IEEE Transactions on Nuclear Science, Vol. NS-26, No. 3, June 1979

POSSIBLE LOWER LIMIT TO LINAC EMITTANCE*

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Abstract

Numerical calculations made by several groups have always shown that an asymptotic lower exit emittance exists for linacs operating with high beam current as the input emittance is reduced to zero. In this paper, a mechanism for this limit is shown to be spread in the betatron frequencies of the individual particles due to the combination of space charge and r.f. gap forces, causing different transverse trajectories correlated with the instaneous longitudinal position of the particle. These trajectories cannot all be simultaneously matched to the average restoring forces, resulting in an overall emittance increase if the space charge force is a large fraction of the restoring force. In principle, equilibrium distributions may exist which would not grow. Raising the linac frequency or reducing longitudinal emittance improves the situation, but higher injection energy does not.

Introduction

Chasman[1] showed in 1968, using a particle tracing code, that the normalized output emittance of a 10 MeV 200 MHz linac reached an asymptotic lower limit as the input emittance was reduced to zero. She also showed that this limit decreased with beam intensity and that the longitudinal-transverse coupling due to the r.f. fields alone was not responsible for this lower emittance limit. More recently, Jameson and Mills[2] conducted a parameter search, also with a particle tracing program (PARMILA), confirming Chasman's result and further showing the surprising result that this lower limit is substantially independent of the injection energy but is directly correlated with operating wavelength, and to a lesser extent, synchronous phase and accelerating gradient. This result was analytically confirmed by Lysenko[3], working from an equilibrium distribution analysis. This paper offers an explanation of these phenomena.

Recent Observations

Several groups, primarily at LBL and LASL, have been investigating this blow-up phenomenon for heavy ion fusion and other applications. These groups observed several consistent phenomena:

(a) There is a lower limit to the output emittance of a linac (fig. 1)
(b) The growth in emittance is initially very rapid -- in the first 2 or 3 cells
(c) There is a slow subsequent growth
(d) The r.f. longitudinal-transverse coupling without space charge is not responsible for the majority of this growth
(e) The growth is almost independent of injection energy over reasonable limits
(f) Reducing the longitudinal emittance helps
(g) There is a strong positive correlation of emittance limit with operating wavelength
(h) The effect is not strongly dependent on the initial particle distribution (KV or non-KV) with the same rms properties

In tracking up to 1000 particles it is observed that the periphery of the transverse phase space quickly becomes filamented and that the lobes are correlated with the instantaneous longitudinal position of the particle. Furthermore, upon tracing orbits of individual particles, it is seen that particles near ϕ_s have a strong transverse tune depression due to space charge forces and particles near the ends of the bunch have lesser transverse space charge effects but are differentiated by stronger or weaker r.f. defocusing. Figure 2 shows the filamented phase space at the second cell of a typical linac, along with the transverse phase space for five longitudinal slices, each 10 degrees wide. The normalized transverse emittance here has almost doubled from the initial value of .05 π cm-mrad.

Matching

The average beam can be matched in the "smooth" sense if no envelope oscillations occur over the strong focussing flutter. If $\bar{a}$ and $\bar{c}$ represent the averaged matched transverse and longitudinal beam radii in a linearized approximation, then in an Alvarez structure operating in the $\beta\lambda$ mode

$$\left(\frac{\Delta\mu}{2}\right)^2 - \frac{(\varepsilon_t^n\lambda)^2}{\bar{a}^4} = 90\Omega\left(\frac{e}{mc^2}\right)\frac{I\lambda^3}{2\bar{a}^2\bar{c}}\,(1-f') \qquad (1)$$

$$-2\pi\left(\frac{e}{mc^2}\right)\frac{E_0\,T\,\sin\phi_s\bar{c}}{\beta} - \frac{(\varepsilon_L)^2\lambda^3}{(2\pi mc^2)^2\,\bar{c}^3} = 90\Omega\left(\frac{e}{mc^2}\right)\frac{I\lambda^2 f'}{\bar{a}^2} \qquad (2)$$

where ε_L is the initial longitudinal emittance area in radian-eV, ε^n_t is the initial normalized transverse emittance area, $f = \bar{a}/(\bar{a}+2\bar{c})$ is a geometric form factor, I is the average beam current, and $\Delta\mu$ is the transverse phase advance per quadrupole cell without space charge. Figure 3 shows the matched transverse beam radius in the smooth approximation as a function of emittance for various frequencies, currents and injection velocities. This method gives a fairly good approximation to a matched beam, defined as an absense of envelope oscillations. Small adjustments to the parameters are made to eliminate any residual mismatch. As the initial tranverse emittance decreases the beam size approaches a constant,

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*This work was supported by the High Energy & Nuclear Physics Division of the U.S. Department of Energy under contract No.W-7405-ENG-48

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0018-9499/79/0600-3698$00.75 © 1979 IEEE

dependent upon the transported current, and the maximum angular divergence decreases in direct proportion to the emittance. These beams are "delicate" -- a small non-linearity can cause large relative changes to the divergence, increasing the area of its trajectory. An even more important mechanism, is the strong effect the varying balance of restoring forces at various values of ϕ have on the axis ratio (x/x') of the trajectories. This indicates that in the presence of space charge, a particle's transverse trajectory is highly correlated with its longitudinal position and that a simultaneous match for all particles does not exist, at least for the mathematical distributions we usually test, which are often uncorrelated or only constrained to be within multi-dimensional hyperelliposids. Work presently in progress[5] is exploring the existence of truly matched beams in accelerator systems. Present real beams appear not to be matched in this sense and therefore the hypotheses are useful. The lower emittance asymptote of a linac can be estimated to be near the region where the beam size begins to be independent of the beam emittance.

Reducing the Emittance Asymptote

If transverse emittance, $\bar{a}$ and $\bar{c}$ are scaled with λ, and E_0 inversely, Equations (1) and (2) remain invariant for constant I and $\Delta\mu$. This implies that the asymptotic emittance scales directly with wavelength if the accelerating and focussing gradients are scaled correctly. The transverse tune depression a beam of a given current in a periodic focusing lattice decreases with decreasing cell length of the lattice. The velocity dependence in Equations (1) and (2) is weak, explaining the relative insensitivity to injection energy. Reduction in longitudinal emittance tends to reduce the phase spread slightly, thereby reducing the range of r.f. defocussing. This reduces the transverse emittance growth, but the effect is not large. Other matching procedures which account for the particular input distribution and the complete nature of the accelerator are now being studied, and we also hope to be able to better simulate actual beams.[6] These procedures also allow matching to a point further along the accelerator when the emittance has stabilized. However, it is also possible that only the growth rate for non-equilibrium distributions will be affected.

Technological Implementation

Reductions in emittance growth would be achieved most successfully if we were able to prepare distributions for injection into the accelerating system which were in six-dimensional equilibrium with it. A new low-β accelerator structure may make this possible to a degree not previously achieved in linear accelerators. This structure, invented in the USSR, is under development at LASL[7,8] and is named the "space-uniform-focusing" or "rf quadrupole" structure in which a beam can be taken from the ion-source/injector system at very low energy ($\lesssim$100 keV), and adiabatically subjected to transverse strong focusing. At at few MeV, a bunch well fitted to the strong-focusing accelerating system is formed, and a smooth transition is made into an Alvarez structure. Strong transverse focusing is achievable in the space-uniform-focusing structure; the practical limits for this type of structure are now under intensive investigation at LASL.

Exploiting higher operating frequencies for better control of transverse emittance brings attendant problems, because stronger focusing forces are required and tolerances becomes tighter. Recent technological advances have significantly extended the range of feasibility. Tests[9] on a 6 cell, $\beta = 0.3$, 440 MHz Alvarez structure achieved 8 MV/m field gradients.

Very strong quadrupole focusing, on the order of 50 kG/cm for apertures of a few millimeters, are possible using rare-earth cobalt permanent magnets using the techniques of Halbach.[10] Studies and tests of such quadrupoles, including tolerance requirements, are in progress at LASL. These gradients are appropriate for 100mA of protons at 440 MHz.

Finally, it is conceivable that transverse focusing for even higher frequency accelerators could be provided by inserting a longitudinal accelerating structure into a superconducting solenoid.

It is clear that the successful combination of these technologies and an increased understanding of emittance growth will find many applications, including, for example, heavy-ion fusion, precision radiography, and high-quality high-intensity beams for experimental physics and chemistry.

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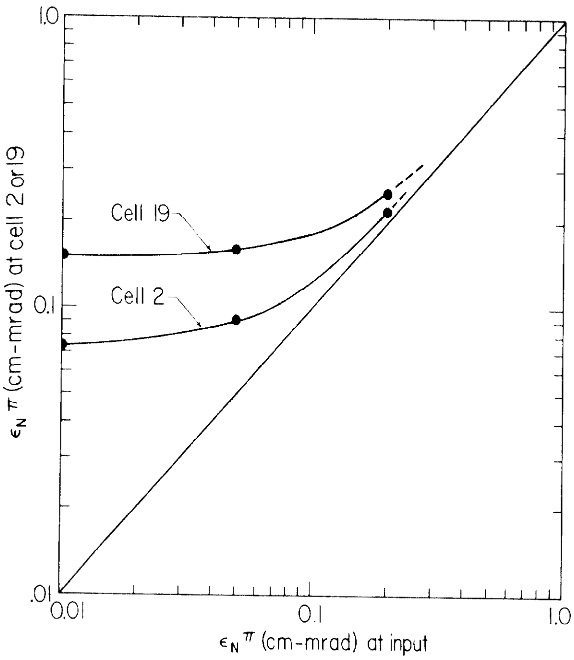

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Fig. 1 Normalized transverse exit emittance as a function of entrance emittance in a 200 MHz linac carrying 100 mA of current. The injection energy is 750 keV and the beam is matched transversely and longitudinally.

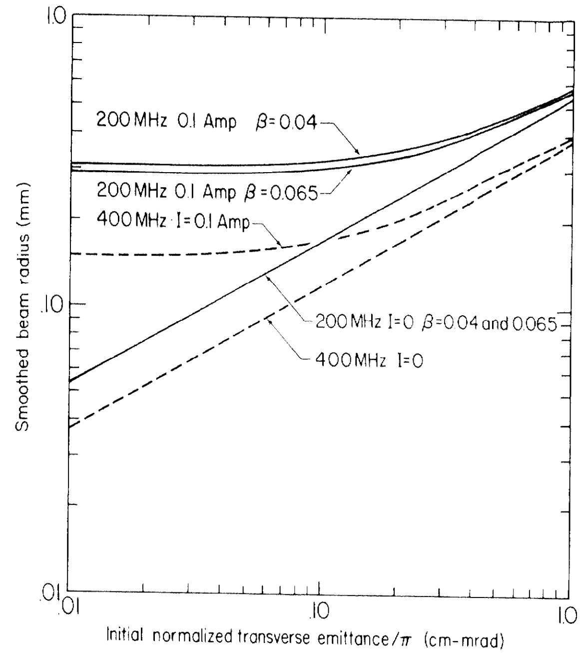

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Fig. 3 Matched smoothed beam size for 200 and 400 MHz linacs carrying 100mA. The corresponding zero-current matched beam sizes are also shown; the phase advance per period is 60°. The injection velocity β is .04 in all cases, and also .065 in the case of the 200 MHz linac. The average axial field is 2.0 MV/m at 200 MHz and 3.3 MV/m at 400 MHz.

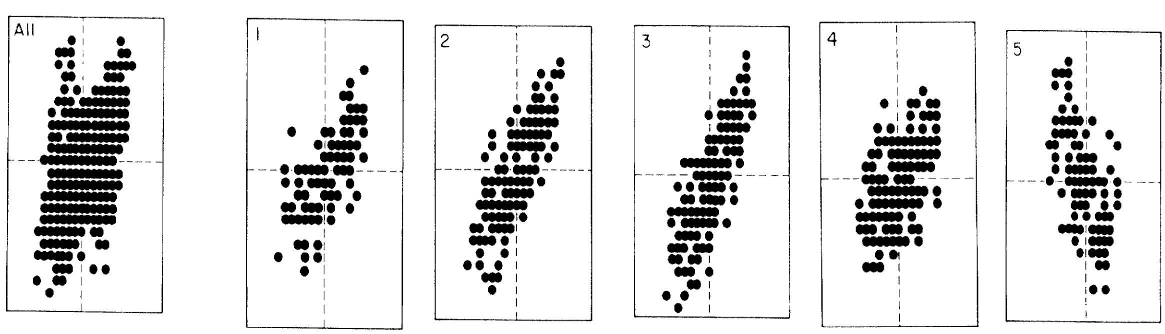

Fig. 2 Transverse yy' phase space at cell 2 of a 200 MHz linac injected at 750 keV. The first diagram shows the total phase space population. The next 5 diagrams show the yy' phase space for 10° longitudinal slices: diagram 1 for $\Phi \leq -45^{\circ}$, 2 for $-45^{\circ} \leq \Phi \leq -35^{\circ}$, 3 for $-35^{\circ} \leq \Phi \leq -25^{\circ}$, 4 for $-25^{\circ} \leq \Phi \leq -15^{\circ}$, and 5 for $-15^{\circ} \leq \Phi$. The scales are $\pm$ 0.5 cm by $\pm$ 15 mm.