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DEPENDENCE OF MULTI-PASS BEAM BREAKUP ON ACCELERATOR PARAMETERS*

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Abstract

A theory has been developed to describe beam breakup in recyclotrons and racetrack microtrons. This theory has been used to calculate $I_S Q_L$ values for numerous model accelerator configurations. The dependence of $I_S Q_L$ on breakup mode frequency, beam transport optics, number of orbits, and accelerating gradient is shown.

Regenerative beam breakup is the result of an interaction of the electron beam and a transverse deflecting mode in an accelerator structure. Because the return orbit intervenes between the deflective and energy-coupling parts of the breakup process, multi-pass beam breakup depends on many accelerator system parameters (e.g., number and length of orbits, breakup mode frequency, and orbit focussing characteristics) in addition to those which influence the single-pass process.

The construction and planning of a number of multi-pass electron accelerators [1,2,3] has created a need for detailed understanding of regenerative beam breakup in such machines. Earlier work on multi-pass breakup at Mainz[3] and Illinois[4] has been directed towards understanding the effect in a particular proposed or existing machine. The present efforts at High Energy Physics Laboratory[5,6] have sought to analyze the breakup process in terms of a generalized model which can be varied in many ways.

A theory for multi-pass beam breakup assuming a TM_{11}-like breakup mode has been developed. In this theory the product of threshold beam current for breakup I_S and loaded Q is given by

$$I_S Q_L = \frac{1028}{g_1^{\,2}} \, (\lambda/\ell)^2 \, \frac{1}{P(\ldots)}$$

where g_1 is the Fourier coefficient of the fundamental space harmonic and λ is the free space wavelength of the breakup mode, ℓ is the structure length, and the factor P contains the dependence of the beam-coupled power on phase slip, breakup mode frequency, orbit length, focussing, etc.

A computer program incorporating this theory has been developed to calculate P, given a description of the accelerator system and breakup mode. The program is capable of calculations for a broad range of model machines; its use so far has been in two areas: first, to calculate the starting current for the Stanford superconducting recyclotron (SCR) in one of its operational modes; second, to calculate P for a number of substantially similar models which are variants of a generalized multi-pass accelerator model, and so to find the dependence of the process on the basic parameters which describe the accelerator system.

The beam breakup limitation in the most recent operation of the SCR is caused by a hybrid dipole breakup mode with a frequency of 2.3 GHz. A calculation of the expected starting current for this mode has been made using the measured values for the orbit phase shift, the space harmonic coefficient g_1 (derived from field profile measurements), the value of Q_L, and other quantities describing the SCR. For the case of inverted image focussing the calculation gave a starting current of 2.65μA for one orbit of recirculation as compared with a maximum starting current of 12μA. As discussed in more detail by Lyneis et. al.[2] the 12μA starting current was attained using a "reflection mode" for the beam optics. For other operating modes the starting currents were 3-5μA, in reasonable agreement with the calculation. The low starting currents obtained to date will improve in the next SCR run with the installation of additional loading probes.[2]

The generalized model accelerator consisting of nine accelerator structures and bending magnets for recirculation is shown schematically in Fig. 1. (Only three orbits of recirculation are shown for clarity.) Thin focussing elements may be provided optionally in various combinations (see Fig. 4) for each orbit as shown. The parameters of the model may be varied systematically, enabling the dependence of the breakup process on them to be determined. Typical parameters are given in Table 1.

The frequency dependence of P, which represents the power coupled from the beam to the breakup mode in arbitrary units, can be separated into two components: (1) a rapidly oscillating component which is amplitude modulated by (2) a low frequency envelope. The rapid oscillations are most easily understood as being a dependence on a phase shift introduced in the return orbit. The phase shift δ produced by an orbit of given geometrical length depends sensitively on the frequency of the breakup mode. (For example, if 500 breakup mode cycles elapse during completion of an orbit, then a frequency change of 0.2% results in a 360° change in δ) The power coupled after one orbit due to a deflection received on the first pass is proportional to $\sin(\delta)$. Ignoring small differences in the orbit lengths, the power coupled after the second orbit due to that initial deflection is proportional to $\sin(2\delta)$, and so on. If there are n orbits the power coupled in all passes generally sums to a function with 2n zeros over a 360° range in δ.

$P(\delta)$ is shown for four orbits in Fig. 2 for three focussing schemes. For the two schemes where P is not zero for all δ, there are eight zeros. The breakup mode to accelerator mode frequency ratio was chosen to be 3/2 in this case enabling constructive interference of the power coupled in all passes at a particular phase shift (near 0° for erect image focussing and near 180° for inverted image focussing).

The low frequency envelope which modulates the rapid oscillations is determined by the degree to which the deflections and coupled powers of all passes can add constructively. Constructive interference is possible if the breakup mode frequency and accelerator mode frequency are in a ratio of small integers as in Fig. 2. The peak amplitude of $P(\delta)$ is shown in Fig. 3 for the resonance frequency ratios between 1 and 2. For the frequency ratios between 5/4 and 3/2 the envelope is shown at intermediate points.

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The focussing properties of the recirculation transport system have been shown to be very important in reducing beam coupled power. From the point of view of regenerative beam breakup, the ideal focussing scheme reinjects the electrons so that they cross the axis at the midpoint of the breakup structure regardless of the location of that structure along the accelerator. In the limit of small energy gain per orbit this can be realized with a system of three lenses as shown in Fig. 4(c). Figs. 4(a) and 4(b) show two simpler focussing schemes which meet the above specification only if the breakup structure is at the accelerator midpoint. The efficacy of these schemes in reducing P is shown in Fig. 2.

Another scheme, denoted as the "weak focussing" scheme, employs lenses in the same positions as those of the inverted image scheme, but with focussing power arbitrarily set at 1/10 the power of the inverted image scheme lenses. This weak focussing scheme is comparitively ineffective if only a few orbits are made, but is an interesting possibility in machines using several tens of orbits.

In order to eliminate effects dependent on energy gradient in the accelerator the results shown up to this point have assumed no energy gain in the machine. The predominant effect of acceleration is the reduction in the growth rate of coupled power as the number of orbits increases. This is a kinematic effect arising from the reduced transverse drift of the deflected electrons as the Lorentz factor grows.

Table I.

Typical Parameters for Generalized Model Accelerator

Number of Structures	9
Structure length	5 m
Interstructure drift length	1 m
First orbit harmonic number	550
Harmonic number increment/orbit	2
Frequency ratio (f_{BBU}/ f_{ACC})	1 to 2
Phase slip	0, π
Injection energy	90 MeV
Energy gain / pass	0 to 135 MeV
First orbit phase shift	0 to 2π
Focussing	Various schemes
Breakup structure	First structure

The injection energy of 90 MeV used in the model accelerator defines a natural unit of energy gain per pass. Calculations of coupled power for up to 80 passes have been made with energy gain per pass from 0 to 1.5 times the injection energy. The peak value of P over a 360° range in δ is plotted logarithmically as a function of number of orbits for no focussing in Fig. 5, and for weak focussing in Fig. 6. Zero phase slip and a 3/2 frequency ratio are assumed in each figure.

At this frequency ratio, which is a worst case (c.f. Fig. 3), P grows approximately as n^3 in the absence of both acceleration and focussing. As accelerating gradient increases, the growth curve approaches a slope corresponding to n^2 growth.

The presence of weak focussing lenses results in growth as n^2 with no acceleration. Increasing accelerating gradients force the growth curve to approach a linear growth slope.

The growth of beam coupled power in the case of inverted image focussing shows the same dependence on number of orbits as the no focussing case, although the interaction is an order of magnitude weaker for inverted image focussing. For five orbits or less, inverted image focussing yields starting currents at least five times as great as weak focussing. This result is consistent with the design of the SCR which allows for quadrapole focussing in the return leg of each orbit.

Acknowledgements

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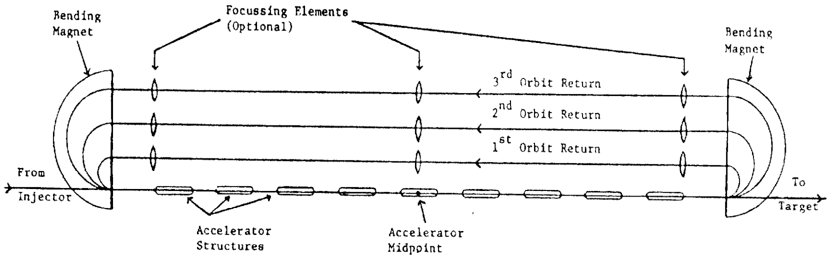

Figure 1. Schematic of generalized multi-pass accelerator. Thin focussing elements may be provided optionally for each orbit as shown.

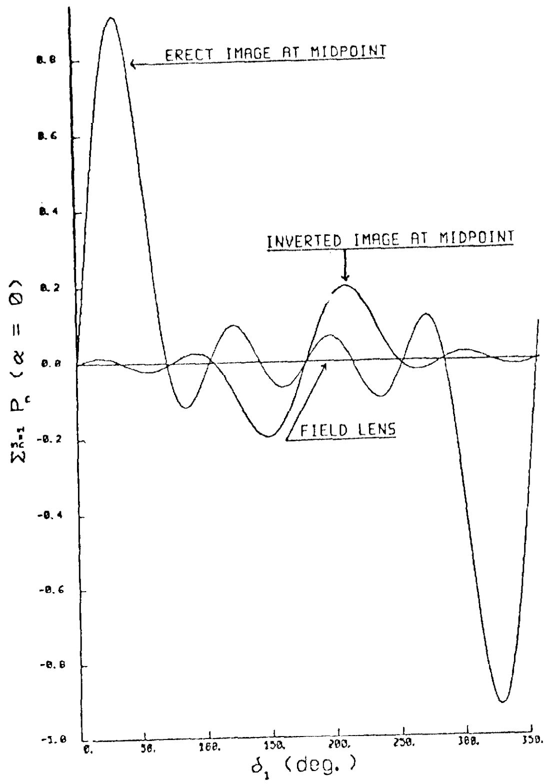

Figure 2. Calculated values of P for 5 passes vs. 1st orbit phase shift for three focussing schemes. Zero phase slip and frequency ratio of 3/2 are assumed.

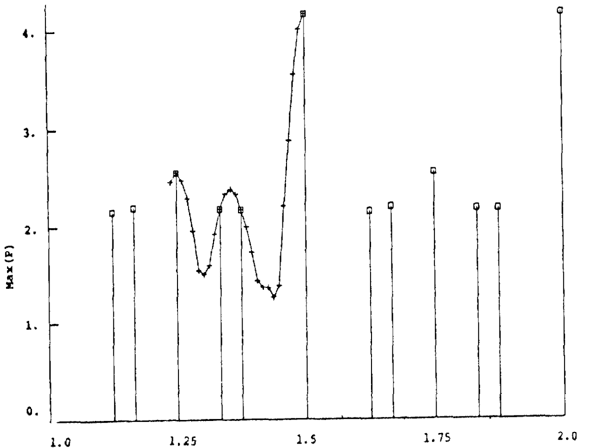

Figure 3. Max(P) for 6 passes over a 360° range in δ vs. frequency ratio f_{BBU}/f_{ACC}. Squares indicate resonance frequencies. No focussing and zero phase slip assumed.

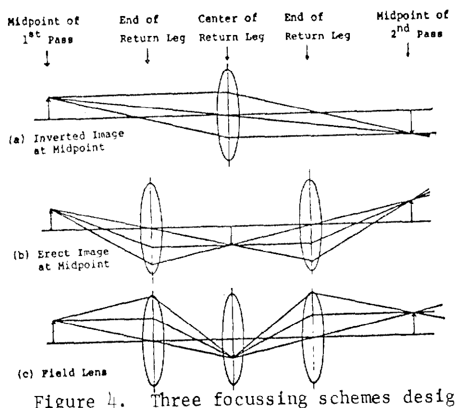

Figure 4. Three focussing schemes designed to superpose image on the object.

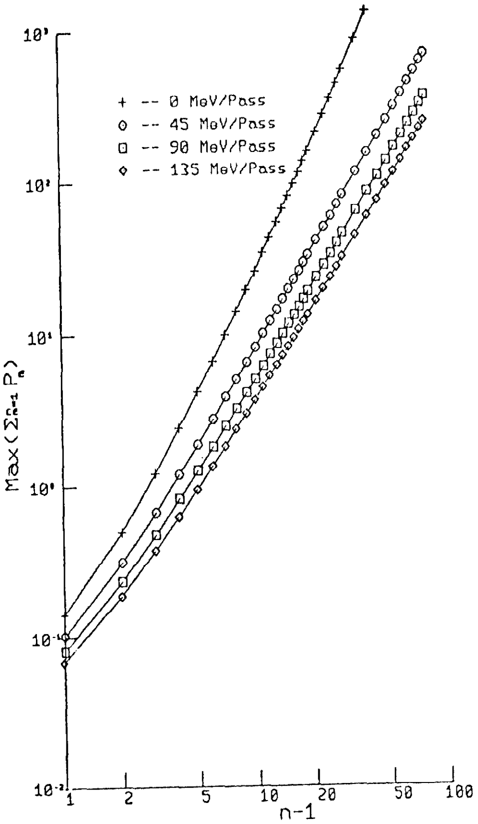

Figure 5. Max(P) over a 360° range in δ vs. number of orbits for four gradients. No focussing, zero phase slip, and frequency ratio of 3/2 are assumed.

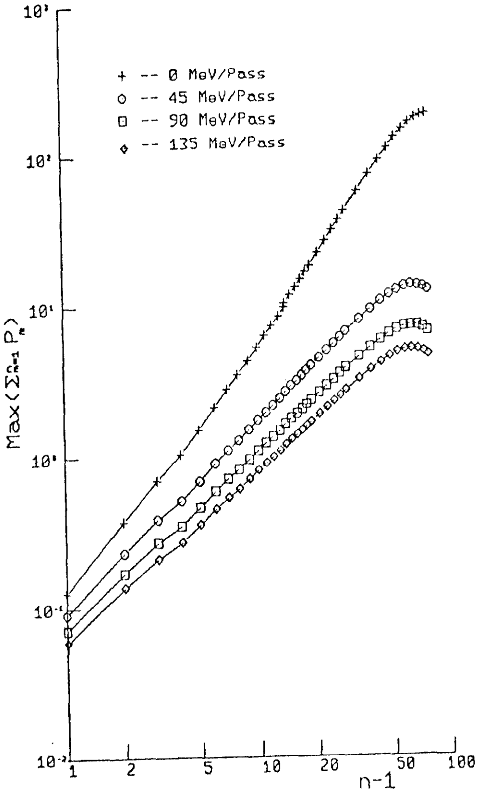

Figure 6. Max(P) over a 360° range in δ vs. number of orbits for four gradients. Weak focussing, zero phase slip, and frequency ratio of 3/2 are assumed.