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 THE LONGITUDINAL COUPLING IMPEDANCE OF STEP DISCONTINUITIES IN A CIRCULAR BEAM TUBE*
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Abstract

The longitudinal coupling impedance presented by a single wall discontinuity to the circulating beam in a circular accelerator or storage ring is usually analyzed by considering a "developed" periodic structure. However, the typical parameters are often such that it becomes adequate to treat the discontinuity as a nonperiodic problem. Using modal field matching methods, solutions were derived for the cases of a single as well as a double-step discontinuity in a circular beam tube. Numerical results are presented in this paper and the typical behavior at low frequency, at resonance, and above cut-off is discussed.

I. Introduction

The coupling impedance presented by a single wall discontinuity to the circulating beam in a circular accelerator or storage ring is usually analyzed by considering a "developed" periodic structure.^{1,2} However, the typical parameters are such that it becomes adequate to treat the cavity as a nonperiodic problem. For example, below the cut-off frequency of the vacuum chamber the circumference of the machine is very much larger than the cut-off wave length, and the circular character of the machine becomes inconsequential. The same reasoning holds above cut-off frequency, if the circumference is also much larger than the attenuation length, typically on the order of a few 100 m. Furthermore, since measurements are conveniently performed only on structures of small dimensions the coupling impedance of a single discontinuity must be understood. These arguments suggest the need for an analytical treatment of the nonperiodic case.

In the present paper the simple, yet representative case of a double-step cross section change in a circular vacuum chamber is analyzed (Fig. 1). The case of a modulated beam interacting with a single cavity at resonance has been previously considered by various authors.^{3,4} The present study was inspired by Eberward who analyzed a single-step cross section change in a very wide rectangular chamber.⁵ His result was that a single-step discontinuity represents a nonresonant inductance below cut-off but that it exhibits also a lossy resistive component above cut-off. The scope of this paper is chosen to cover the single as well as double-step discontinuity in order to include the study of resonant effects.

The single-step solution could be obtained by considering very long lossy cavities. The analysis is simplified by treating the two cases separately; the losses then enter only as perturbation of the ideal solution. The mathematical approach taken here employs field expansions in subregions and field matching along common surfaces. A judicious choice of the subregions is important to achieve rapid convergence and transparency of the formulae. Matching of fields on transverse planes⁶ appears particularly suited to the problem on hand. However, space limitations made it necessary to omit all mathematical derivations.

The scope of this paper is limited to the case of extreme relativistic particles. Nevertheless, the resulting solution exhibits all expected qualitative features of a single cavity: Low-frequency inductance, cavity resonances below cut-off damped only by wall losses, cavity resonances above cut-off damped by radiation into the vacuum chamber. A quantitative comparison with published results, as far as available, indicates general but not complete agreement. In particular, it was found that an earlier version of this paper⁸ in which only the dominant space

harmonics were retained yielded marginal results. Two aspects of the single-step solution are worth pointing out: (1) The Z/n can show a resonance-like enhancement at the cut-off frequency of the larger tube, and (2) the Z/n well above the cut-off frequency of the larger tube is mostly resistive in nature and decreases inversely with frequency.

II. Fields in a Smooth Vacuum Chamber

For the purpose of the subsequent analysis we at first consider a modulated particle beam traveling with velocity $v=c$ on the axis of a smooth vacuum chamber. The beam is assumed to be stiff, that is, fields generated by the environment do not change the initial motion or charge distribution. To simplify the expressions a filamentary current of unit strength is assumed, $i = \exp(-jkz)\exp(j\omega t)$. Wave number k and frequency ω are related by $\omega = kc$ in natural units ($c = \omega_0 = 1$). Omitting the common time factor $\exp(j\omega t)$, one can write for the three field components $E_z = 0$ and $E_{zB} = H_{zB} = (2\pi r)^{-1}$. The field configuration of the filamentary beam is that of a coaxial transmission line, which is at the basis of bench measurements of the coupling impedance.⁹

The well-known expression for the coupling impedance of the smooth vacuum chamber¹⁰ cannot be derived using the assumption of a filamentary beam (and certainly not if $v=c$). However, it is thought that the calculation of the additional coupling impedance resulting from the discontinuities in the vacuum chamber will not be affected by these simplifications. It must be expected that the extreme relativistic solution loses its validity at frequencies at or above 30 times the cut-off frequency of the beam tube.¹¹

III. Single-Step Solution

General Expressions. The general expressions for the fields in the presence of a single-step are represented as the sum of (1) the smooth-wall beam fields and (2) the additional fields required to satisfy the boundary conditions, $E = E_B + E_{TM}$ and $H = H_B + H_{TM}$. The beam fields are given above, whereas the induced fields are written as series with expansion coefficients c_m^I and c_n^{II} . The solution is obtained by the usual modal field matching which leads to the matrix equations in the expansion coefficients $(\hat{A}k^I A + K^{II})c^{II} = v$ and $c^I = A c^{II}$. Having determined the expansion coefficients, one obtains the longitudinal coupling impedance, $Z = \rho + j\omega L$, from

$$Z = - \int_{-\infty}^{\infty} (E_{zTM})_{r=0} e^{jkz} dz$$

Note that the usual stability criterion¹ involves the coupling impedance, Z , divided by the mode number, $n = kr$, with R the average radius of the machine. Evaluating the integral yields ρ and L as weighted sums over all c_m^I and c_n^{II} .

In a circular machine, geometry dictates the exclusive presence of double-step discontinuities albeit far removed from each other and noncommunicating. As a consequence the ρ terms due to an up and down step cancel and the beam sees only the L term, i.e., $Z/n = jL/R$ per single step.

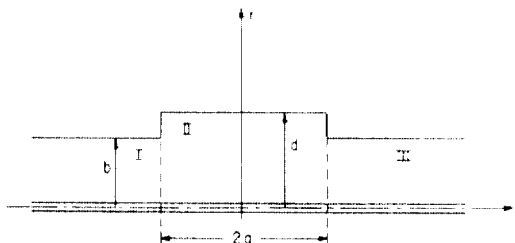


Fig. 1. Geometry.

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Low-Frequency Limit. The expressions for the coupling impedance of a single step in the low-frequency limit were derived and numerical results obtained. In Fig. 2 the step inductance, actually $\Lambda/(d-b)$, is plotted as function of the step size, $S=d/b$. Also shown are results published by Keil and Zotter.¹ The problem considered here is comparable to the scattering of waves by a step in the outer conductor of a coaxial waveguide, for which Marcuvitz¹² quotes the result ($\delta=S-1 \ll 1$)

$$\Lambda/(d-b) = (2\pi)^{-2} \delta(1 + 2 \ln 2\delta^{-1})$$

Good agreement is found. On the other hand, the first order approximation⁸

$$\Lambda/(d-b) = \delta/[2\pi J_0^2 J_1(j_0)]$$

shows only a marginal agreement.

Frequency Dependence. In Fig. 3 the behavior of the step inductance, normalized to its value at zero frequency is shown. Below cut-off of the larger tube Λ is strictly real, above cut-off an imaginary component of Λ corresponding to beam loss into the vacuum chamber appears. Since the analysis is based on lossless structures, the energy is dissipated at infinity. In practice, the energy is dissipated within about one attenuation length from the discontinuity. Whereas the character of Λ changes at the cut-off frequency $\omega=j_0/d$ of the larger tube, passing through cut-off $\omega=j_0/b$ of the smaller tube produces only minor ripples.

The absolute value of $|\Lambda|$ over a wider frequency range is shown in Fig. 4. A strong resonance-like enhancement can be seen at all frequencies corresponding to $\omega=j_0/d$. Strictly speaking, this is not a resonance since the impedance is inductive above as well as below the peak. The enhancement is strongest at cut-off ($n=1$) and its value dependent on the step size (shown as solid curve in Fig. 5). At very high frequencies, $\omega \gg j_0/d$, the character of Λ is essentially imaginary (i.e. the coupling impedance is resistive). The value of the "resonant" peaks decreases like $|\Lambda| = A_0 j_0 d^{-1/\omega}$, which is depicted by the dashed curve in Fig. 5.

IV. Double-Step Solution

The Low-Frequency Limit. The general solution for the lossless double-step (Fig. 1), valid at all frequencies, is obtained by the same approach used above. The results for the low-frequency limit are given in Fig. 6, where the quantity $ZR/n(d-b)$ is plotted as a function of the step size with the cavity length, $2g/d$, as parameter. As expected, a long cavity ($2g \gg d$) behaves like two single steps. A comparison of the results in Fig. 6 with the approximation⁸

$$\frac{Z}{n} \frac{R}{d-b} = \frac{g}{\pi d} \frac{S}{S-1} \ln S$$

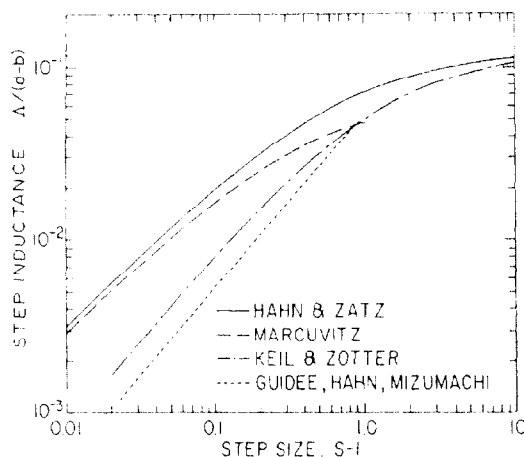


Fig. 2. Single-step inductance at low frequency.

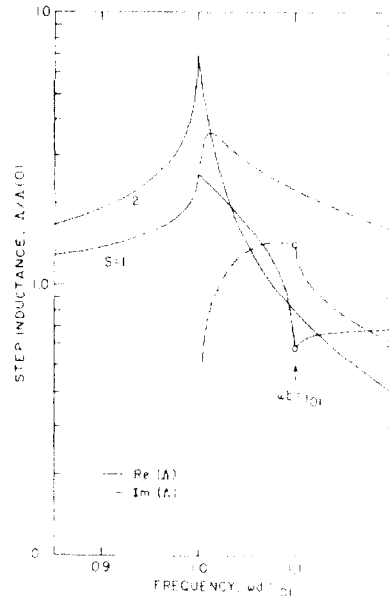


Fig. 3. Single-step frequency dependence.

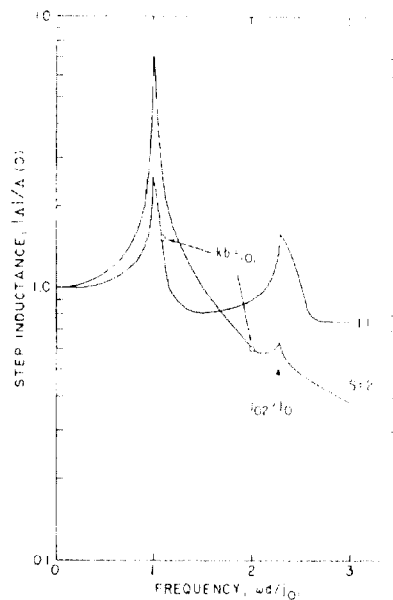


Fig. 4. Single-step frequency dependence.

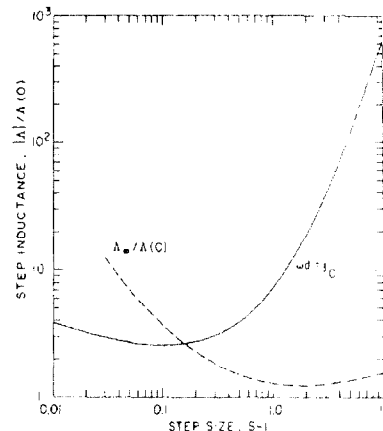


Fig. 5. Enhancement of single-step inductance.

shows reasonable agreement for sufficiently large step sizes ($S \gg 1$) and short cavities ($2g \ll d$).

Cavity above Cut-off. Above the cut-off frequency $\omega = j_{01}^2/b$ of the beam tube, the coupling impedance shows numerous peaks corresponding to the cavity resonances. The presence of the beam tube provides a natural damping of the resonances; nevertheless, the first resonances above cut-off can show a marked enhancement over the low frequency value. A full parametric study is beyond the scope of this paper. The results shown in Fig. 7 may serve as typical example, where the frequency dependence of $A/A(0)$ is mapped onto a Smith chart. The reduction of the resonant peaks with frequency has been studied numerically and, grosso modo, a variation with $\omega^{-1.5}$ was found. However, the sampling of cases was not large enough to preclude a somewhat different dependence (ω^{-1} to ω^{-2} are expected to be the limits).

Cavity at Resonance. Below cut-off of the beam tube, the cavity created by the double-step discontinuity will have one or more sharp resonances. The largest coupling impedance is presented by the dominant TM_{010} -like resonance. The relevant parameter, coupling impedance Z divided by quality factor Q , may be derived from the homogenous field equations, i.e. with the beam absent,¹³ according to

$$\frac{Z}{Q} = \frac{UU^*}{\int_{HH} dV} \quad \text{with } U = \int_{-\infty}^{\infty} (E_{zTM})_{r=0} e^{jkz} dz$$

The results are conveniently presented by normalizing them to the value for a pure TM_{010} cavity without beam tube, for which

$$\frac{Z(b=0)}{Q} = \frac{2}{\pi j_{01}^3 J_1^2(j_{01})} \frac{\sin^2(j_{01}g/d)}{g/d}$$

The variation of $Z/Z(b=0)$ with the beam tube radius is shown in Fig. 8 for the TM_{010} -like resonance. The cavity length $g/d = \frac{1}{2} \pi / j_{01}$ corresponds roughly to the worst case of strongest coupling.

Acknowledgments

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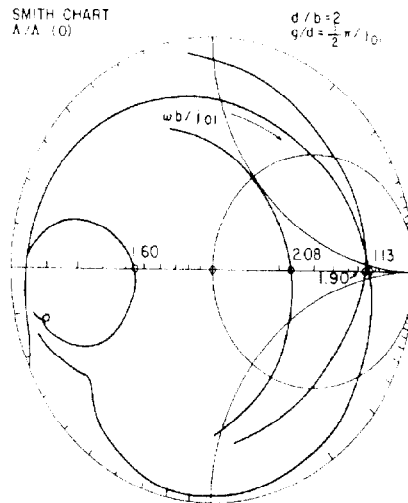


Fig. 7. Double-step frequency dependence (Smith chart).

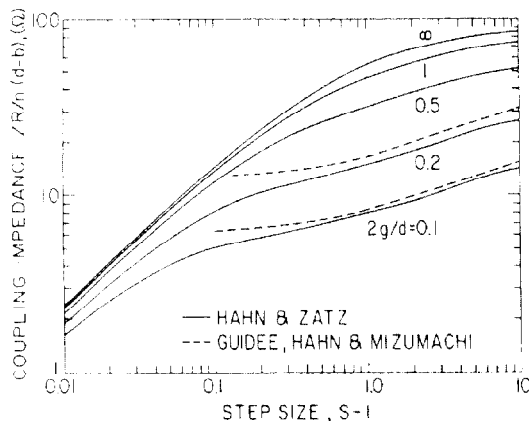


Fig. 6. Double-step inductance at low frequency.

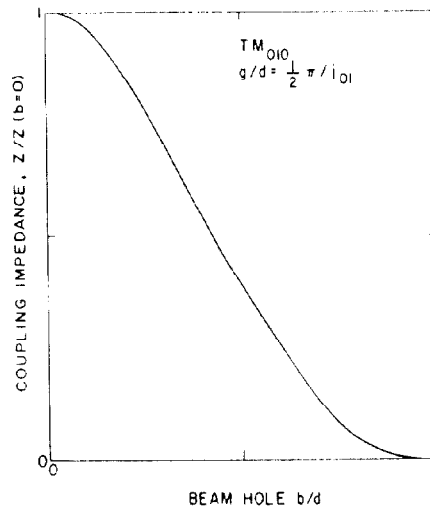


Fig. 8. Z/Q of TM_{010} -like resonance.