IEEE Transactions on Nuclear Science, Vol. NS-26, No. 3, June 1979

A MEASUREMENT OF THE LONGITUDINAL COUPLING IMPEDANCE IN THE BROOKHAVEN AGS*

F. Pedersen† and E.C. Raka††

Summary

The imaginary part of the longitudinal coupling impedance has been measured as a function of energy from 5 to ~ 28 GeV. This impedance is proportional to $\Delta f = (f_q - 2f_d)$ where f_d is the coherent dipole frequency and f_q the coherent quadrupole frequency. These frequencies are obtained by stimulating coupled bunch oscillations. If the dominant impedance is due to inductive wall plus space charge effects, then one has $(Z/n) = j[\Omega_0 L - g_0 Z_0/2\beta\gamma^2]$[1] where L is the inductance per turn and $\Omega_0 = 2\pi f_0$ the particle rotation frequency. The expression $(Z/n) = 4j\Delta f \pi^2 h V_0 \cos\varphi_s B^3/3I_0 f_d$ can be used to find the impedance if the synchrotron phase space distribution is proportional to $(1 - r^2)^{\frac{1}{2}}$. I_0 is the current per bunch, $B = f_0 \times \tau_\ell$ the bunch length and V_0 is the external voltage. For a distribution given by $(1 - r^2)$ the right hand side should be multiplied by $27/4\pi^2$. If the latter is assumed, an inductive impedance of 20.4 Ω is obtained with a null at ≈ 6 GeV (γ_{tr} = 8.5) for a transverse emittance of 22 πμ rad-m. At 5 GeV the reactance is negative but larger than the simple relation assumed for (Z/n) would predict. If the bunches are parabolic, then the inductive impedance would be 29.7 Ω with a null again at 6.6 GeV but only for an emittance of 2.5 μrad-m. Again the 5 GeV reactance is much too large. The significance of these results is discussed.

Introduction

The longitudinal coupling impedance is the induced voltage per turn at a given frequency divided by the beam current modulation causing it. It is usually normalized with the harmonic number $n = f/f_0$ and written as (Z/n). The induced voltage can affect both the incoherent or single particle phase oscillation frequency[1] and coherent bunch frequencies for the modes m = 1, 2, 3, etc.[2,3] If the reactive part of the impedance is due to space charge plus inductive wall effects only, then a measurement of the coherent frequencies ω_{1c} (dipole) and ω_{2c} (quadrupole) can be used to determine its value. (The real part of Z/n is assumed to be << $I_m(Z/n)$).

In order to obtain $I_m(Z/n)$ from these measurements, an expression for $\Delta\omega_m = (\omega_m - m\omega_s)$ the coherent frequency shift of the mode m from the incoherent frequency $m\omega_s$ is needed (see Fig. 1). If we assume a distribution in synchrotron phase space (r,θ) proportional to $\sqrt{(1 - r^2)}$, which gives a parabolic line charge or bunch shape, we can use the results of Ref. 1. For the m = 1 or dipole case we rewrite their Eq. 4 as

$$\omega_s^2 = \omega_1^2 \; [1 - jA(Z/n)] \tag{1}$$

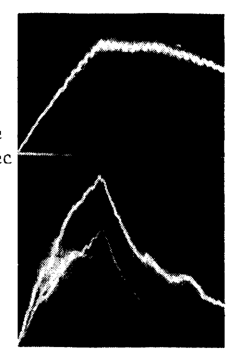
where ω_s is the incoherent synchrotron frequency which depends upon the slope of the total voltage V_T seen by the beam, ω_1 is the coherent dipole frequency, $A = 3I_0/\pi^2 h V_0 \cos\varphi_s B^3$, I_0 is the current per bunch, h the harmonic number, V_0 the external applied voltage, $B = \ell/2\pi R$, ℓ is the bunch length and $(Z/n) = (j\Omega_0 L - jg_0 Z_0/2\beta\gamma^2)$ with Ω_0 the angular rotation frequency, Z_0 = 377 Ω and $g_0 = 1 + 2\ln(b/a)$ where b is the radius of the vacuum chamber and a, the beam radius. Note that ω_1 does not depend upon the induced voltage for rigid dipole motion and is hence equal to ω_{s1}, the zero intensity value in Fig. 1 where also $\omega_{1,inc} = \omega_s$. If we solve for $\Delta\omega_1 = (\omega_1 - \omega_s)$ we obtain

$$\Delta\omega_1 = j\omega_1(Z/n)\;(A/2) \tag{2}$$

which is (-2π) times the Δf_s of Ref. 1. That is for rigid dipole oscillations the coherent shift $\Delta\omega_1$ is the negative of the incoherent frequency shift caused by the induced voltage. Since the higher order modes are characterized by non-rigid bunch motion, they will be affected by this voltage and will, in fact, modulate it in such a way that the $\Delta\omega_m$ will be less than the corresponding incoherent shift[1] (in Fig. 1 for m = 2 we have $\Delta\omega_{2,inc} = \Delta\omega - \Delta\omega_2$). Sacherer[4] has calculated $\Delta\omega_m$ for I_m (Z/n) and a parabolic bunch for which Legendre modes are exact solutions for the oscillating part of the charge distribution. For m = 2 he obtains $\Delta\omega_2 = 1.5\,\Delta\omega_1$ so that in Fig. 1, K = - .5 and since $\Delta\omega = (\omega_2 - 2\omega_1)$ we have

$$(Z/n) = 4j\Delta\omega/A\omega_1 \tag{3}$$

where V_0 can be obtained from the measured value of ω_1 and the beam energy. Note that the equations in Fig. 1 are valid only for a (Z/n) that is independent of frequency (hence n).



n=2
m=1
Lin.
scale
50msec
div.
n=2
m=2

on off

Fig. 3

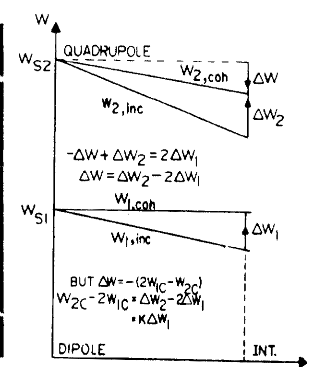


Fig. 1

Next we consider bunches with a phase space density proportional to $(1 - r^2)$ for which the oscillating part of the charge density can be approximated by sine waves.[2,3] In this case the $\Delta\omega_m$ are given by[2]

$$\Delta\omega_m \simeq \frac{jm\omega_1 I}{(m+1)\,3B^2 h V_0 \cos\varphi_s} \sum_p F_m \frac{Z(\omega_p)}{n_p} \tag{4}$$

where I is the current in M bunches and F_m for sinusoidal and Legendre modes is plotted in Fig. 2 for m = 1,2: hence $f_p = |(n + pM)f_0 + mf_1|$, $-\infty < p < \infty$ and $n_p = |n + pM|$. If (Z/n_p) is reactive and independent of ω_p then Eq. 4 becomes

$$\Delta\omega_m = j(m/m+1)A^*(Z/n_p) \tag{5}$$

since $\Sigma F_m = 1/BM$; here $A^* = (9/\pi^2)A$. From Eq. (5) we find that $\Delta\omega_2 = (4/3)\,\Delta\omega_1$ and that $\Delta\omega_1$ for the sinusoidal modes is $(9/\pi^2)$ the $\Delta\omega_1$ for Legendre modes. In Fig. 1 then K = - 2/3 and we can write

$$(Z/n_p) = 3j\Delta\omega/A^*\omega_1 \quad . \tag{6}$$

---
*Work done under the auspices of the U.S. DOE.

†CERN, CH-1121, Geneva 23, Switzerland.

††Brookhaven National Laboratory, Upton, N.Y. 11973.

0018-9499/79/0600-3592$00.75 © 1979 IEEE

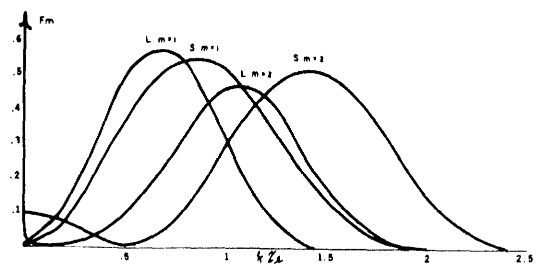


Fig. 2

Thus the (Z/n) given by Eq. (3) will be $(4\pi^2/27)$ than that given by Eq. (6) for the same observed $\Delta\omega$. In principle, one can determine which equation to use by observing the relative amplitudes of the frequencies f_p in the line spectra for a given mode m and comparing their square to the form factor F_m plotted in Fig. 2. Another method of discrimination is by how well the resulting fit of the data to a frequency independent (Z/n) agrees with other beam measurements (see below).

Measurement of ω_1 and ω_2

Measurement of ω_1 and ω_2 can be made by exciting the beam with an rf voltage in the neighborhood of one of the frequencies f_p where n is the coupled bunch mode number and $2\pi n/M$ is the phase shift of the oscillation mode m from bunch to bunch. Use of the n = 0 mode is precluded because of possible interaction with the high impedance of the 40 accelerating gaps on the AGS ring. Instead, the mode numbers n = 1, 11; 2, 10 (h = 12) were stimulated by rf excitation around the frequencies $f = 13f_o \pm f_1$ and $2f_1$ and $14\ f_o \pm f_1$ and $2f_1$. This voltage was added to the main rf drive signal that powers the ten accelerating cavities. Because of the location and phasing of these cavities and a rapid gain reduction in the rf system above 5 MHz, effective excitation at other values of n is not possible.[5]

These measurements were made on flat topped magnet cycles at energies of 5.2 GeV, 6.64 GeV, 10.2 GeV and 27.4 GeV after phase locking the beam frequency to a frequency synthesizer. A second synthesizer is used for the excitation signal which is gated on for .2 to .3 sec. Figure 3 shows the output of a spectrum analyzer tuned to the lines at $(26\ f_o + f_1)$ and $(26\ f_o + f_2)$ obtained by driving at the n = 2 dipole (m = 1) and quadrupole (m = 2) frequencies (p = 1). The latter mode is strongly damped as the decay is rapid after the excitation is removed while the dipole mode is close to its threshold since the decay is very slow at first. Slightly positive slopes do not interfere with the measurement but the intensity is always adjusted so that no rapid spontaneous growth is observed. Generally both the upper and lower dipole and quadrupole frequencies are obtained at 13 f_o and 14 f_o. This insures that a reasonable part of the frequency spectrum is sampled and serves as a check on any impedance that may vary rapidly over the regions covered.

On all the runs the n = 0, m = 2 mode was suppressed by a feedback system acting on V_o. Also for all the energies except 27.4 GeV a feedback loop between the beam radial position and the magnet voltage on the flat top was closed. This is required since $\omega_1 \sim 1/\sqrt{\eta}$, where $\eta = (1/\gamma^2 - 1/\gamma_{tr}^2)$, so $d\eta/\eta$ depends upon $d\gamma/\gamma$ directly and indirectly through the fact that γ_{tr} varies with the radial position because of non-linearities in the magnetic field. This latter effect is about four times the direct effect and of the same sign. Thus, one could, in principle, determine the variation of γ_{tr} with radius by measuring the change of ω_1 across the aperture at fixed field and V_o.

Once the resonant frequency is found, the excitation signal is reduced to the minimum level required to obtain a useful output from the spectrum analyzer which is operated in the zero scan mode. The frequencies ω_1, ω_2 were determined to within ± 2 cycles or of less which is consistent with the ≈ 3 cycle bandwidth of a .3 sec. excitation period and the fact that the synchrotron frequency spread within the bunches was ≤ 1.1 cycle for all but one set of measurements. Dipole modes are much easier to stimulate than quadrupole modes which is evident from Fig. 2. The value of F_m for a given $f\tau_\ell$ (see Table I) is a measure of how easily a mode m can be excited by a frequency f. The low frequency mode for m = 2 and sinusoidal oscillations along the bunch is not often plotted[2] since it is a result of the approximations made[4] and F_m for even m is usually assumed to be very small for $f\tau_\ell < (\frac{m-1}{2})$.

Results and Conclusions

Table I lists the calculated values of (Z/n) using the measured values of $\Delta\omega$. The first (Z/n) column is for Eq. (3) or Legendre modes and the second is obtained using Eq. (6). The errors quoted include those for the frequencies ω_1, ω_2 and for the bunch length τ_ℓ which enters in A as τ_ℓ^3 but not for the energy determination. Also listed is 13 $f_o\ \tau_\ell$ which along with Fig. 2 can be used to estimate the range of frequencies sampled. Assuming sinusoidal waves along the bunch and m = 2 at 27.4 BeV, the maximum frequency is approximately (2.4/.093) 13 f_o ≅ 124 MHz while for the large bunches at 5 BeV the range is only approximately 65 MHz.

Table I

E(BeV)	(Z/n)(jΩ)	$(Z/n)^*$(jΩ)	$13f_o\tau_\ell$
5.05	-153±9.2	-104±6.3	.174
5.05	-51.9±2.9	-35.5±2	.132
6.64	5.5±3.2	3.8±2.2	.114
10.2	18.5±2.9	12.7±2	.107
27.4	27.9±5.6	19.1±3.8	.093

The lowest of the two values obtained at 5 BeV is an average of four separate runs where $\bar{\tau}_\ell$ was approximately 28 ns and the larger value is the average of two runs where $\bar{\tau}_\ell$ = 34 ns. Although the cause of the different bunch lengths is not known, no reason for doubting the (Z/n) determination has been found. The value at 6.6 BeV is the average of two runs and the errors are larger here since one is near a null of (Z/n). The remaining two points are the results of single runs only.

In Fig. 4 we have plotted both the positive and negative values of (Z/n) and their sum for the following parameters: $\Omega_o L$ = 20 Ω, b = 6.5 cm and $a = \sqrt{\epsilon\bar{\beta}/\beta\gamma}$ where $\bar{\beta}$ = 14.6 m and ε = 22 π μrad-m. Also plotted are the calculated values of (Z/n) from Eq. (6). As can be seen, the fit is good for the high energy points and the assumed value of ε, the normalized emittance. If the other set of (Z/n) values are used, an equally good fit is obtained for $\Omega_o L$ ≅ 30 Ω but with ε = 2.5 π μrad-m. Since the normal AGS emittance at low intensity $(2.5\text{-}4 \times 10^{12})$ is approximately 30 π μrad-m this value is much too small. The value of b is only approximate but it would have to be about 3 times larger for a 22 π emittance to fit the Eq. (3) values. A measurement of F_m for the dipole mode was made by observing the amplitude of the line spectra out to 122 f_o or $f\tau_\ell$ ≈ 1.55 but only for the wide bunches at 5 GeV. The result was closer to the sinusoidal mode curve than the Legendre mode curve.

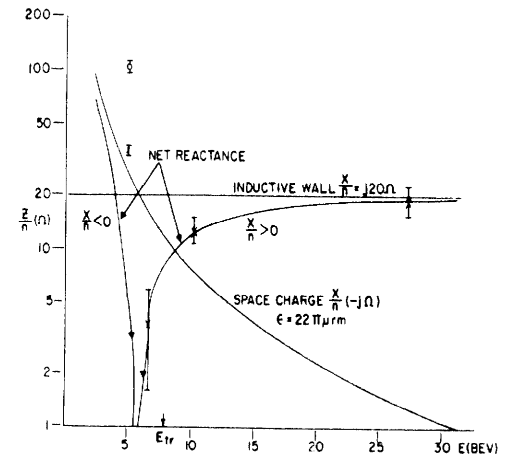


Fig. 4

No matter what choice is made for b, ε or the oscillation mode the large deviation of the points at 5 GeV cannot be fitted with the assumed expression for (Z/n). Nor can the apparent effect of bunch length be explained by a (Z/n) that is independent of frequency. However, if this assumption is abandoned, then many of the expressions used here for the frequency shifts are no longer valid and a determination of (Z/n) by this method becomes quite complex. Still it is reasonable to assume that on the basis of these results the impedance at transition energy in the region below a few hundred MHz is inductive. This is the opposite sign to what has been normally used in the past.[6] However, a mismatch of the bunch will still occur as transition is passed and blow-up due to the negative mass instability can take place. The latter requires a negative I_m (Z/n) after transition but the fastest growing modes occur for very large values of n or extremely short wavelengths. Since the impedance in the many hundreds of MHz to giga Hz region will change sign as one will be above the resonances of the vacuum chamber discontinuities, flanges, etc., the low frequency results do not represent a true conflict with past experience. An investigation of transition phenomena plus additional impedance measurements particularly at low energy are planned.

Acknowledgments

The authors have benefited from discussion with F. Sacherer and B. Zotter and appreciate the assistance of L. Ahrens in the measurement program. Also the technical aid of E. Gill and S. Naase has been of great value.

References

1. S. Hansen et al., IEEE Trans. Nucl. Sci. NS-22, 138 (1975).
2. F. Pedersen and F. Sacherer, IEEE Trans. Nucl. Sci. NS-24, 1393 (1977).
3. F. Sacherer, IEEE Trans. Nucl Sci. NS-24, 1393 (1977).
4. F. Sacherer, CERN Int. Rep./SI-BR/72-5 (1972).
5. F. Pedersen and R. Raka, BNL Int. Rep. ISA Tech. Note 69 (1978).
6. A. Sorenssen, Particle Accelerators, Vol. 6, 14 (1975).
7. W. Hardt, Proc. IX Int'l. Conf. on H.E. Accel., 434 (1974).

Addendum

A measurement has been obtained at 4.08 BeV which gives -j 137.2 ± 19.7 Ω by Eq. (3) and -j 93.8 ± 13.5 Ω by Eq. (6). For this run 13 f_o τ_ℓ = .16 which means the bunch area was somewhat smaller than the average area of the bunches for the four runs that gave the lower value of (Z/n) at 5.2 GeV. This result is at least consistent with that value and implies a very large impedance at lower energies.