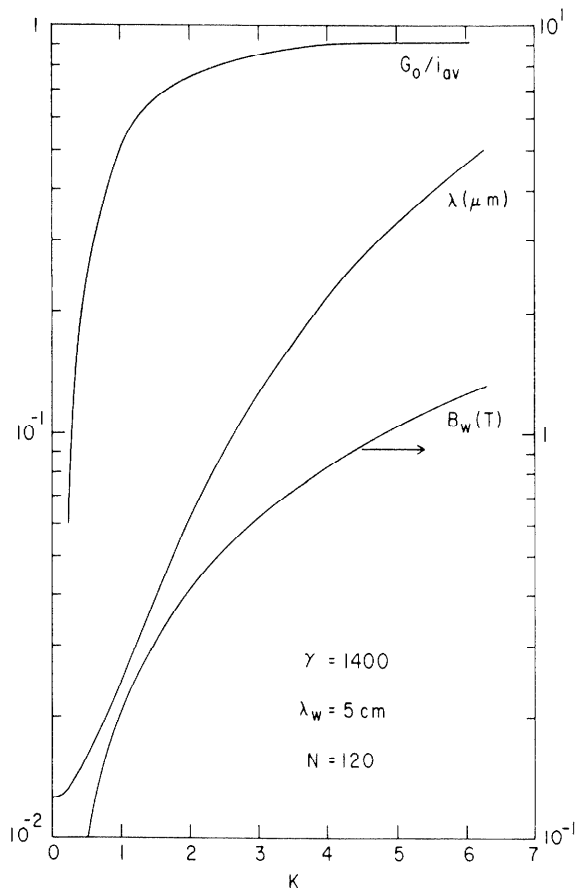


Fig. 6. Wavelength, gain and wiggler magnetic field versus the wiggler parameter  $K$ , for a free electron laser operating in a storage ring.

The idea of prebunching the beam has been proposed by Skrinky and Vinokurov<sup>17</sup> as a way to increase the gain for a given amount of electron current. The wiggler system would be split into two parts, one for producing the bunching, the other for transferring the energy to the radiation field.

Longitudinally variable wiggler fields in which the electrons are captured in the equivalent of a radio-frequency bucket and then decelerated, might allow to increase by one order of magnitude the energy transfer to the radiation field. Such a system, operated with the beam from a linear accelerator, might produce very large laser power, of the same order of the electron beam power, with high efficiency.<sup>18</sup>

All these ideas, and other possible new developments, might greatly increase the capability of the free electron laser, increasing either the output power or the efficiency or both.

However, also without these improvements, a free electron laser - electron storage ring system as discussed in Section VI, is a unique source of electromagnetic radiation: it can provide synchrotron radiation as well as coherent laser radiation continuously tunable over a wide wavelength region. This system might have useful applications in many areas of research, such as solid state physics, photochemistry, biology and molecular physics.

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